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THE ESTIMATION OF THE VIBROACOUSTIC PARAMETERS OF THE SOUNDING BOARDS OF MUSICAL INSTRUMENTS

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ABSTRACT

The paper presents the results of numerical investigations of the vibrational and acoustical fields of the sounding boards of musical instruments. The steady-state sounding board oscillations arising from a harmonic driving force are considered. The structured model of the sounding board is created on the base of the finite element method. The calculation relationships take into account the anisotropy of physical-mechanical characteristic and layer structure of the material. The problem of dynamics for the sounding boards is solved by decompositions of the motion on the eigenmodes. The vibrodisplacements and the vibrovelocity are defined with the help of the Raleigh model of the proportional damping. The analysis of the results shows that complicating of the configuration of a plate contour makes frequency modes less definite and "equalizes" its resonance curve in a low frequency area of a spectrum. The got vibrovelocity field is used as raw data for the numerical solution of the system of the gas dynamics equations. The pressure is determined in some locations of the near field of the air ambience on resonance frequencies of the designs. The comparative analysis of vibrational and acoustic fields allows to estimate dependence of the features of the acoustic field from dynamic characteristic of the sounding board within the low frequencies range.

INRODUCTION

The most widespread sound-radiating members of devices and gears are thin-walled designs like plates and shells. The problem of vibrations of that simple mechanic systems are well-known and studied. However the process of transfer of mechanical oscillations energy to the environment is not clearly understood. In the case of vibrations in acoustic frequencies range the energy transfer means the radiation of sound. The sound radiation of such vibratory systems shows special nature caused by nodal patterns grid. Establishing the basic relationships of these phenomena is an important problem, as it gives a key for development of the most expedient methods of management of the radiating ability of thin-walled designs. The analytical solutions of the problems about radiation of the sound by a canonical forms plate are known [1]. As the contour configuration complicates and heterogeneities occur the defined correlations may vary significantly [2]. The sounding boards are acoustic radiating elements of such type used in designs of string musical instruments (MI). The sounding board is a thin-walled braced plate. Its configuration, size and kind of structural materials have been selected intuitively during the evolution of a musical culture. The most successful samples of MI have been manufactured and duplicated. Refusal from the adopted canons, caused by the new qualitative requirements to MI, frequently gives negative results. The reason is the absence of the scientifically grounded approaches for designing of MI. The present paper focused on investigating of the influence of sounding board configuration on its dynamic properties and conducting the comparative analysis of vibrational and acoustic fields, induced by mechanical oscillations.

The sounding board producing forced vibrations caused by harmonic force is deformed. Thus, each small segment on the plate surface deviates from equilibrium position on the definite value which is the amplitude of oscillations, displacing thereby some air volume. Summarizing elementary volumes on the entire sounding board surface, a general volume (***V***), displaced by the plate is found. The value of ***V*** may be more common characteristic of vibrations energy distribution on frequency than the frequency curve built for the definite point.

Four configurations of plate are investigated such as rectangular (further referred as No.1), elliptical (No.2), shaped like a guitar soundboard of a "Dreadnaught" model (No.3) and Orchestral model (No.4). The size of the plates are picked out so that their area (0,1267 m^2),

height (0,46 m) and thickness (0,003 m) remain constant for all forms. The dissipate coefficients and elastic constants are the same for all configurations.
According to [3, 4] values of physical-mechanical characteristics of the plate are considered as: density ρ = 494 kg/?^{3}, Young modulus $?_1$ = 15 GPa, $?_2$ = 0,335 GPa, Poisson coefficient n_{12} = 0,44, n_{21} = 0,009, shear modulus G_{12} = 0,622 GP?, G_{13} = 0,422 GP?, G_{23} = 0,043 GPa; the same for the braces are: r = 590 kg/?^{3}, $?_1$ = 16,6 GPa, G_{13} = 1,18 GPa. Here, subscript 1 corresponds to the direction that is parallel to grain, the subscript 2 denotes the across direction, subscript 3 determines the direction of the normal to the median surface of the plate. Each brace is trapeziform beam with rectangle cross-section which dimensions are thickness of 5 mm and maximum height of 12 mm. The calculations were conducted for isotropic design also made of plastic material. Its physical-mechanical characteristics are considered as: ? = 2,1 GP?, ρ = 1200 kg/?^{3}, n = 0,3. The swing joint of the outline is taken in the computation. The calculation is made for two points of application of a driving force. The point K will be used for the symmetrical modes, point L for the skew-symmetric ones.

COMPUTATIONAL MODEL

The problem of dynamics is described with the following system of differential equations

$$[M]\{\ddot{q}\} + [B]\{\dot{q}\} + [K]\{q\}=\{F(t)\}. \quad \text{(Eq. 1)}$$

Here $[M]$, $[B]$ and $[K]$ are the mass, damping and stiffness matrices of the construction; $\{\ddot{q}\},\{\dot{q}\},\{q_m\}$ are the generalized acceleration, velocity and displacement vectors, respectively. The stiffness and mass matrices and the load vector are constructed using a modificated variational Hellinger-Reissner formulation. Independent approximations of generalized displacement and deformation are used. The asymmetry in the disposition of braces is taken into account by the matrices of transformations of the generalized coordinates. Integration is executed in Gauss quadrature formulas. The load vector has the form of $\{F(t)\} = F_0 sin(W\ t)$, where W is an angular excitation frequency, t is time. The order of matrices is equal to the number of degrees of freedom: n = 7629 for the plate No.1 and n = 8205 for the others. The lowest eigenmodes and eigenfrequencies are computated by the subspace iteration method. The oscillating motions of a sounding board are modeled as the superposition of the lowest eigenmodes like $\{q(t)\} = [?]\{Z(t)\}$. Here $[?] = [j_1, j_2\ldots, j_{30}]$ is a matrix consisting of thirty lowest eigenmodes, $\{Z(t)\}$ is the vector of the main or normal coordinates. In such a case, the equations (Eq. 1) written down in the main coordinates assume the form of

$$\ddot{Z}_i + 2x_i w_i \dot{Z}_i + w_i^2 Z_i = f_i(t), \quad (i = 1, 2\ldots, r). \quad \text{(Eq. 2)}$$

The energy dissipation complies with the Raleigh model of proportional damping [5]. Hence equations of the motion (Eq. 1) become decoupled and damping matrix is expressed through matrices of the mass and stiffness like

$$[B]=M\sum_{i=1}^{r} g_i\left[M^{-1}K\right]^{i}, \quad \text{(Eq. 3)}$$

where g_i – are Raleigh parameters. In general the amount of damping factors x_i is equal r. Under r = 2 equations (Eq. 3) happen to classical Raleigh formula like

$$[B] = g_1[M] + g_2[K] \quad \text{or} \quad 2x_i w_i = g_1 + g_2 w_i^2. \quad \text{(Eq. 4)}$$

The first summand in (Eq. 4) takes into account the "external" friction (e.g. the resistance of radiation), the second one takes the "internal" friction (e.g. the energy dissipation in the material). The damping factors value x_i on resonance frequencies w_i for varied element designs turn out to be different. In condition of limited information it is possible to use as a first approximation the extrapolation [6]:

$$x_i = x_1 \left(\frac{w_i}{w_1}\right)^{0,6}, \quad \text{(accordingly to [4] } ?_1 = 0{,}08\text{).} \quad \text{(Eq. 5)}$$

VIBRATIONAL FIELD OF THE SOUNDING BOARD

On the base of constructed mathematical model the vibrational fields of the sounding board are determined. The obtained graphic curves of the displaced volume (***V***) versus an oscillation frequency for wood sounding boards are shown in fig.1. The area bounded by curves in the range up to 1000 Hz is determined (the total value of a displaced air volume is ***S***). To estimate the displaced air volume according to frequency the table is showed in which this value is calculated for narrower intervals (100 Hz). The value of ***V*** is given for each interval< where the upper line corresponds to the point ***K***, the low line corresponds to the point ***L***. The vibrodisplacement spectrums of the design No.1 are submitted on Fig.2 for some locations as ***?***$_0$(125, 137.5, 0), ***?***$_0$(230, 225, 0), ***?***$_0$(375, 137.5, 0), ***D***$_0$(230, 75, 0), ***E***$_0$(250, 137.5, 0).

Table I. - The distribution of *V* to frequency intervals in percents

Frequency range [Hz]	1		2		3		4	
	Plastic	Wood	Plastic	Wood	Plastic	Wood	Plastic	Wood
0-100	63,3	9,3	65,3	9,6	64,5	8,2	63,2	8,7
	58	8,7	61,7	9,2	60,6	7,7	59,6	8,3
100-200	15,3	38,5	14,7	34	13,5	36,5	13,1	36,4
	20,4	35,6	18,5	33	18,4	33,8	18,6	34,5
200-300	9,2	17,5	9,1	20	9,9	18,2	11,2	12,8
	8,8	21,9	8	21	7,8	18,2	8	15
300-400	4,4	9,1	3,7	9,5	3,9	7,4	4	9,5
	4,4	8,4	4,4	9,7	4,7	10,2	5,2	9,5
400-500	2,9	8,1	2,8	8	2,7	9,8	2,7	11
	2,9	7,5	2,6	8,3	3,3	8,6	3,2	10,2
500-600	1,9	5,2	1,9	6,3	2,3	5,9	2,5	6,7
	2,1	6,8	1,8	6,3	2	7,8	2	7,8
600-700	1,3	4,2	1,2	4,3	1,5	4,9	1,5	4,8
	1,5	3,7	1,3	4,2	1,4	4,4	1,4	4,9
700-800	0,9	2,9	0,7	3,2	0,9	3,4	0,9	4,3
	0,9	2,6	0,8	3,6	0,9	3,4	1	3,7
800-900	0,5	2,9	0,4	3	0,5	3,3	0,5	3,3
	0,5	2,6	0,5	2,8	0,5	3,2	0,6	3,3
900-1000	0,3	2,3	0,3	2,3	0,4	2,4	0,4	2,4
	0,4	2,2	0,3	2,1	0,4	2,5	0,4	2,8
0-1000 (100%)	**187**	**178**	**194**	**173**	**181**	**167**	**173**	**158**
	180	**181**	**194**	**172**	**178**	**168**	**171**	**158**

The analysis of the results shows the weak influence of the position of the excitement point of the oscillations on the distribution of ***V*** on frequency range. It is typical for all configurations and structure materials under study. The sounding boards No.3 and No.4 have the most even distribution of ***V*** in the reviewed frequency range. In the case of the isotropic structure the low frequencies are responsible for the value of a displaced air volume. Already in range up to 100 Hz the amount of the displaced air volume is 58-70% of the total air volume ***V*** (0-1000 Hz). Anisotropic structures show more even distribution of oscillation energy on frequency range.

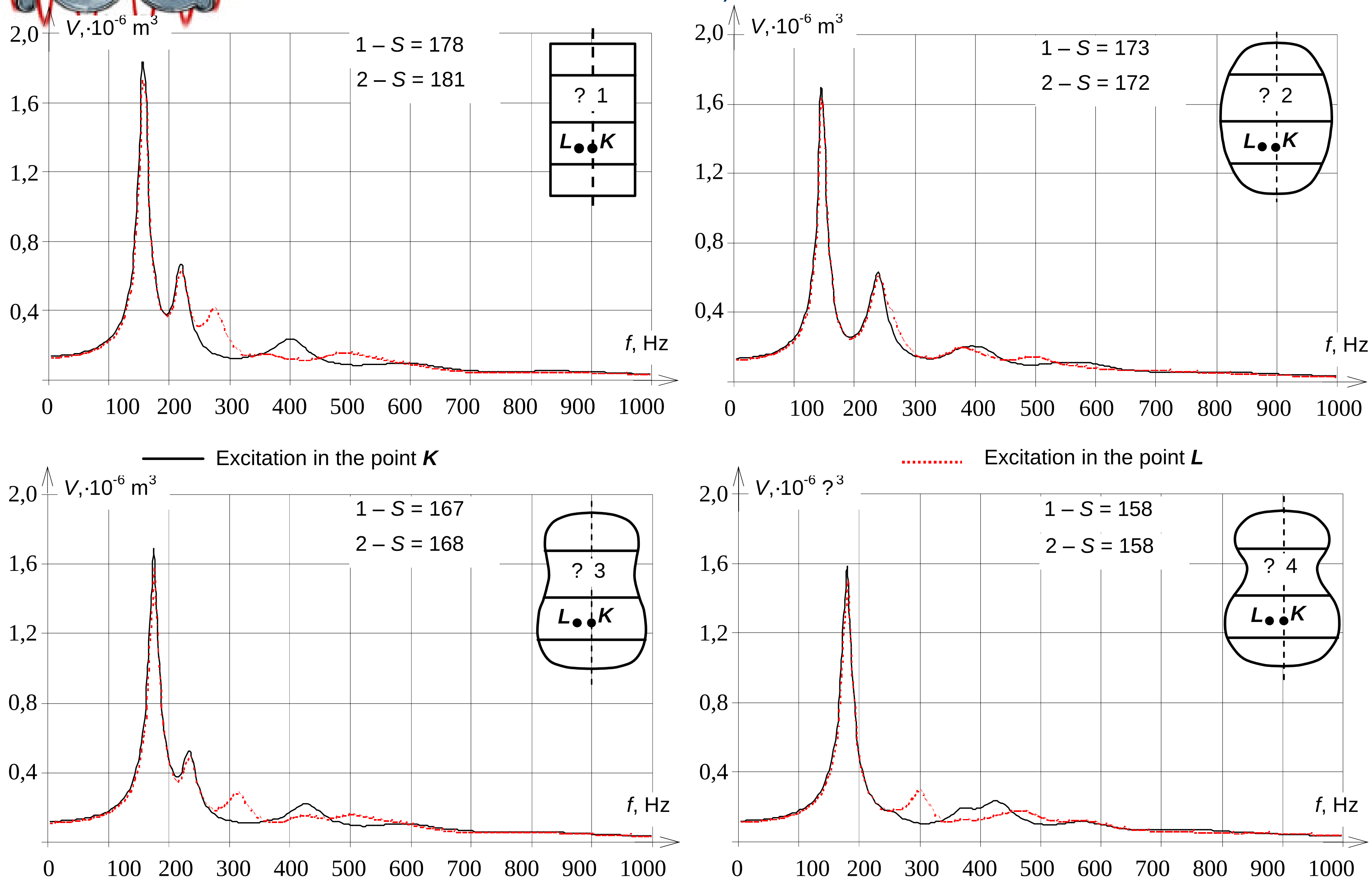

Figure 1. - The curves of a displaced air volume versus oscillation frequency for the sounding boards

SOUND FIELD OF THE SOUNDING BOARD

Based on obtained amplitudes of vibrodisplacements the field of vibrovelocities was defined for each resonance frequency. In the case of harmonic oscillations there is the simple relation between vibrovelocity and vibrodisplacement like $u_0(x, y) = w(x, y)\cdot 2\cdot p\cdot f$. The tube of rectangular cross-section of the following configuration is studied. The oscillator representing the configuration No.1 excited in-phase with driving force is located at the start of the tube. Velocity of the median surface is the harmonic function of $u = u_0(x, y)\cdot \sin(Wt)$. The tube is considered to have a perfect rigid walls and filled with air (fig. 2). The braces on fig.2 are hatched and numbered by 1, 2 and 3. Its coordinates are $?_1$ = 90 mm, $?_2$ = 255 mm and $?_3$ = 370 mm respectively.

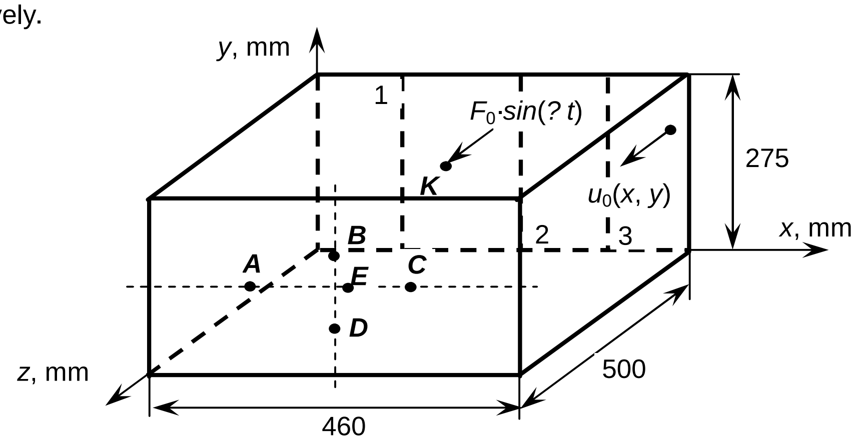

Figure 2. - Scheme of the sound oscillations excitation by vibrations of the wall (sounding board)

The sound field of plane oscillators always consists of the near filed, caused by cross-flowing of medium and pressure leveling, and the far field, being responsible for energy transfer of elastic waves. In the case of bending vibrations wave-length being less than the wave-length of elastic wave in the air there is no sound radiation and the plate oscillates in acoustic shortcut mode. Non-radiated energy stays linked to the plate in the form of kinetic energy of added air mass. The frequency range of 64-200 Hz being studied hence the wave-length of elastic wave in the air varies from 6 to 1,5 meters respectively. At the same time the wave-length of bending vibrations is less by order therefore only the near field is generated, which is responsible for low-frequency sounds. The air pressure is derived from points ***?***(125; 137,5; 0,5), ***?***(250; 137,5; 0,5), ***?***(375; 137,5; 0,5), ***D***(125; 75; 0,5), ***E***(230; 225; 0,5) by gas dynamic equations numerically solved by “FlowVision” application. For a graphical representation of results the logarithmic scale was used. On an ordinate axis the difference of levels of vibrodisplacements and air pressures in dB was sidetracked like

$$\tilde{w} = 20Lg\left(\frac{w}{w_0}\right), \qquad \tilde{P} = 20Lg\left(\frac{P}{P_0}\right). \qquad \text{(Eq. 6)}$$

Here w_0 is minimal obtained value of vibrodisplacement, P_0 = $2\cdot 10^{-5}$ H·?$^{-2}$ is threshold level of acoustic pressure.

The analysis of results derives that low levels of vibrodisplacements causes in most cases low values of overpressure. Nevertheless there is the contrary fact. Slight vibrodisplacements on the sixth resonance frequency for points ***?***$_0$, ***?***$_0$ and on the fourth resonance frequency for points ***?***$_0$, ***D***$_0$ exite acoustic field intensity of 15-30 dB for points ***B***, ***C***, ***D***, ***E***. As it is showed on fig. 2 the increasing of frequency causes the decreasing of overpressure amplitude. Thus the near field reduces so the far acoustic field begin to prevail. Also the pressure field generated by the oscillated plate built in fixed rigid wall of big-sized wall has been investigated. For that case pressure spectrums were made for some points located 0,5 meters far from the ***?xy*** plane. Obtained results noticeably differ from fig. 3 and show the homogeneity of pressure levels as for points ***A***, ***B***, ***C***, ***D***, ***E***, so for others on the same plate which are more distant from the oscillator hence possessing less intensity of acoustic field. However, as the frequency grows the overpressure levels become approximately equal for all points.

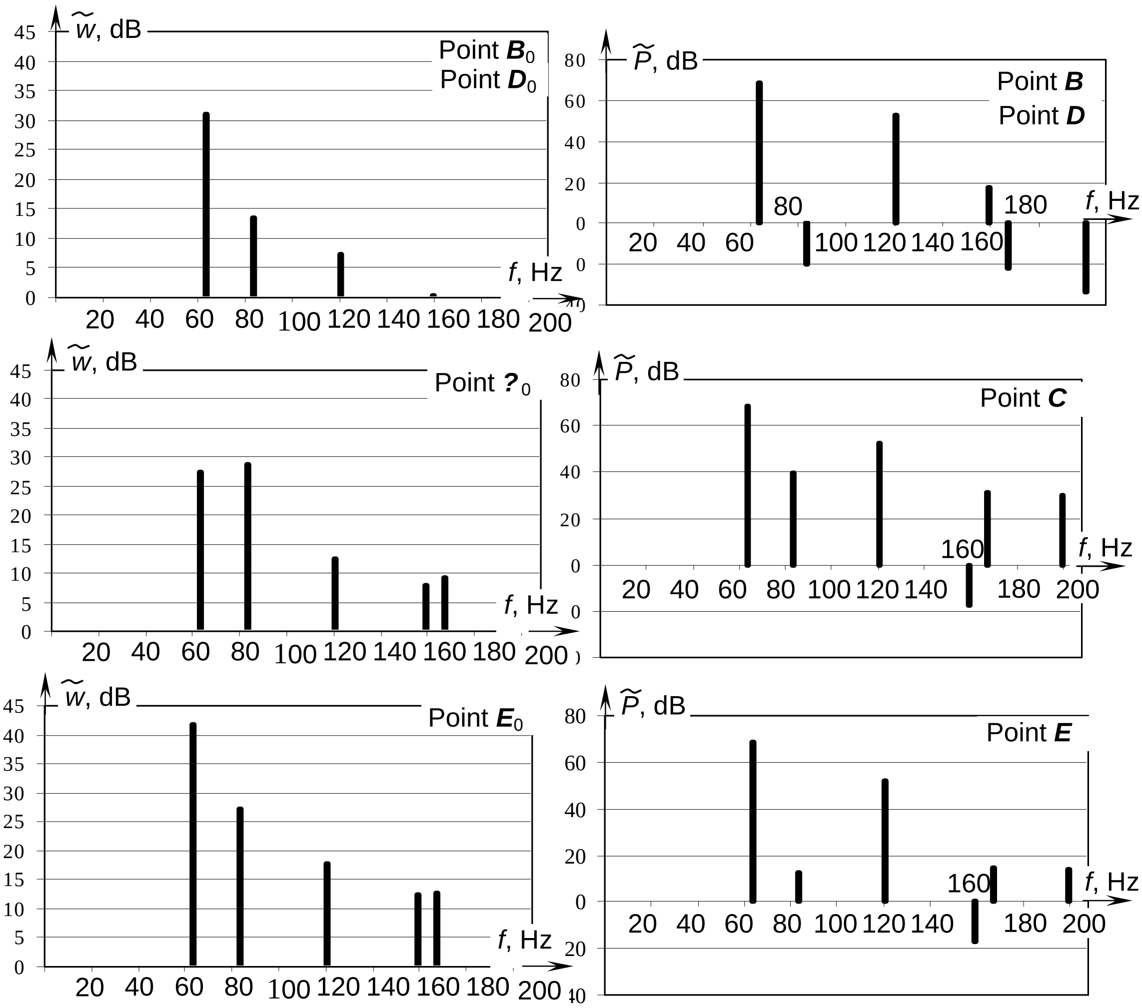

Figure 3.- The vibrodisplacement spectrums (left) and the acoustic pressure spectrums (right)

CONCLUSIONS

Complex configuration of the plate causes the resonance curve to be more even. According to [7] homogeneity of resonance curve is an important characteristic of MI quality. Hence plates of complex configurations are more suitable for producing of sounding boards. Dynamic characteristics of a sounding board serve as a sufficient condition for estimation of generated acoustic fields only in definite conditions such as vibrations occur in low-frequency range and a room for sound expansion is limited.

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