19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

INFLUENCE OF THE PROFILE OF SOME VIBRATING STRUCTURES ON THEIR SOUND RADIATION EFFICIENCY AND DIRECTIVITY

PACS: 43.40.Rj; 43.20.Rz

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ABSTRACT
Basically, this research reports the effect of making slight changes to the side view of some vibrating structures on both, their sound radiation efficiency and directivity. For undertaking this, two numerical methods can be used: 1) Romberg's based numerical integration of the surface integrals involved in the problem and 2) Finite Difference Time Domain (FDTD) method. To find the directivity, the sound pressure field is obtained at different points from the source using direct numerical integration. On the other hand, the sound radiation efficiency is estimated from the integration results for the real part of the mechanical radiation impedance, which is necessary to determine the total sound power radiated. The FDTD method can be used as a help to visualize the sound field. The numerical results are compared with the ones obtained by other methods reported in the literature. In addition, the methodologies are applied to commercial loudspeakers. [Work partially supported by FONDECYT No 7060073].

INTRODUCTION
In designing sound radiators for audio applications, the approximation for a circular plane piston mounted flush in an infinite baffle is usually considered to perform the calculations at low frequencies. This is done because of its simplicity and the existence of closed form and analytical expressions for the sound pressure at field points in the far-field, on the axis, and on the piston face. However, it is well-known that, in practice, most of radiators are not perfectly flat and they have certain profile which is a function of depth. In addition, the simple calculation is performed taken into account the far-field approximation. In this paper, different transverse profiles of circular pistons are considered, by performing the spatial numerical integration, in order to estimate their effects on the sound efficiency and directivity.

THEORETICAL BACKGROUND
The sound radiation in the far-field produced by a flat circular piston mounted in an infinite baffle can easily be obtained. This is attained by assuming that the vibration velocity on the surface of the piston is constant and the sound field is a superposition of elementary simple sources of surface dS, according to the geometry shown in Fig. 1(a). The mathematical expression can be found in classical textbooks [1, 2].

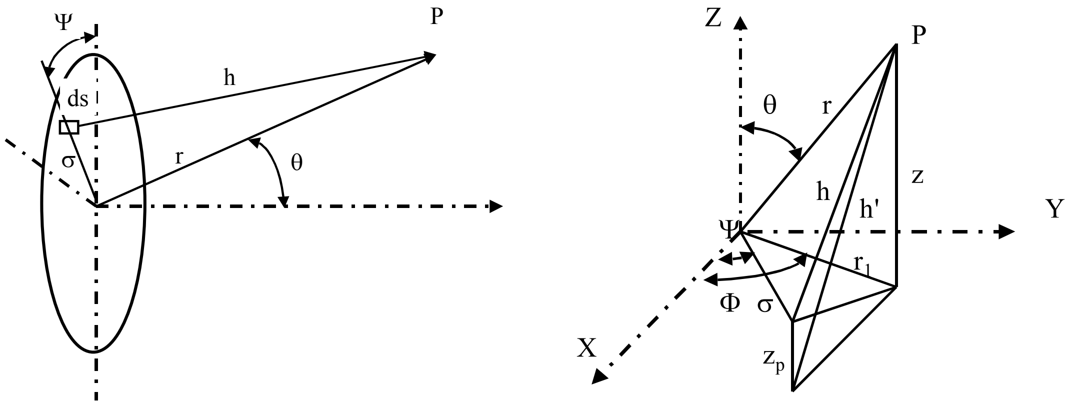
For a circular piston having a given axisymmetric transverse profile, the differential sound pressure produced by a simple source at a distance r, vibrating with a maximum surface velocity u_o and frequency f, in a fluid medium of density ρ_o, can be written as

$$dp = -j\frac{\rho_o f u_o dS}{h'} e^{jk(h'-ct)}, \tag{1}$$

where k is the free-field wavenumber, c is the speed of sound, and h' is given by

$$h'^2 = r^2 + \sigma^2 + z_p^2 - 2r(\sigma\sin\theta\cos\Psi\cos\Phi + \sigma\sin\theta\sin\Psi\sin\Phi + z_p\cos\theta). \tag{2}$$

The values of each parameter in Eq. (2) are given in Fig. 1(b). Here, z_p is the value of the variation in the Z-axis for a given profile. Integration of Eq. (1) results in the sound pressure at point P. Clearly, Eq. (1) has to be numerically integrated.



(a) (b)

Figure 1.-Geometries used to describe the problem: (a) Circular plane piston; (b) Circular piston having a given transverse profile.

In the far-field $h' \approx r$, and for small profile depth values

$$h' \approx r + z_p - \sigma \sin\theta \cos\Psi \,. \tag{3}$$

Thus, the total sound pressure can be approximated by

$$\begin{aligned} p(r,\theta,t) &\approx -j\frac{\rho_o f u_o}{r} e^{-jk(r-ct)} \int_{\sigma=0}^{\sigma=a} \sigma e^{jkz_p(\sigma)} d\sigma \int_{\Psi=0}^{\Psi=2\pi} e^{-jk\sigma\sin\theta\cos\Psi} d\Psi = \\ &= -j\frac{\rho_o f u_o}{r} e^{-jk(r-ct)} \frac{4J_1(ka\sin\theta)}{ka\sin\theta} \int_{\sigma=0}^{\sigma=a} \sigma e^{jkz_p(\sigma)} d\sigma, \end{aligned} \tag{4}$$

where J_1 is the Bessel function of order 1. Solution of Eq. (4) depends on the given profile.

Mechanical radiation impedance

The mechanical radiation impedance is related to the forces that the fluid exerts on the vibrating surface. These are a force in phase with the surface velocity that accounts for radiation losses and a force proportional to the surface acceleration that accounts for an increase of the surface mass. Mathematically, for time-harmonic vibration of the surface, the mechanical radiation impedance is given by [2]

$$Z_{MR} = \frac{1}{u_o e^{jkct}} \iint_{S'} p \, dS' \,. \tag{5}$$

Therefore, by integrating Eq. (1) and then integrating Eq. (5), the value for Z_{MR} can be obtained.

APPLICATION TO A DYNAMIC LOUDSPEAKER

The real part of the radiation impedance is very important to predict the efficiency of vibrating systems, such as a dynamic loudspeaker [3]. In fact, it can be shown that the efficiency η for such a device, when mounted flush in a baffle (sound power radiated/total power supplied by the electric generator), is given as [4]

$$\eta(\%) = 100\frac{(Bl)^2 R_{MR}}{(R_g + R_E)|Z_{MT}|^2 + (Bl)^2(2R_{MR} + R_{MD})}, \tag{6}$$

where is Z_{MT} (kg/s) is the total mechanical impedance, B (T) is the steady air-gap flux density, l (m) is the length of wire on the voice-coil winding, Rg (Ω) is the generator resistance, R_E (Ω) is the resistance of the voice coil, and R_{MD} (kg/s) is the mechanical resistance of the diaphragm. The total mechanical impedance can be written as

$$Z_{MT} = Z_{MD} + Z_{MR}, \tag{7}$$

where Z_{MD} (kg/s) represents the pure mechanical impedance:

$$Z_{MD} = R_{MD} + j\left[\omega M_{MD} - \frac{1}{\omega C_{MD}}\right], \tag{8}$$

where ω (rad/s) is the circular frequency, M_{MD} (kg) is the mass of the diaphragm and the voice coil, C_{MD} (m/N) is the mechanical compliance of the diaphragm, and Z_{MR} (kg/s) is the mechanical radiation impedance [see Eq. (5)]:

$$Z_{MR} = 2R_{MR} + j2X_{MR} = 2R_{MR} + 2j\omega M_{MR}, \tag{9}$$

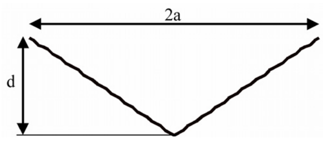
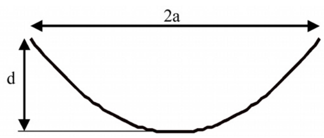
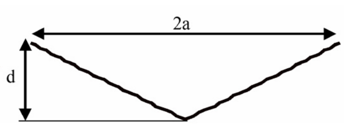
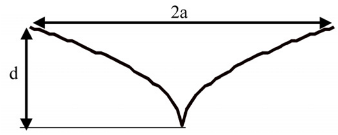
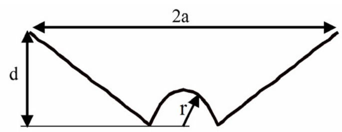
where R_{MR} (kg/s) is the mechanical radiation resistance, X_{MR} (kg/s) is the mechanical radiation reactance, and M_{MR} (kg) is the mechanical radiation mass. In general, the efficiency is calculated at 1 kHz. It is observed that both increasing R_{MR} and reducing M_{MR} enhances the efficiency.

NUMERICAL EXAMPLES

Mechanical radiation impedance

Five transverse profiles corresponding to typical loudspeaker diaphragm were considered as examples to apply the method. The profiles are presented in Table I.

Table I.- Profiles considered for the numerical examples

Profile	Geometry	Equation
linear		$z_p = d\frac{x}{a} - d$
quadratic		$z_p = d\left(\frac{x}{a}\right)^2 - d$
exponential		$z_p = \frac{d(e^x - e^a)}{e^a - 1}$
square root		$z_p = d\sqrt{\frac{x}{a}} - d$
linear + circular		$z_p = \begin{cases} \sqrt{r^2 - x^2} - d & x < r \\ \frac{d(x-a)}{a-r} & x \geq r \end{cases}$

The Romberg's method was used to perform the numerical integrations and the method was implemented into a Matlab© computer code. The vibrating surfaces were discretized in radial and angular directions, having a total of 16384 elements. This choice is because the Romberg integration method uses power-of-two nodes of the function. Details on the integration scheme can be found in the literature [5, 6].

Figure 2 presents the computed results of normalized mechanical radiation impedance for the five transverse profiles and the plane circular piston as a function of ka. The numerical results have been normalized by the product of the surface S and $\rho_o c$. A depth d=0.25a has been considered for all the numerical examples.

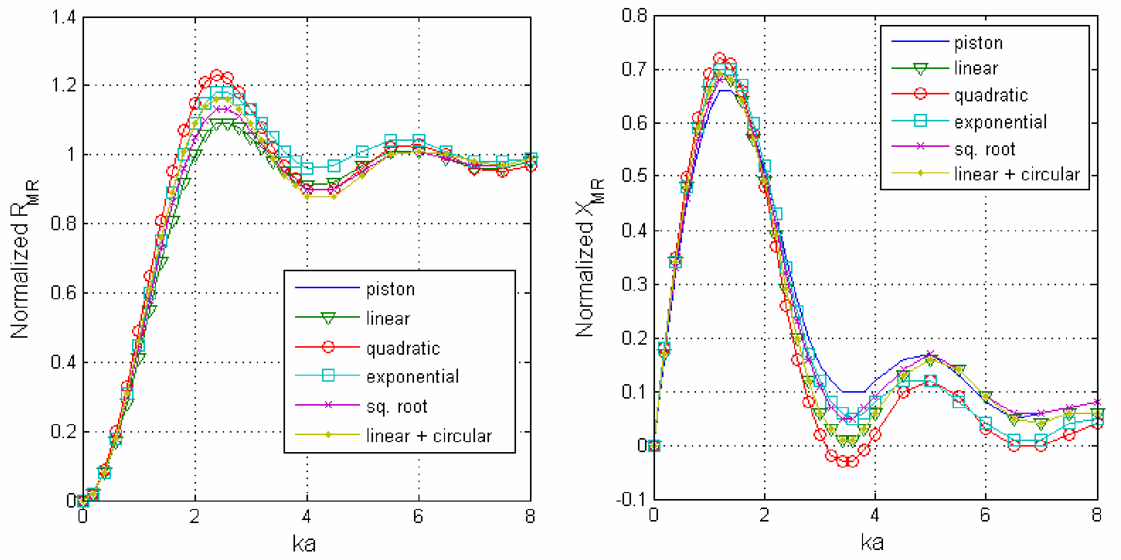


Figure 2.- Real (R_{MR}) and imaginary (X_{MR}) parts of the normalized mechanical radiation impedance (Z_{MR} / $S\rho_o c$) for different transverse profiles calculated by numerical integration. The results are compared with the classical results for a circular plane piston.

It is observed in Fig. 2 that the profile can produce significant changes in the real and imaginary parts of the mechanical radiation impedance. The maximum value of mechanical radiation resistance is achieved by the quadratic profile for ka=2.5, approximately, while the minimum value is observed at the same frequency for the linear profile. Therefore, it should be expected that the directivity patterns and values of efficiency can be quite different when compared to the results obtained by using the plane piston model. As expected, for all cases, the normalized resistance tends harmonically to unity at high frequencies.

Figure 2 (right) shows that, for frequencies above ka=3, negative results of reactance are obtained. It is expected that, for very high frequencies, the value of reactance must go to zero. This can be due to the number of integration steps and mesh used to numerically evaluate the integrals. Better results should be obtained by increasing the discrete mesh points or reducing the integration steps. However, this can have a huge cost in computational time.

Directivity patterns

Since sound pressure radiated by the vibrating structure can be obtained at a fixed distance from the baffle, it is possible to compute the sound pressure on a plane perpendicular to the vibrating structure. Thus, it is possible to plot the directional characteristics for each profile.

For comparison, Figs. 3 and 4 show the numerical results for directivity patterns of a loudspeaker of diameter 8" having different transverse profiles, at frequencies 1kHz and 4kHz, respectively. The sound pressure field is plotted at a fixed distance of 1 m every 2.5^{O}.

It can be noticed the effect of the profiles on the directivity patterns. There are several differences in the sound field radiated for a non plane profile when compared with the one produced by a plane piston.

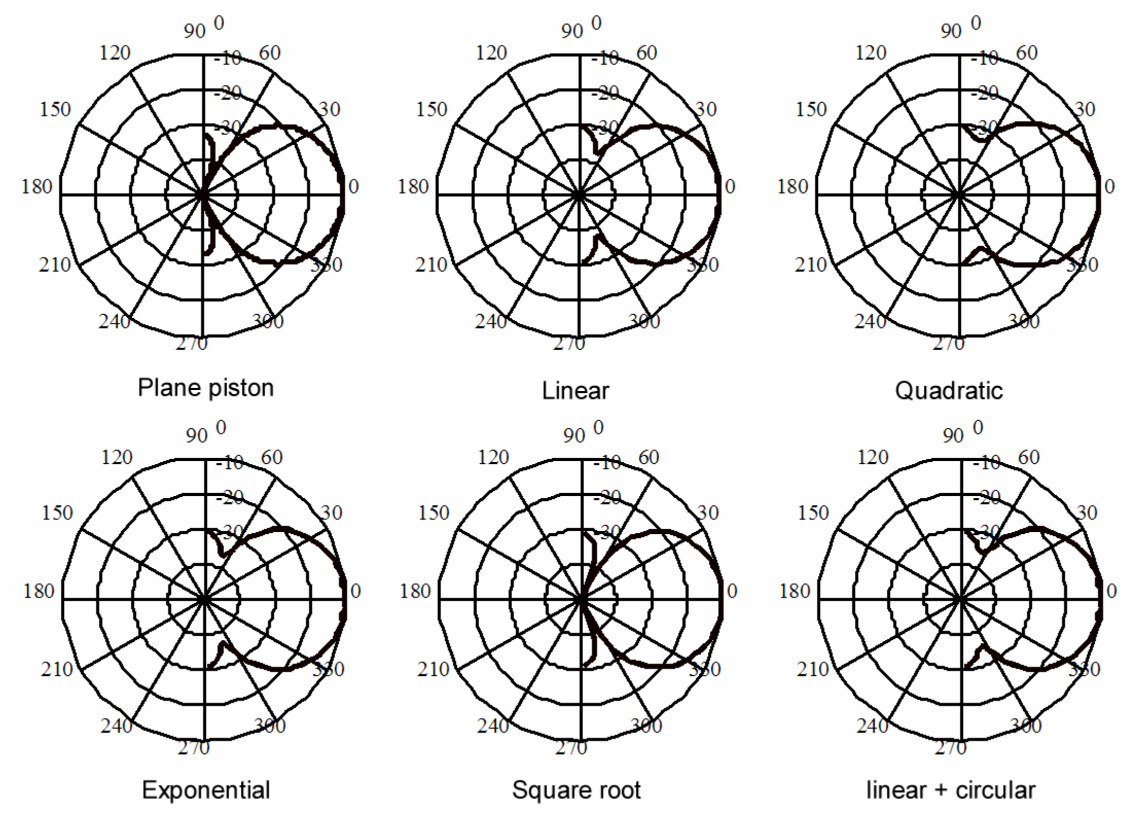


Figure 3.- Directivity patterns for a loudspeaker (a=4”) having different transverse profile calculated at 1kHz.

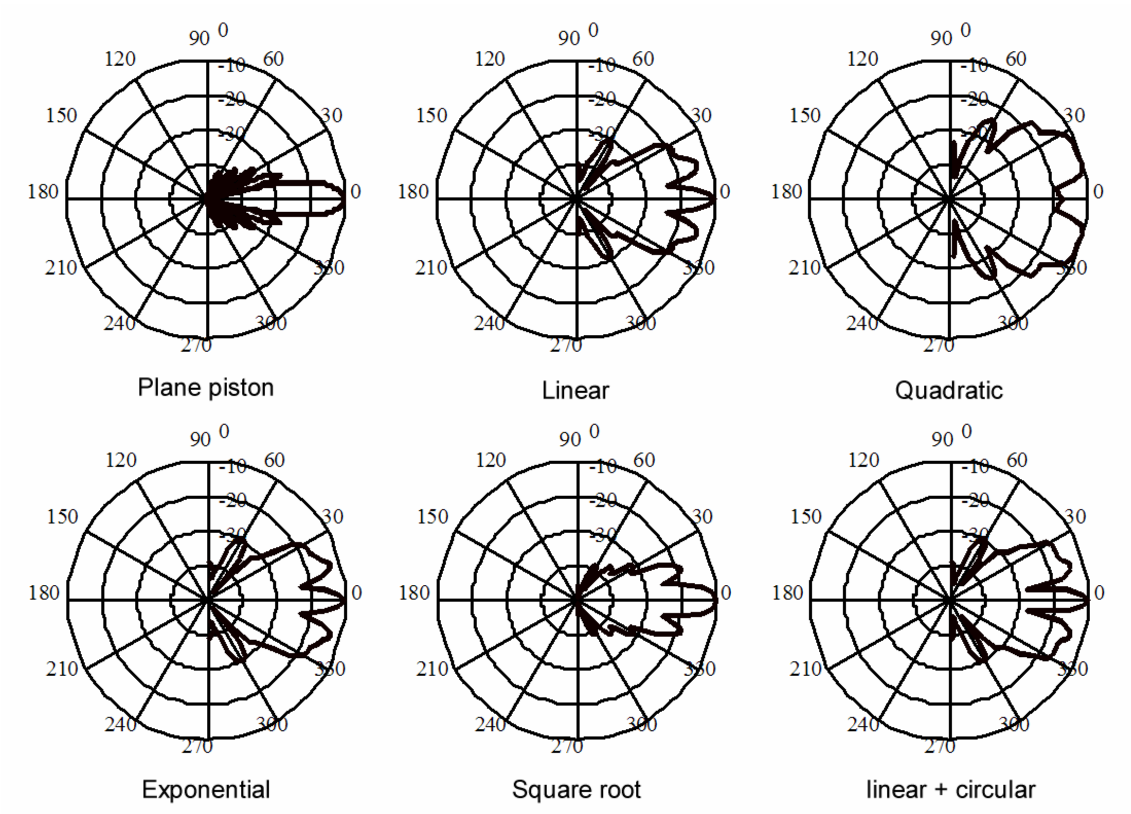


Figure 4.- Directivity patterns for a loudspeaker (a=4”) having different transverse profile calculated at 4kHz.

However, it is observed that for 1kHz the beamwidth of the main lobes remains constant and it is almost independent of the profile. Some changes on the side lobes can be observed. For 4kHz, the profiles show larger beamwidth when compared with the plane piston. Then, it is suggested that modification of the profile of a loudspeaker could lead to important changes in the directivity pattern. This fact can be used to design better radiation devices.

Efficiency

From the results of mechanical radiation impedance obtained by numerical integration of Eq. (5), it is possible to evaluate the efficiency of a dynamic loudspeaker as given in Eq. (6), taking into account its transverse profile.

As an example, Table II shows the results of efficiency η at 1kHz for a commercial 8" loudspeaker model 8M70 made by Beyma in Spain. Efficiency has been calculated using the impedance of a plane piston, the method presented by Panzer [7], and the numerical method presented above. In addition, the results are compared with the measured efficiency in laboratory. The efficiency stated by the manufacturer was 1.1. According to the physical dimensions and shape of the loudspeaker diaphragm, the numerical results were obtained using a cubic profile having 3 cm-depth.

Table II.- Efficiency at 1kHz obtained by four different methods for a commercial loudspeaker.

Loudspeaker	Efficiency η			
	by Piston	by Panzer	Numerical	Experimental
8M70	1.05	1.37	1.2	1.2

It can be observed that the efficiency calculated by the present method agrees quite well with the measured result. The efficiency calculated by using the plane piston model underestimates the efficiency by 12.5%. The efficiency calculated by the method reported by Panzer overestimates the value of efficiency by 14.2%.

CONCLUSIONS

A numerical method to consider the effect produced by a transverse profile of a vibrating surface mounted flush in a baffle has been presented. The method uses numerical integration to find the sound pressure field and the mechanical radiation impedance. The results can be applied to the practical case of a dynamic loudspeaker in order to estimate its directivity patterns and efficiency. The numerical results show that modification of the transverse profile of a loudspeaker can produce important variation in the sound field radiated. The result of efficiency calculated by the method presented here matches perfectly with the result obtained by experimental method. Of course, this methodology applies when we can assume that the loudspeaker diaphragm behaves as a membrane [8]. Further studies should be performed in order to test the methodology with other commercial loudspeakers.

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