3-Dimensional violin corpus motion and far-field radiativity

PACS: 43.75.De

Bissinger, George
Physics Department, East Carolina University, Greenville, NC 27858 USA; bissingerg@ecu.edu

ABSTRACT
In- and out-of-plane (IP-OP) corpus mobilities for the “Titian” Stradivari and the “Plowden” Guarneri del Gesu violin were extracted from >500 point tri-laser scans using bridge force hammer excitation. Partial scans of the “Willemotte” Stradivari and a 2006 Joseph Curtin violin were also performed. Acoustic radiativities (pressure/force) were measured over a sphere in an anechoic chamber for these, plus a 2006 Samuel Zygmuntowicz violin. As expected for a shell structure there was significant IP motion: average back-plate OP/IP rms mobility ratios dropped over 0.2-5 kHz from ~4 to ~2, while for the top-plate the ratio dropped from ~1.5 to ~1, although the Titian remained near 2 over this interval. The Titian also had the highest directivity (top/back hemisphere radiativity ratio) overall of 17 violins tested to date, while the Willemotte had the lowest. Total damping for these old Italians generally was somewhat higher than for modern instruments at f < 2 kHz, becoming similar above. Effective critical frequencies for the Titian and Plowden were ~3.3 and ~3.6 kHz resp. somewhat below the average of 4.0±0.6 and 4.2±0.3 for “good” and “bad” violin averages, resp. Generally, 3-old-Italian radiativity was less than 2-modern-violin below 3 kHz, becoming somewhat larger above.

INTRODUCTION
There is little to date in the violin literature about in-plane (IP) motion. The pioneering work of Runnemalm, Molin and Jansson in 2000 employed TV holography of a violin excited at a number of single frequencies over the 400-600 Hz range to examine in-plane and out-of-plane (OP) motion [1]. A recently developed commercial tri-laser scanning system, has made true 3-dimensional motion analysis of broad-band excited violins possible over a broad frequency range [2]. In addition to the usual frequency, total damping $\zeta_{\text{tot}}$ (in % of critical) and out-of-plane OP mode shape vibrational information obtained from previous laser scans [3], important information about the in-plane IP mobility was obtained. Since OP motion should be more effective at producing acoustic radiation than IP, comparisons were then be made with allied radiativity measurements over a sphere around each violin to compare radiation into the top and back hemispheres. Combining the vibration and radiation measurements provides important additional information such as radiation efficiency $R_{\text{eff}}$, radiation damping $\zeta_{\text{rad}}$, internal damping $\zeta_{\text{int}}$, effective critical frequency $f_{\text{crit}}$, and fraction-of-vibrational-energy-radiated $F_{\text{RAD}}$.

MEASUREMENTS
The 3-D mobility $Y(\omega)$ (velocity/force) measurements over a 0-5 kHz range were performed during a period of 3 days on 4 violins: the Titian (1715) and Willemotte (1734; back plate only) Stradivari and the Plowden Guarneri del Gesu (1735) as well as a 2006 Joseph Curtin violin (top plate only). Complete far-field acoustic radiativity $R$ (pressure/force) measurements over a sphere in an anechoic chamber were performed for these and an additional 2006 Samuel Zygmuntowicz violin over the same period. Vibrational and acoustical measurements utilized the same force hammer excitation, striking the G-string corner of the bridge parallel to the plane of the bridge and top plate on a violin suspended in the same support fixture providing approximate “free-free” condition used for previous VIOCADEAS measurements [3].

XYZ Mobility
The 3-D measurements provide immediate post-scan vibrational feedback in the form of animations of the mobility magnitude and phase over the measured surface at a particular
frequency. The frame of reference has its origin at the top plate between the bridge feet: the y-direction is perpendicular to the plane of the violin, the x-direction is across and the z-direction along. (Approximations used top and back: OP = y-direction, IP = x- and z-directions combined, although it is possible to isolate each; ribs OP = x-direction). Figure 1 shows the stitched XYZ 3-D Plowden motion for the 1st longitudinal cavity mode A1 at 458 Hz (coupled to A0 the Helmholtz-like mode near 280 Hz [4]) and XZ IP motion only for the lowest CBR corpus mode at 383 Hz. These examples emphasize vibro-acoustic aspects of violin vibrations where the A1 forces corpus wall motion as well as strong shear-like IP motion between top and back plates.

Figure 1 – (L) Plowden XYZ A1-induced corpus motion; (R) XZ IP motion for CBR mode shows strong shear-like IP motion between top and back. Wireframe shows undeformed surface.

The rms top-ribs-back mobility magnitude \( <Y(\omega) > \) for the Plowden is shown in Figure 2. The A1-induced peak at \(~ 455 \text{ Hz} \) has the same magnitude as the 1st corpus bending modes B1 and B1*, at \(~ 465 \text{ and } 510 \text{ Hz} \), resp. The averaged radiativity \( <R> \) for A1, also shown in Figure 2, is very similar to that of the 1st corpus bending modes. The Plowden A1 radiativity relative to the B1 modes is the largest of 17 violins measured to date.

Figure 2 – Rms magnitudes from 200-1000 Hz for violin radiativity \( R(\omega) \) (upper curve; top hemisphere) and OP mobility \( Y(\omega) \) curves for top, ribs, back as well as area-weighted corpus average (legend identifies separate Y curves) for Plowden. Two lowest cavity modes A0, A1 and three lowest corpus modes CBR, B1 and B1* notated. Arrows indicate string harmonics. Spacing between \( <R> \) and \( <Y> \) curves is indicative of radiation efficiency. A1 cavity-mode-induced indirect surface radiation mechanism very strong in this violin.
Figure 2 includes cavity modes that radiate directly through the f-holes - A0, indirectly through induced cavity wall vibrations - A1, corpus modes that radiate directly from the surface – B1 modes and indirectly through the f-holes – also B1. These four radiation mechanisms all have different frequency dependences, with cavity-related direct and indirect contributions falling in narrow frequency bands, while corpus related contributions cover the entire 5 kHz range.

RESULTS

OP vs. IP Mobility
With the vibration directions isolated in the proper frame of reference it was straightforward to extract rms $Y_{\text{OP}}$ and $Y_{\text{IP}}$ values from the corpus mobility data. Band averages were then applied to help simulate the strong damping effect of holding/playing on the various spectra and simplify the plots. Normally these averages were over 250 Hz bands, but A0 is a strongly radiating cavity mode near 270 Hz whose corpus mobility was quite small (see Fig. 2), so the lowest band was reconfigured somewhat. For corpus mobilities only, the lowest band runs from 300-500 Hz and contains generally only one strongly radiating mode $B_1$, all higher band then run in 250 Hz steps. From 500-750 Hz there is generally only one strongly radiating mode $B_1$.

The ratio $Y_{\text{OP}}/Y_{\text{IP}}$ was computed and the results shown in Figure 3 for all the old-Italian and modern violin data (top and back plate data were available for the Titian and Plowden only). Back plate OP/IP ratios seem consistent for all three old-Italian samples showing a gradual falloff from roughly 4 at low frequencies to 2 at the highest. The top plate results were quite a bit more variable, with the Plowden and Titian lying near 1 at the lowest frequencies, peaking about 2 kHz, and then the Plowden falls back toward 1 and the Titian plateaus near 2. Compare this behavior with the 2006 Curtin violin, which starts near 2 and falls off to 1, essentially duplicating the behavior of the Plowden curve above 1375 Hz.

Violin Radiativity
Radiativity measurements were averaged over 266 microphone positions on a $r = 1.2$ m sphere for the rms radiativity $<R(\omega)>$, which was then band-averaged as for mobilities. However, one lower band was added for A0, the only strongly radiating mode below ~400 Hz; $<R>$ was averaged over ~20 Hz region centered on the peak, isolating $<R>$ for A0 only. Higher bands were the same as for mobility. The 300-500 and 500-750 Hz bands gauge overall radiative strength of the $B_1$ modes relative to all the other bands, and the relative strength of each.
Figure 4 – Rms band-averaged radiativity for three old Italian (3 OI), and two 2006 violins of Curtin and Zygmuntowicz (new). Also shown is <R> for three “bad” violins from VIOCADEAS database (up to 4 kHz only). A0 band highlighted. (Standard deviation error bars for 3 OI and bad violins are a measure of inter-violin variability only.)

Figure 4 provides band-average radiativities for the three old Italian (3 OI), the two 2006 violins (new) of Curtin and Zygmuntowicz, and for comparison three modern, “bad” violins from the VIOcadeas database [5]. To give a measure of inter-violin variability standard deviations have been computed for the three old Italian violins and the three bad violin data. Figure 4 makes two matters clear: 1) the 3 OI-bad error bars generally overlap, indicating no significant statistical difference between the respective band-averaged violin radiativity and 2) a violin’s efficiency in converting mechanical motion into sound does not vary substantially between quality classes. One statistically significant exception appears for the A0 band, however, where the ability to couple mechanical motion to a cavity mode oscillation is much better for the 3 OI violins. The Plowden in fact also has one of the largest A1-mode contributions to sound of 17 violins measured, significantly increasing its 300-500 Hz band average.

Figure 5 – Directivity for 3 violins with top plate OP/IP ratio. Titian Stradivari has highest overall directivity of 17 violins measured, the Willemotte has the lowest.

**Violin Directivity**

The rms radiativity over the sphere <R(ω)> was easily separated into top and back components to compute a simple measure of sound directivity <D(ω)> = <R_{top}>/<R_{back}>. These results are
presented in Figure 5. As expected at lower frequencies <D> is close to 1. All instruments show relative maxima of ~1.5 near 1 kHz and then a slow general increase with frequency above 2 kHz. The Titian shows the greatest overall directivity of 17 violins measured to date, while the Willemotte shows the least.

A recent patch near-field acoustical holography experiment on radiation from the f-holes only has provided some insight into the prominent contribution of these ports to the overall “corpus” radiation above A0: 1) low-lying corpus modes can have nominally >50% of the far-field radiativity from the f-holes, with an erratic falloff toward higher frequencies, 2) f-hole directivity increases more rapidly than corpus, reaching ~2 by 1 kHz, 3) a number of modes near 1 kHz had almost all of their radiation from the f-holes [6]. Consideration of all three factors together could imply enhanced directivity near 1 kHz due to the f-hole contribution to overall radiativity.

**Total and Radiation Damping**

The modal analysis program provides a fitting routine for the mobility spectra that straightforwardly gives global frequencies and total damping (%critical) output for each mode of vibration. The radiation damping however comes from an entirely different source, requiring the radiativity/mobility ratio to compute the radiation efficiency, violin mass, and a frequency to calculate [3]. The radiation efficiency can be computed directly from data such as in Fig. 2 if the modal scan covers the top, ribs, and back to compute a reliable corpus rms mobility, and the radiativity is averaged over a sphere. This was possible only for the Titian and Plowden violins.

![Figure 6](image.png)

Figure 6 – Total damping (▲) from computer fits to mobility frequency response functions; radiation damping (■) estimated from computed radiation efficiency for 2 old Italian (solid points) and 9 modern instruments. Error bars shown indicate variability among band values only.

Heretofore $R_{eff}$ had been computed mode by mode and then band-averaged to provide the necessary data points to run a trendline through. Since there were only a few modes per 250 Hz band this approach resulted in erratic small-number-statistics behavior heretofore only partially remedied by averaging over multiple violins. Our analysis was done in an alternative manner, band-averaging <Y> and <R>, computing modal-average $R_{eff}$ values for each band, and computing radiation damping for each band. The total and radiation damping results for the Titian and Plowden are presented in Figure 6. For comparison 9-modern-violin results [5] with a recomputed modal-average $R_{eff}$ analyzed in the same way are superimposed.

**Critical frequency**

The critical frequency $f_{crit}$ influences violin sound because the fraction-of-vibrational-energy-radiated is a maximum in this frequency region [3]. Earlier band-averaging of normal mode $R_{eff}$ had shown a significant difference between quality classes but these “effective” $f_{crit}$ values,
obtained by solving trendline equations for $R_{\text{eff}} = 1$, suffered from small-number statistics as noted above. An alternative method was employed here for all violins, computing $<R>/<Y>$ ratios for each band, fitting a trendline through these values, and solving for a modal-average $R_{\text{eff}} = 1$. The Plowden-Titian average $f_{\text{crit}} \approx 3.4 \pm 0.3$ kHz was significantly lower than that for “bad” violins, $f_{\text{crit}} \approx 4.2 \pm 0.3$ kHz. Figure 7 shows average results for different quality classes.

![Figure 7 – Effective critical frequency for “bad”, “good” (incl. 1 bent wood), and old Italian violins.](image)

**CONCLUSIONS**

The 3-dimensional vibration measurements on 3 old Italian and 1 modern violin reveal: 1) significant in-plane motion for all violin top or back plates, 2) all old Italian measured mobility and directivity properties consistent with lesser-reputation violins (with the Titian having the highest directivity and the Willemotte the lowest of 17 violins tested), 3) slightly higher total damping <2 kHz for the old Italians due to increased radiation damping in this region, but similar fractions-of-vibrational-energy-radiated, 4) old Italian violins had averaged radiativity magnitudes similar to previously measured “good” and “bad” modern violins but a somewhat different spectral balance, 5) lower average critical frequency for the two exemplary old Italian than newer “bad” violins.

**Acknowledgments**

The results presented here are based on measurements performed at the East Carolina University Acoustics Laboratory over a three day period by four separate teams. I would like to acknowledge the essential participation of Polytec, Inc. (with David Oliver and Vikrant Palan running the 3-D laser scans), violinmakers Sam Zygmuntowicz and Joseph Curtin, the Violin Society of America (with Fan Tao and Joseph Regh) facilitating and providing funds to cover insurance, and my graduate student Danial Rowe in the Acoustics Laboratory. I would especially like to thank the owners of the Stradivari and Guarneri del Gesu violins who graciously allowed us to measure them.

**REFERENCES**