19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

NON-LINEAR PROPAGATION CHARACTERISTICS IN THE EVOLUTION OF BRASS MUSICAL INSTRUMENT DESIGN

PACS: 43.75.Fg

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ABSTRACT
The capacity of brass instruments to generate sounds with strong high-frequency components is dependent on the extent to which the bore profile supports non-linear propagation. At high dynamic levels some instruments are readily sounded in a *cuivré* (brassy) manner: it is now recognised that this phenomenon is due to shock wave generation. Non-linear propagation is also evident to some extent in playing at lower dynamic levels and contributes to the overall tonal character of the various kinds of brass instrument.

INTRODUCTION
Pyle and Myers[1] suggested a parameter related to non-linear propagation in brass musical instruments as a tool in the study of scaling in families of brasswind instruments, using the term "brassiness" for this parameter. Gilbert[2] further developed the theory and carried out initial experimental work. The "brassiness" concept has been refined by the present authors, who also carried out further experiments to confirm its validity[3]. The geometries of some 900 brass instruments belonging to museums worldwide and to individual musicians have been measured in sufficient detail to give useful estimates of the values of the brassiness parameter *B*. This paper critically evaluates the utility of parameter *B* in characterising the various kinds of brass instrument.

DEFINITION OF BRASSINESS PARAMETER *B*
Weakly nonlinear propagation in ducts has been treated by Hamilton and Blackstock[4], and the application of this theory to the present case has been developed by the present authors in a companion paper[5]. A coordinate stretching function *z(x)* with the dimension of length is introduced:

$$z(x) = \int_0^x \left(D_o \big/ D(y) \right) dy \qquad (1)$$

Where *D* is the duct diameter and D_0 is the diameter at its initial point of wave propagation, in our case the the mouthpiece end of a brass instrument, and x is the distance along the duct from this initial point.

Pyle and Myers[6] suggested that a brassiness parameter might be based on the coordinate stretching function defined Equation (1) at the geometric length *x=L* where *L* is the overall tube length of an instrument. The stretched co-ordinate approach can be seen as defining a new equivalent length *z(L)* of a cylinder in which the non-linear distortion of a sine wave propagating without losses develops to the same extent as it does in the actual instrument of tube length *L*. Thus, for example, if we have a cylinder of length L_{cyl} and a cone of length L_{cone}, the ratio $z(L_{cone})/z(L_{cyl})$ lower than 1 defines in a sense the lower development of non-linear distortion in cone compared to the cylinder. In other words, the cone of length L_{cone} is "equivalent" to a cylinder of a shorter length $z(L_{cone})$.

We wish to define a brassiness parameter *B* which will allow us to predict the ability of different instruments to obtain *cuivrés* or brassy sounds. A simple approach is to define this parameter as proportional to *z(L)*. To obtain a dimensionless parameter, the constant of proportionality could be chosen as the inverse of the geometric length *L*. However, this choice would not satisfy the desirable condition that for two instruments *a* and *b*, $B_a/B_b = z(L_a)/z(L_b)$, since in general L_a ¹ L_b even for two instruments with the same nominal pitch. We therefore propose that the brassiness parameter is defined as:

$$B = {z(L)} \Big/ {L(ecl)} \tag{2}$$

where *L(ecl)* is the equivalent cone length, $L(ecl)=c/2f_1$, f_1 being the fundamental frequency of the harmonic series best matching the playable notes.

Indeed, we can consider *z(L)* as being a new equivalent length for non-linear propagation: *L(nlp)=z(L)*. Thus although $L(ecl)_a=L(ecl)_b$, since they play the same notes, and $L(nlp)_a > L(nlp)_b$ for example. In fact, the increase is just proportional to the ratio of the brassiness parameters: $L(nlp)_a=(B_a/B_b)L(nlp)_b$. Moreover the brassiness parameter *B* defined in Equation (2) allows a comparison of brassiness between brasswinds of different pitch.

The brassiness parameter is related to effects which are independent of other important features of the instrument such as the player, the mouthpiece and the reflection/radiation properties of the bell. It is very convenient to ignore mouthpieces, since they are interchangeable and in the case of historic instruments, often of uncertain association or absent. Length measurement is therefore made from the proximal end of the mouthpiece receiver, as a well-defined point on all instruments (except the very few with integral mouthpieces). The minimum bore of the instrument, generally located within the first 100mm of the instrument, is for consistency taken as the initial value D_0 in the calculation. Bends in the tube are ignored, distances from the proximal end of the instrument being measured along the local axis of the tube, which in most instruments is approximately of circular cross-section (Myers & Parks[7]). For consistency, measurements are made with tuning-slides fully inserted.

Accuracy of calculations of *B* from instrument geometry

The data needed to calculate *B* are the diameters of the bore at the beginning of the instrument and at sufficient points along the length of the bore to give useful accuracy. Axial length measurements can be done with an accuracy of the order of 3%. The minimum bore is usually accessible to direct physical measurement, and can be determined to 0.2mm (typically within 3% accuracy). The minimum bore in a few cases, however, can be disguised by repairs. Bore measurements along the length of the tube can be estimated to an accuracy of 1% at tuning-slides and other points of separation, and by external diameters with estimates of tube wall thickness elsewhere (typically giving bore diameters accurate to 0.2mm). Alternatively, bore reconstruction by pulse reflectometry[8] can provide data for inaccessible parts of the bore. The wider tubing of the bell flare makes relatively little contribution to non-linear propagation effects, and measurement precision is less important here than in the proximal (narrower) part of the bore. Over short distances the expansion of the bore can be well approximated by a linear interpolation of the reciprocal of the bore diameter, allowing efficient calculation of the contribution of elements of the bore to the overall value of *B*. If the bore of an instrument is divided into n elements, brassiness *B* is approximated by the sum

$$B \approx \sum_1^n \frac{\ln}{L(ecl)} \left({2D_0} \Big/ {(Dn + D(n-1))} \right) \tag{3}$$

where for each element ln is the tube length and *Dn* is the bore diameter at the distal end of the element. We have found that for most instruments, measurements of bore diameter at as few as four well-chosen points give an approximate value of *B* which differs by only 1-2% from the value derived from measurements of the bore when divided into 30-40 elements.

The final factor in the calculation of *B* is the equivalent cone length *L(ecl)*. For instruments in playing regimes the intended playing pitch is well known and the equivalent cone length can be derived from knowledge of the nominal pitch and the design pitch standard. For a given instrument (say in a museum) we can calculate the equivalent cone length either after playing tests (introducing player, mouthpiece and temperature uncertainties) or from knowledge or assumptions about the nominal pitch of the instrument and the pitch standard it was made to be used at (introducing organological uncertainties). In most cases we can estimate the playing pitch by one means or another to within half a semitone, giving an uncertainty in equivalent cone length of plus or minus 3%. Since the brassiness parameter varies lies within a range of 0.3 to 0.9 depending on the instrument, its utility is not compromised by errors of the order of 3%.

Effects of operating valves and slides

Operating the valves on a valved instrument or the slide on a trombone clearly affects the bore geometry considerably. Players' experience correspondingly shows an increase in *cuivré* effect when cylindrical tubing is introduced into the bore by valves or a slide: this is sometimes utilised by players sounding notes where alternative fingerings or positions are possible. For example, Kaiserbaryton (EUCHMI 3412) with no valves operated is in Bb with B=0.37; with first and third valves operated it is effectively an instrument in F with B=0.46. Baroque trombone (EUCHMI 3205) with the slide closed is in Bb with B=0.81; with the slide extended to increase the overall length from 2711mm to 3711mm it is effectively an instrument in F with B=0.85. The greater effect on brassiness of extending the length of instruments with low values of B is supported by experimental results[9]. It should be noted that the other main determinants of tone quality (player's vocal tract, lip musculature, mouthpiece geometry and bell flare geometry) are not directly affected by the operation of slides and valves. It is normal for audiences to perceive instruments as having recognisable and identifiable tonal characteristics although their brassiness varies slightly between consecutive notes. With the most usual playing techniques, players generally select the shortest available tube length for maximum accuracy and best intonation, so in comparing instruments using the brassiness parameter in general we compare B for their basic (shortest) configurations. One common exception to this is the french horn in its most common double horn model: here players use the Bb "side" of the horn for accuracy and response, but in passages of music where tone colour is the priority will often use the F "side" of the horn with its spectra richer in harmonics deriving from non-linear propagation.

TYPICAL VALUES OF *B* FOR RECOGNISED KINDS OF BRASSWIND

Brass instruments with equivalent cone lengths in the range 2500mm - 3500mm (8-ft and 9-ft nominal pitches) show the greatest variety in taxonomy. A few typical calculated values of B (from a much larger population measured) are shown below:

Instrument, Nominal Pitch		Maker, Place, Date	*B*
EU3590	Ophicleide, keyed for A	Gautrot, Paris, c 1860	0.31
EU3412	Kaiserbaryton, 9-ft Bb	Cerveny, Königgraetz, c 1900	0.37
EU3414	Euphonium, 9-ft Bb	Boosey & Co, London, 1928	0.46
EU2946	Baritone, 9-ft Bb	Besson, London, c 1925	0.48
EU2726	Baritone, 9-ft Bb	Boosey & Co, London, c 1920	0.53
JG2	Bass trombone, 9-ft Bb	Courtois, Paris, 2000	0.67
EU3207	Tenor trombone, 9-ft Bb	R. Schopper, Leipzig, c 1910	0.73
EU3747	Tenor trombone, 9-ft Bb	Courtois, Paris, 1865	0.77

Instruments recognised as euphoniums (or similar) have values of B in the range 0.37 to 0.47. Instruments recognised as baritones (or instruments similar to narrower bore saxhorns) have values of B in the range 0.44 to 0.60. Trombones have values of B in the range 0.63 to 0.80 (the variations in trombone design are discussed below). French horns with basic tube lengths in this range have values of B in the range 0.45 to 0.60, so consideration of factors other than brassiness (such as the distinct difference in minimum bore) is required to differentiate the shorter french horns from baritones.

Instruments with equivalent cone lengths in the range 1700mm - 2500mm (6-ft and 7-ft nominal pitches) can be considered together. Some typical calculated values of *B* are shown below:

	Instrument, Nominal Pitch	Maker, Place, Date	*B*
EU3886	Tenor horn, 6½-ft Eb	Boosey & Hawkes, London, 1962	0.48
EU3881	Tenor horn, 6½-ft Eb	Besson, Paris, c 1892	0.58
JW	Alto trombone, 6½-ft Eb	Courtois & Mille, Paris, c 1880	0.70
MSM67-5	Trumpet, 7-ft D	Hainlein, Nürnberg, 1632	0.83

Instruments recognised as tenor horns (alto horns) or similar have values of *B* in the range 0.45 to 0.65: a considerable variety of models fall within this range. Trumpets and alto trombones have values of *B* in the range 0.65 to 0.85, with baroque natural trumpets enjoying the highest brassiness of all instruments which have at some time been in regular use.

Instruments with equivalent cone lengths in the range 1250mm - 1700mm (4-ft nominal pitches) can also be considered together. Some typical calculated values of *B* are:

	Instrument, Nominal Pitch	Maker, Place, Date	*B*
KM1221	Flugelhorn, 4-ft C	Lorenz, Linz, c 1880	0.44
EU4654	Flugelhorn, 4½ft Bb	Boosey & Hawkes, London, 1959	0.54
EU3710	Cornet, 4½-ft Bb	Courtois, Paris, 1862-71	0.56
EU3475	Cornet, 4½-ft Bb	Courtois, Paris, 1856-58	0.61
EU3212	Trumpet, 4½-ft Bb	Boosey & Hawkes, London, 1933	0.68
EU2438	Trumpet, 4½-ft Bb	Micol-Montagna, Trieste, m.20	0.79

Instruments recognised as flugelhorns have values of *B* in the range 0.43 to 0.56. Cornets have values of *B* in the range 0.56 to 0.67. Valved trumpets have values of *B* in the range 0.65 to 0.79.

We finally consider instruments with equivalent cone lengths in the range 3400mm - 5000mm (12-ft and 14-ft nominal pitches). Typical values of *B* are:

	Instrument, Nominal Pitch	Maker, Place, Date	*B*
EU2992	Tuba, 13-ft Eb	Hawkes & Son, London, c 1913	0.36
BM1281	Bass Tuba, 12-ft F	Moritz, Berlin, c 1840	0.48
DMC	French horn, 12-ft F	Boosey & Hawkes, London, 1938	0.56
EU2492	Natural horn, 14-ft D	Winkings, London, c 1760	0.65
EU519	Bass trombone, 11-ft G	Besson, London, c 1890	0.70
BE0567	Bass trombone, 13-ft Eb	W W Haas, Nürnberg, c 1745	0.79

Tuba family instruments have values of *B* in the range 0.36 to 0.52. Instruments of the french horn family have values of *B* in the range 0.52 to 0.65. Bass trombones have values of *B* in the range 0.62 to 0.79.

Instruments of smaller basic tube length (soprano cornets, piccolo trumpets etc) show less variety in brassiness, and also fewer well-characterised species. Instruments in very large sizes also show less variety: those with equivalent cone lengths in the range 5000 - 6800mm (16-ft and 18-ft nominal pitches) are either from the tuba family with values of *B* in the range 0.31 to 0.46, or contrabass trombones with values of *B* in the range 0.59 to 0.82.

Instrument families (utility of *B* in scaling discussion)

It can be seen from the above that where instrument models are designed in families (for example, alto, tenor, bass and contrabass trombones; the saxhorns) the parameter *B* is rather similar for all sizes: brassiness is kept constant as designs are scaled. This satisfying result does not obtain in the smallest sizes, however: trumpets and soprano cornets in high Eb have largely overlapping ranges of values of *B* (0.55 - 0.70).

How *B* can illuminate the evolution of recognised instruments

The brassiness parameter can be used to study the evolution of instruments. A good example is the trombone, where the early forms are so different from present-day models that they enjoy their own name ("sackbut") and reproductions of old models are widely used in period-instrument performance rather than modern trombones. Typical examples from the various stages in the history of the trombone are:

	Instrument, Nominal Pitch	**Maker, Place, Date**	***B***
EU2695	Tenor trombone, 9-ft Bb	Schnitzer, Nürnberg, 1594	0.71
EU3205	Tenor trombone, 9-ft Bb	Huschauer, Vienna, 1794	0.88
EU3747	Tenor trombone, 9-ft Bb	Courtois, Paris, 1865	0.77
EU3207	Tenor trombone, 9-ft Bb	R. Schopper, Leipzig, c 1910	0.73
DMC	Tenor trombone, 9-ft Bb	King, Cleveland, c 1943	0.64
JG2	Bass trombone, 9-ft Bb	Courtois, Paris, 2000	0.67

The results seem counter-intuitive, with Renaissance period trombones (sackbuts) having higher values of *B* than modern trombones, although the early trombones are perceived as being less brassy. It has to be remembered that although at high dynamic levels some instruments are readily sounded in a *cuivré* (brassy) manner, non-linear propagation is also evident to some extent in playing at lower dynamic levels. Typical sackbut playing is at a low dynamic level and the high value of *B* engenders some non-linear propagation, resulting in a rich timbre. Modern trombones with their lower value of *B* are not "brassy" at low dynamics, but for these models *B* is not so low that *cuivré* playing is out of reach of the player: it develops at the high dynamic levels which characterise modern trombone playing. Nineteenth-century trombones, represented here by the French model (Courtois 1865) and the German model (Schopper) accommodated playing styles where *cuivré* playing was part of the idiom, but dynamic levels were lower than is common today.

CONCLUSION

A "brassiness parameter" *B* has been defined which can be derived from the bore geometry of brass instruments. Typical values characterise the recognised kinds of brasswind, resulting acceptable consistency in timbre of instruments even when the overall tube length is changed by valves or slides. The concept of brassiness is useful in discussing the scaling of instrument families and the evolution of recognised instruments.

Aknowledgments: Raymond Parks, Eugenia Mitroulia (University of Edinburgh) for help with measurements. The professional staff of some thirty museums, and individual musicians for access to instruments. The instruments discussed here were made available for measurement by the Berlin Musical Instrument Museum (BE), Brussels Musical Instrument Museum (BM), Edinburgh University Collection of Historic Musical Instruments (EUCHMI), Musica Kremsmünster (KM), Munich Stadtmuseum (MSM), John Webb (JW) and the authors (JG, DMC).

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