ABSTRACT

If a sound source is located near a large plane-surface reflector, the total acoustic power radiated from the source including the reflection depends on the source position. Interference is the cause of this phenomenon, and the musical instruments for bass sound are particularly influenced. The surface diffusion treatment of the stage reflectors may be a solution for improving the problem. In this report, from the viewpoint of acoustic power from the stage area to the audience area, the effect of surface diffusion of a stage rear-wall is examined by using an analytical model of two-dimensional wave theory.

1. INTRODUCTION

The power output radiated from a sound source varies depending on the position of the source. This phenomenon has been observed in measurements of the acoustic power using a reverberation chamber; see, for example, references [1-3], and is caused by interference between the direct and the reflected waves. Then, the frequency range affected by this phenomenon is restricted, which is relatively low. From this point of view, there have been some works regarding the design method for the layout of musical instruments for bass sounds.

In this paper, the effects of the configuration of a stage rear-wall on the power output to the audience area are investigated by using fundamental models of the stage enclosure. This model consists of two semi-infinite plane reflectors joined at right angles. The vertical and horizontal planes correspond to the stage rear-wall and the stage floor, respectively. The discussion is given to the problem whether any modification of the reflecting surface can alleviate the tonal effects at low frequencies.

2. ACOUSTIC RADIATION OF A POINT SOURCE NEAR REFLECTORS
Consider a sound field composed of two semi-infinite plane reflectors joined at right angles, having a point source $Q$ located near the corner as shown in Fig.1. The response at a point $P$ as an expression of the velocity potential $\phi$, due to a point source $e^{ik_0 r}/4\pi r_0$ with the wave number $k$, can be written as

$$\phi = \sum_{j=1}^{3} \frac{e^{ikr_j}}{4\pi r_j},$$  

where $r_j (j=1,2,3)$ is the distance of the images due to each plane reflector. When the two planes cross at right angles, this expression is mathematically rigorous.

If the sound source is located near the reflector, the output characteristics of the source change due to change in the position. Figure 2 shows the calculated results of the sound pressure level (SPL) in the vicinity of the source, averaged over a sphere of radius $r_0$ and normalized by the sound pressure of the same source and same position in the free field. For reference, the results for the case of one plane-reflector of infinite extent are given in the same graph. It is seen from these results that a source-reflector distance of one-quarter wavelength or a multiple of odd numbers gives rise to the level attenuation. Moreover, the case $d_i = d_z$ shows the strongest effect of attenuation, which implies the interactive effect of two orthogonal plane reflectors. The same situation can be seen in the results obtained by smoothing over 1/3-octave bands, which seem to be more compatible with our auditory sense.

The characteristics in the far field of a sound source can be evaluated by the directivity index and the total properties can be evaluated by the acoustic power. In the far field, the directivity index $L(\theta, \phi)$ can be written in the form

$$L(\theta, \phi) = 10\log_{10}[1 + e^{ikR_1} + e^{ikR_2} + e^{ik(R_i - R_2)}]^2,$$  

where

$$(3) \begin{align*} R_1 &= 2d_i \cos \theta \sin \phi, \\
R_2 &= 2d_z \sin \theta. 
\end{align*}$$
The calculated results are shown in Fig.3. It is seen from this graph that the directional pattern is strongly affected by change in the source position. A source point of \( d_1 = \frac{\lambda}{4\theta} \) and \( 3\lambda / 4 \) causes the attenuation effect for the direction normal to the vertical plane. For the direction normal to the horizontal plane, the same effect can be seen in the case \( d_2 = \frac{\lambda}{4\theta} \). Considering this effect as an interactive effect due to a stage floor and a stage rear-wall, if a musical instrument with the fundamental tone corresponding to the wavelength \( \lambda \) located at a position \( d_1 = d_2 = \frac{\lambda}{4} \), at seats in front of the stage, the contribution of the direct sound cannot be expected.

The power output can be calculated by integrating the squared sound pressure over the 1/4-sphere with respect to \( \theta \) and \( \phi \). The calculated results of the power level (PWL) are shown in Fig.4, in which the power is normalized by that of the same source in the free field. As is expected from the previous discussion, the power output changes depending on the source position. The tendency is similar to the case of SPL variation in the vicinity of the source.

A composition of the direct sound and its images can be regarded as one sound-source, which generates the variable distribution of the sound pressure. Then the frequency characteristics at a receiving point vary depending on the positions, a source and a receiver. The numerical examples of SPL variation including the experimental results are shown in Fig.5, in which the source is located at \( d_1 = d_2 = 1.7m \) and the receiver is 7.5m in Fig.5(a) and 10m in Fig.5(b), away from the source normal to the vertical plane. A slight level reduction can be seen at frequencies around 50Hz, which is related to the one-quarter wavelength for the distance 1.7m. However, another attenuation effect also appears in the frequency range 125-500Hz depending on the receiver distance. Another examples of calculation are shown in Fig.6, in which the source is located at \( d_1 = d_2 = 1.7m \) in Fig.6(a) and \( d_1 = d_2 = 1m \) in Fig.6(b), and the receiving point is located 20m away from the source, the vertical angles are 25 degrees for both cases. In these cases, more serious level reduction appears at low and mid frequencies.
The sound field generated from fundamental structures of reflection herein make us imagine that of an open-air theater. In such case, there is a possibility that the attenuation effect really occurs and comes to be serious. The attenuation effect due to a stage enclosure, which is here called “stage enclosure effect (SEE),” is similar to that of “seat dip effect (SDE),” and occurs at rather earlier time of reflection than suffering from SDE. Thus the combination of both effects may give rise to more serious tonal effects.

3. EFFECT OF SURFACE DIFFUSION OF THE STAGE REAR-WALL

As described in the previous section, the reflection due to two large plane-reflectors joined at right angles, has a possibility of tonal distortion at low and mid frequencies. One of the most realistic methods for improving this acoustic deterioration may be to design some diffuse-reflection treatment regarding the surface configuration of the stage rear-wall. In this section, we investigate the potential of diffuse reflection for improving the stage enclosure effects. It is seen from the discussion in section 2 that the interactive effect of two orthogonal reflectors seriously appears. In this section, therefore, the discussion is given to the two-dimensional problem for scattering in the vertical direction.

3.1. Method of calculation

The geometry of an analytical model with the co-ordinate system is shown in Fig.7. Combining the expression of Helmholtz-Kirchhoff integral formula for the velocity potential at a receiver \( P \) and that of a receiver \( P' \), which is the image of \( P \) with respect to z-axis, yields

\[
\phi(P) = \phi_0(P,Q) + \phi_0(P^*,Q) + \int_{S_1} \phi(P) \left[ \frac{\partial G(P,P_0)}{\partial n} + \frac{\partial G(P^*,P_0)}{\partial n} \right] ds + \int_{S_2} \phi(P) \left[ \frac{\partial G(P,P_0)}{\partial n} + \frac{\partial G(P^*,P_0)}{\partial n} \right] ds, \tag{4}
\]

where \( G \) is the two-dimensional free space Green’s function for outgoing waves, which is given by

\[
G(P,P_0) = \frac{i}{4} H_0^1(kR), \quad R = PP_0. \tag{5}
\]

Considering the condition that the integration over \( S_2 \) is 0, and

\[
\phi_0(P^*,Q) = \phi_0(P,Q^*), \quad G(P^*,P_0) \bigg|_{S_1} = G(P,P_0^*) \bigg|_{S_1}, \quad \frac{\partial G(P,P_0)}{\partial n} \bigg|_{S_1} = \frac{\partial G(P,P_0^*)}{\partial n^*} \bigg|_{S_1}, \tag{6,7,8}
\]

one obtains

\[
\phi(P) = \phi_0(P,Q) + \phi_0(P,Q^*) + \int_{S_1} \phi(P) \left[ \frac{\partial G(P,P_0)}{\partial n} + \frac{\partial G(P^*,P_0)}{\partial n} \right] ds + \int_{S_1} \phi(P^*) \left[ \frac{\partial G(P,P_0^*)}{\partial n^*} - \frac{\partial G(P^*,P_0)}{\partial n} \right] ds.
\]

This expression shows that the response at a
receiver $P$ can be obtained by summation of each solution for a source $Q$ and a source $Q'$, respectively, for the sound field shown in Fig. 7(b). For the images with respect to $x$-axis, in a completely analogous way, the velocity potential at a receiver $P$ can be eventually written in the form

$$\phi(P) = \phi_0(P,Q) + \phi_0(P,Q') + \int_s \phi(P_2) \frac{\partial G(P,P_2) \partial n}{\partial n} ds + \int_s \phi(P_2') \frac{\partial G(P,P'_2) \partial n}{\partial n} ds,$$

where a prime denotes the image with respect to $x$-axis.

3.2. Numerical results and discussion

The reflector configurations treated here are shown in Fig. 8, in which the height of uneven reflectors (diffusers) is fixed at 4 m. The configurations for Types 1-3 correspond to the tilted walls, and diffusers of a succession of triangular profile with the different projection are prepared for Type 4 and Type 5. The effect of surface corrugation is here evaluated by the characteristics of the acoustic power, which is calculated by integrating the squared sound pressure in the far field over the 1/4-cylindrical surface.

Figure 9 shows the characteristics of the calculated PWL. In the level calculation, the power output is normalized by that of the same source in the free field, and is averaged by smoothing over 1/3-octave bands. It is seen from these results that noticeable effects owing to the diffuse treatment cannot be expected, except for the case Type 1, which is the tilted rear-wall with the vertical angles of 97.5 degrees. However, the degree of the improvement for the level reduction decreases as the distance from the surface increases, i.e., the improvement seems to be restricted in the case where the sound source is placed near the corner. As can be seen in Fig. 6, the response at a receiver generated from a sound source near the corner of two orthogonal plane-reflectors varies depending on the receiver position, especially at low frequencies. At some points low frequencies are seriously attenuated. It is interesting whether the tilted rear wall can improve this attenuation. The calculated results regarding this matter are shown in Fig. 10. It is seen from these results in comparison with
those of Fig.6 that the level reduction at frequencies around 80-300Hz recovers due to the tilted wall of 97.5 degrees for the case of a source located at a position $d_1=1m$ and $d_2=1m$. In the case, where $d_1=1.7m$ and $d_2=1.7m$, a similar improvement in the frequency range 50-200Hz can be seen, however the effect becomes less in comparison with the former case.

4. CONCLUSIONS
To investigate the characteristics of the acoustic radiation from a stage enclosure, we analyzed the sound fields generated by a point source located near the corner of two semi-infinite plane reflectors joined at right angles. As a result of this simple analysis including some experimental verification, it is clear that a source located at a distance corresponding to one-quarter wavelength or its multiplication of odd numbers causes power reduction at the corresponding frequencies. The effect has directional characteristics, in which stronger effect can be seen for the direction in the median plane. Also a level reduction over rather wide bands of frequency, which is similar to “seat dip effect,” can be seen at some receiving points in the audience area. For the purpose of improvement in this phenomenon, we discussed the possibility of the effect of changes in reflector configuration, which includes surface diffusion and tilted walls. As a result of some numerical investigations using Helmholtz-Kirchhoff integral formula, it was shown that a rear wall tilted more than right angles has a potential of improving the attenuation effect due to a sound source located near the corner.

REFERENCES