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SOME COMMENTS ON COMPUTATIONAL ISSUES IN MODULARIZED PHYSICAL MODELING SOUND SYNTHESIS

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Bilbao, Stefan
Acoustics and Fluid Dynamics Group, Room 7306B, James Clerk Maxwell Building., King's Buildings, University of Edinburgh, Mayfield Rd., Edinburgh, EH9 3JZ, United Kingdom; sbilbao@staffmail.ed.ac.uk

ABSTRACT

Modularization has always been a final goal of physical modelling sound synthesis algorithm developers. The idea is to allow the user to combine, at will, various basic physical building blocks, such as stings, tubes, membranes, plates, and lumped objects into a virtual instrument which may well lack a counterpart in the real world. Many techniques, and especially those based on the use of lumped networks, modal decompositions, and scattering networks are undergoing such developments. This article is intended as a general, non-technical discussion of some of the issues which invariably arise when components or modules are connected, with the point of view that all physical modelling techniques are, in essence, different varieties of numerical simulation methods. Particular topics, such as global vs. local representations of the dynamics of individual modules, and numerical stability are addressed with regard to the specific problem of interconnection.

INTRODUCTION

Physical modelling sound synthesis has been approached in many ways over the years. Some of the best known techniques are lumped models [1], modal synthesis [2], finite difference techniques [3], digital waveguides [4], methods based on the use of wave digital scattering networks, as well as a number of hybrids [5]. All such techniques, however, may be viewed as numerical methods to determine an approximate solution to an underlying model problem, generally framed as a set of partial differential equations. In many ways, these techniques are the same as those used in investigations in pure musical acoustics. Distinctions arise, however, with regard to the issue of modularity, which is of special importance in a synthesis environment, in which arbitrary connections among various objects are allowable---such connections might not be of interest in a more strict musical acoustics setting, but are of extreme importance if an instrument designer (i.e., a composer) wishes to create an instrument, through interconnection of basic components, in a novel manner.

It is impossible to present the technical details of the wide variety of methods mentioned above, and thus familiarity with them is assumed here, and the reader is referred to the works cited in the previous paragraph. A complete technical treatment of the many issues discussed here will appear in a forthcoming publication [6]. The purpose here is to highlight, at a general level, many of the difficulties inherent in the development of modular synthesis systems. The primary difficulties, as will be detailed below, are in the combination of components, or modules, which are described globally (versus locally), and in numerical stability of combinations of components. Possible solutions are hinted at as well.

GLOBAL AND LOCAL REPRESENTATIONS OF VIBRATING OBJECTS

The starting point for sound synthesis based on physical models is usually a partial differential equation (PDE) or coupled set of PDEs---a typical example is the wave equation [7], used to describe, to a rough approximation, the vibration of strings, acoustic tubes, membranes, and also room acoustics. It is, essentially, a pointwise or local description of the dynamics of the object under consideration; the behaviour of a medium at a given location is influenced only by the region in its immediate neighborhood.

In the mainstream simulation community, there are many numerical techniques which maintain this principle of local action---explicit finite difference methods [8] and finite element methods (in a non-diagonalized form) [9] employ updating at a given location, often called a grid point, with reference only to previously computed values at neighboring points. In the more specific setting of sound synthesis, various derived or related methods, including finite differences, digital waveguides, and lumped network models also operate in this way. On the other hand, there are many other techniques which do not incorporate this locality principle---some spectral methods [10] are built around global basis functions, and implicit finite difference schemes [8] often involve a coupling among unknown values over the entire problem domain. Perhaps the best example of a non-local method in sound synthesis is the body of modal techniques (which are, in fact, none other than Fourier spectral methods).

Modal Methods and Precomputation

Modal techniques rely on a decomposition of the solution to a PDE system into contributions from modes, each of which oscillates at its own natural frequency; generally, such techniques may be applied with ease only to systems which are linear and time-invariant. Such techniques have been applied for some time in physical modelling sound synthesis, and form the basis for the now-classic Modalys and MOSAIC synthesis systems [2].

A typical modal representation of a solution, for any lossless linear and time-invariant initial boundary value problem in the variable $u(x,t)$ is of the form

$$u(x,t) = \sum_p U_p(x)\left(A_p \cos(w_p t) + B_p \sin(w_p t)\right)$$

t and x are time and space variables, respectively, the functions $U_p(x)$ are the modal functions, and w_p the associated modal frequencies. Such a representation is global, in the sense that each mode extends, spatially, to cover the entire spatial domain, and as such, must take into account boundary conditions. A perturbation in the properties of the medium at a given location, or a connection, affects, in general, all the modes.

Numerical methods based on modal representations, by association with spectral methods, have the advantage of great accuracy, as well as conceptual simplicity; many other methods, including finite difference schemes, for which the modal frequencies are not calculated explicitly, exhibit numerical inharmonicity, or mistuning of modal frequencies, and can be more troublesome to construct. On the other hand, only in relatively few problems may the modal functions and frequencies be expressed in closed form---otherwise, these functions and frequencies must be determined through a precomputation step, which is essentially the solution of a (potentially large) eigenvalue problem.

Leaving aside the problem of storage (dealt with in a companion paper [11]), consider the connection of one object to another. For simplicity, consider the elementary case of a lossless linear string, of length L, and wave speed c, under fixed boundary conditions, connected, at some point along its length, to a mass-spring system, as shown in Figure 1. In the absence of the connection, the modal functions of the string and the frequencies may be expressed simply as

$$U_p(x) = \sin(ppx) \qquad \text{and} \qquad w_p = pcp/L$$

for integer p. Without going into too much detail, it should be clear that the modal functions, as well as the frequencies themselves, will vary as the properties of the spring (such as, say, the stiffness) are varied. See Figure 1. In general, these will need to be computed numerically, and stored, and will be distinct for each possible choice of the parameters of the mass/spring system, again requiring an offline calculation any time such parameters are changed by the (eventual) user. As one can imagine, the amount of precomputation will grow with the complexity of the collection of objects to be interconnected, and can become a formidable task if the components themselves possess a large number of degrees of freedom; see the companion paper [11]. The important point is that in general, a global representation of a collection of interconnected modules can not be derived in a simple way from global representations of the modules themselves.

A related approach, the functional transformation method [12], which is also based on global frequency domain descriptions of objects, but from a transfer function or input/output perspective, is probably better suited to modular interconnection of objects. Again, though, the transfer functions must be computed in an offline step for each separate input/output location pair.

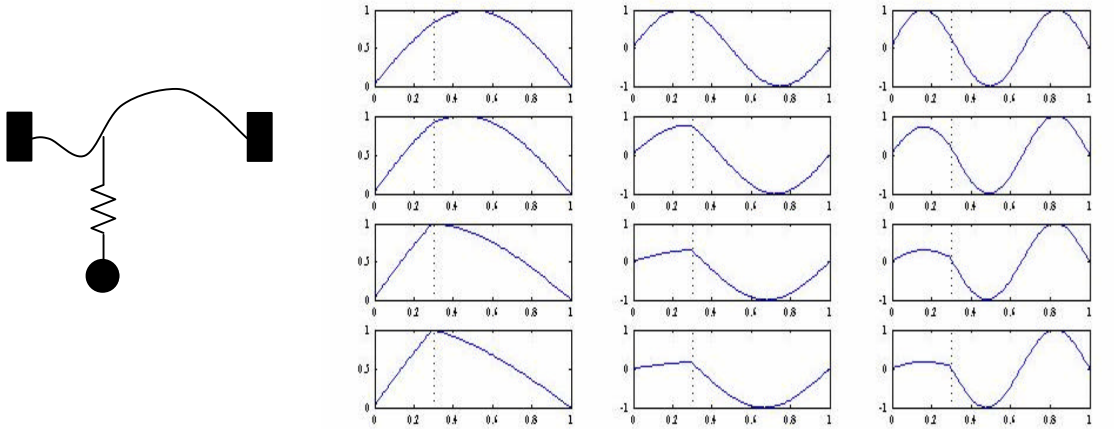

Figure 1 – Left, lossless linear string, between rigid supports, connected to a lumped mass-spring system. Right, resulting modes of vibration of the combined system (in order of frequency, across the column, and for different values of the spring stiffness (down the rows). The modes of a string in isolation appear across the top row.

Difference Schemes and Locality

Time domain update methods, such as finite difference schemes (which appear in synthesis applications directly, and also as digital waveguides, wave digital networks, and lumped network models), are distinct from modal methods in that the observable variables, such as displacement, etc. are represented directly, at a collection of points on a spatial grid. For schemes for linear systems (and for many nonlinear systems), the updates may always be written, in the zero-input case, as an update of the form

$$Au_{n+1} = Bu_n \qquad \text{Eq. 1}$$

where u_n is a vector of observables at time step n, and A and B are update matrices. Generally, for simple schemes, these matrices are sparse. If A is diagonal, the scheme is referred to as explicit, and otherwise implicit. See Figure 2.

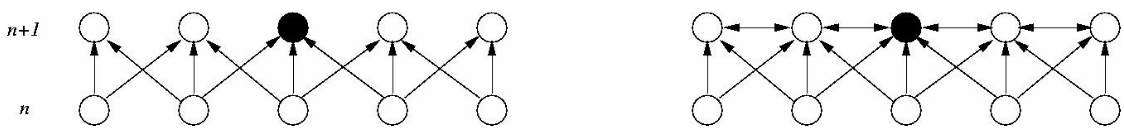

Figure 2 Left, update of a sparse explicit numerical method. Right, implicit update of a sparse numerical method.

In general, implicit methods involve more computation than explicit schemes, i.e. a linear system solution (at least) is required at each time step, but often possess better numerical stability properties. On the other hand, it is also clear that for an implicit method, there is a global coupling among all the unknowns, which can complicate interconnections among

components. This is a slightly different manifestation of the concept of a “global” method, in that the state variables which are, finally, observed, still correspond to physical variables at given spatial locations.
As an example, consider a simple linear array of masses and springs, as shown at left in Figure 3. As is well-known, this system may be integrated readily using explicit finite difference schemes; this is the basis of the approach employed in the CORDIS environment. If, on the other hand, a wave digital network is derived via a circuit representation, as is also common in sound synthesis applications, and as shown at center and right in Figure 3, one may arrive at a structure which possesses so-called “delay-free loops,” or instantaneous connections among elements. Such a network does not lead, as is commonly claimed, to a “non-recursible” or “non-computable” structure, but rather to one for which the entire set of state variables must be computed together. In essence, this is none other than an implicit method, of the form of Eq. 1. The artifice of employing so-called reflection-free ports [13] so as to derive an ordering in which to update the state values is none other than a particular technique for inverting the linear system in Eq. 1; note however, that the variation in the value of any of the parameters in the systems (such as, e.g., the mass of any element), as in the case of a connection, will require a recomputation of the entire path. This is unnecessary in an explicit method, and also must be performed in an offline step.

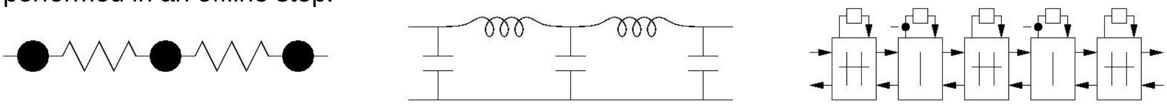

Figure 3 Left, a connection of masses and springs, center, an LC ladder circuit representation, and right, the corresponding wave digital network.

NUMERICAL STABILITY

Modal methods, and those based around the use of scattering blocks possess, as an advantage, somewhat simplified control over the problem of numerical stability, even when disparate elements are connected. In the case of modal synthesis, the issue is transparent, as modal frequencies must be calculated in an offline step (in general numerical), so that any numerically computed spurious unstable modes may be discarded before run time. Methods designed around the use of networks of scattering blocks, at least in the linear case, are stable by construction, due to the preservation of a positive-definite numerical energy function, which is a consequence of consistent use of the so-called trapezoid rule of numerical integration.

For general time domain methods such as finite difference schemes, the problem of stability in a modularized setting as a more delicate one; given stable numerical methods for two objects in isolation, it is certainly not true that a combination leads immediately to a numerically stable implementation. As an example, consider again the simple example of the ideal lossless string between fixed supports, connected to a mass-spring system. Stable explicit numerical methods may easily be developed for both the objects in isolation---see Figure 4, at left. When these same schemes are connected however, instabilities can arise, depending on how the connection itself is implemented---see Figure 4 at right.

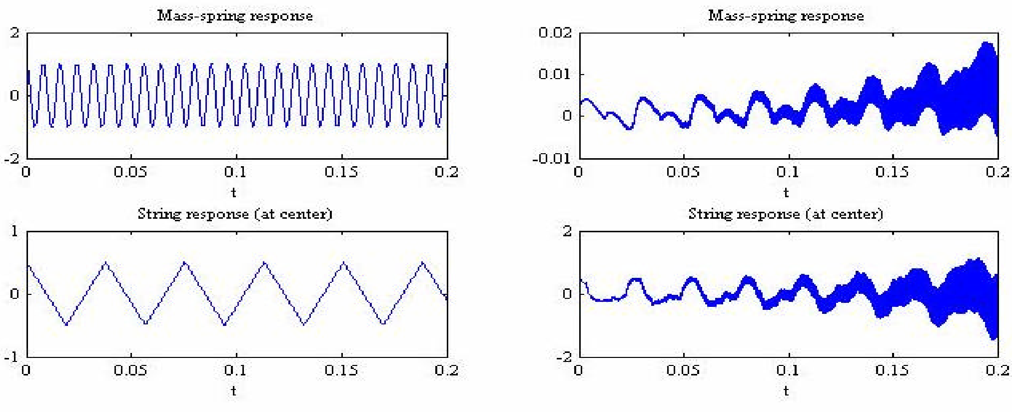

Figure 4 – Responses using a finite difference scheme for the coupled string/mass-spring system shown at left in Figure 1, where the string is initialized in the shape of a triangle, and the

mass-spring to an initial deviation in displacement. Left: responses of the mass/spring and string individually. Right, responses of the coupled system, employing the same numerical techniques, exhibiting numerical instability. This instability causes the solution to diverge within a few time steps of the end of the plots shown.

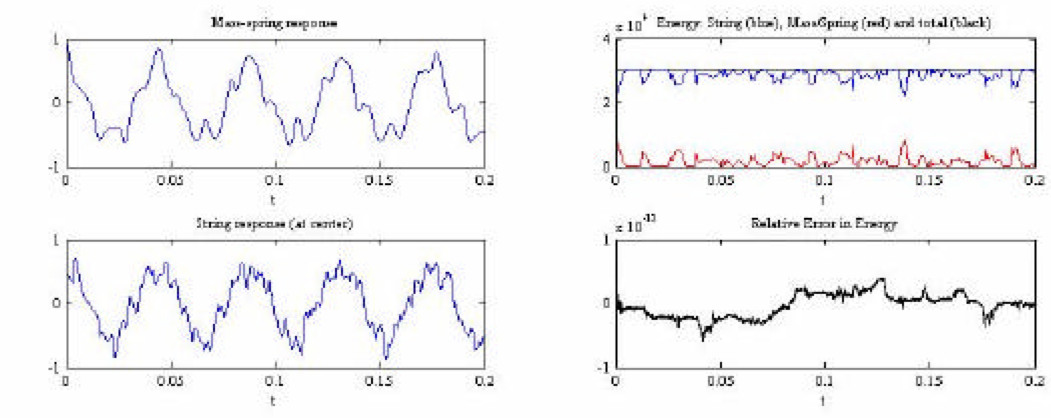

Figure 5 – Responses using a finite difference scheme for the same system discussed in Figure 4. In this case, an energy-conserving method is used to implement the connection, leading to provably stable numerical behaviour. Left, responses of the coupled system. Right: Plots of the variation in numerical energy for the string (blue), mass-spring (red) and in total (black), at top, and at bottom, the variation in the total numerical energy normalized by the total energy, as a function of time, exhibiting numerical energy preservation down to variations in the least significant bits.

One useful design approach, when working with time-domain methods, is to examine numerical energy conservation, a more modern approach to numerical stability analysis [14]. All connections between objects can be viewed as lossless, and, provided one may find a suitable numerical energy function which mirrors its continuous time counterpart, and can be guaranteed positive definite under some easily enforced condition. This is fact relatively simple in many cases: considering the same system of the string and connected spring discussed previously, an energy-conserving numerical connection may be derived which does indeed permit an explicit determination of stability conditions for the combined system; as a bonus, it is possible to monitor energy conservation at the debugging stage, as numerical energy is conserved to machine accuracy. See Figure 5.

As mentioned above, this energy conservation principle is in fact exactly what is behind the good stability properties of scattering networks based on the use of digital waveguides and wave digital filters---but, as mentioned above, it is by no means limited to the special class of numerical methods on which scattering structures are based (namely, the use of the so-called trapezoid rule of numerical integration). Explicit schemes also possess such properties, and are undeniably easier to program, as well as more computationally efficient. For more information on this large topic, see [6].

NONLINEAR ELEMENTS AND CONNECTIONS

The problem of connections among nonlinear elements is an involved one, regardless of the synthesis method employed. Modal methods are, for obvious reasons, difficult, if not impossible to employ in general. But while it is easy to arrive at differencing strategies even for connections between fully distributed nonlinear objects, the stability problem persists.

The most basic problem is the following: even for a nonlinear system in isolation, it is difficult to design a method which is both (a) provably stable, and (b) explicit. Stable implicit methods for nonlinear systems do exist, but the problem then becomes that of ensuring existence and uniqueness of solution; the possibility of this is easily seen upon examination of Eq. 1, in the case where A depends on u_{n+1}. Wave digital networks are no exception in this regard:

nonlinear wave digital elements and networks may indeed be designed, but, in general, port resistances are signal dependent, and must normally be solved for using iterative techniques (which may not converge). As a side note, it is useful to point out that translating the stability behaviour of scattering methods to the nonlinear case requires the use of power-normalized wave quantities, significantly complicating the iterative techniques which must be employed [15]. This problem remains to be solved in general, and is of crucial importance in the synthesis of “interesting” sounds produced by nonlinear systems.

CONCLUSIONS

This article has been devoted to a general study of some of the core issues involved in modularized physical modelling sound synthesis. This is a complex and difficult undertaking, regardless of the synthesis method employed. The good numerical behaviour of modal synthesis and scattering based methods, which are often employed in a modular environment, does not carry over to the nonlinear case in a straightforward way. In addition, modal methods will require a fair amount of precomputation, making rapid experimentation more difficult for the user. Energy-based techniques do offer an alternative methodology for modular connections which, while far from straightforward, can potentially allow for stable behaviour even under strongly nonlinear conditions.

ACKNOWLEDGEMENTS

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References:

[1] C. Cadoz, A. Luciani and J.-L. Florens: CORDIS-ANIMA: A Modeling and Simulation System for Sound and Image Synthesis. Computer Music Journal **17, No.1** (1993) 19-29

[2] J.-M. Adrien: The Missing Link: Modal Synthesis. In G. DePoli, A. Picialli and C. Roads, editors, Representations of Musical Signals, pages 269-297, MIT Press, Cambridge, Massachusetts (2006)

[3] A. Chaigne: On the Use of Finite Difference for Musical Synthesis. Journal d’Acoustique **5, No. 2** (1992) 181-211

[4] J. O. Smith III: Acoustic Modeling Using Digital Waveguides. In C. Roads, S. Pope, A. Picialli and G. DePoli, editors, Musical Signal Processing, pages 221-263, Swets and Zeitlinger, Lisse, The Netherlands, (1997)

[5] R. Rabenstein, S. Petrausch, A. Sarti, G. DeSanctis, C. Erkut and M. Karjalainen: Block-based physical modelling for Digital Sound Synthesis. IEEE Signal Processing Magazine **24, No. 2** (2007) 42-54

[6] S. Bilbao: Numerical Sound Synthesis. Under contract, John Wiley and Sons; due for publication in September, 2008.

[7] P. Morse and U. Ingard: Theoretical Acoustics. Princeton University Press, Princeton, New Jersey**,** (1968)

[8] J. Strikwerda: Finite Difference Schemes for Partial Differential Equations, Wadsworth and Cole, Pacific Grove, California, (1989)

[9] R. Cook: Concepts and applications of finite element analysis, 4th ed., John Wiley and Sons, New York, (2002)

[10] L. Trefethen: Spectral Methods in Matlab, SIAM, Philadelphia, (2000)

[11] S. Bilbao: Modal Representations and Computational Complexity in Physical Modeling Sound Synthesis, International Congress on Acoustics, Madrid, Spain (2007)

[12] L. Trautmann and R. Rabenstein: Digital Sound Synthesis by Physical Modeling Using the Functional Transformation Method, Kluwer Academic Publishers, (2003)

[13] G. Borin and G. DePoli and D. Rocchesso: Elimination of Delay-free Loops in Discrete-time Models of Nonlinear Acoustic Systems. IEEE Transactions on Speech and Audio Processing, **8,** (2000) 597-606.

[14] S. Li and L. Vu-quoc: Finite Difference Calculus Invariant Structure of a Class of Algorithms for the Nonlinear Klein Gordon Equation. SIAM Journal of Numerical Analysis **32,** (1995) 1839-1875

[15] A. Fettweis: Wave Digital Concepts and Relativity Theory. In D. Liu and P. Antsaklis, Eds., Stability and Control of Dynamical Systems with Applications Birkhaeuser , Boston**,** (2003) 1-22