Different Zakharenko waves in layered and quantum systems

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ABSTRACT
This presentation discusses different Zakharenko waves (ZW) such as the non-dispersive ZW, dispersive leaky ZW, slow surface ZW and ultrasonic surface ZW, as well as their possible applications. The non-dispersive Zakharenko waves can be found in complex systems such as different layered and quantum systems. Existence examples of the non-dispersive Zakharenko waves in both fundamental modes of Lamb type waves propagating in [110] propagation direction of non-piezoelectric cubic crystals are introduced. It is also thought that one non-dispersive Zakharenko wave can exist in each energy branch of quantum elementary excitations. The leaky Zakharenko waves with the shear-horizontal (SH) polarization represent an analog of the leaky Sezawa waves possessing the in-plane polarization. The slow surface Zakharenko waves originating at non-zero wavenumber can be found in layered systems in which Love waves can also exist, but using an additional condition for the shear elastic constants. The ultrasonic surface Zakharenko waves were recently discovered in layered systems, consisting of cubic crystals with strong piezoelectric/piezomagnetic effect, and represent an analog of dispersive Bleustein-Gulyaev waves, which cannot propagate in piezoelectric cubic crystals. Also, there are many other possibilities to find different surface and non-surface waves caused crystal anisotropy that is discussed.

PACS: 67.40.-w, 67.40.Db, 67.40.Pm, 67.80.Cx, 67.90.+z

KEYWORDS: dispersive Rayleigh and Bleustein-Gulyaev type waves, Love and Lamb type waves, leaky Sezawa waves, layered and quantum systems, non-dispersive Zakharenko waves, leaky Zakharenko waves, slow surface Zakharenko waves, ultrasonic surface Zakharenko waves, other possible existence of waves with the shear-horizontal and in-plane polarizations.

PRESENTATION
This presentation is aimed to familiar a wide spectrum of Acousticians with recent discovery of several new waves, which supplement already well-known and commonly used dispersive and non-dispersive acoustic waves in monocrystals (surface Rayleigh and Bleustein-Gulyaev type waves) and in different layered structures (dispersive Rayleigh and Bleustein-Gulyaev type waves, leaky Sezawa type waves, and Love and Lamb type waves). Therefore, it is possible to list the recently discovered new waves and the other possibilities.

Ultrasonic surface Zakharenko waves. It was numerically discovered new dispersive shear-horizontal surface waves [1] called the ultrasonic surface Zakharenko waves (USZW), which can exist in some suitable propagation directions of piezoelectric coated crystals satisfying the condition of perpendicularity between the wave propagation direction and an odd order symmetry axis, in which the dispersive surface Bleustein-Gulyaev waves cannot exist, for example, in layered structures, consisting of piezoelectric cubic crystals ([101] propagation direction for both media). The ultrasonic surface Zakharenko waves can be used in different technical devices, like dispersive Bleustein-Gulyaev type waves are used, and in addition to seven-partial Love type waves (LTW7). Also, unusual modes of Love type waves were found in the layered structures consisting of two piezoelectric cubic crystals. Dispersive Bleustein-Gulyaev type waves were recently calculated in Ref. [2], noting that further experimental investigations are required. The non-dispersive USZW-waves in piezoelectric cubic crystals were also studied in Ref. [3].

Possible existence of non-dispersive SH-waves in (multi)-layered structures. Concerning piezoelectric cubic crystals, interesting solutions were analytically found in Ref. [1] studying [101]-propagation direction in the crystals. The solutions correspond to the phase velocity \( V_{ph0} = V_{st}a_K \) with \( a_K = 2[K(1 + K^2)1/2 - K^2]1/2 \) being lower than the speed \( V_t = V_{st}(1 + K^2)^{1/2} \) of the bulk shear-horizontal (SH) wave \( (V_{st} = (C_{66}/\rho)^{1/2}) \). A strong dependence \( V_{ph0}(K^2) \) on the so-called static coefficient of electromechanical coupling (CEMC) \( K^2 \) was found. It was also found that for the phase velocity \( V_{ph} < V_{ph0} \) there are all complex roots for any \( K^2 \) including the special case of \( K^2 = 1/3 \) where \( V_{ph0} = V_t \). And for \( V_{ph0} < V_{ph} < V_t \) the roots depend on \( K^2 \); the roots are pure imaginary in monocrystals with \( K^2 < 1/3 \), but real for strong piezoelectric cubic crystals with \( K^2 > 1/3 \). The interesting feature is a very slow velocity \( V_{ph0} \) in weak piezoelectrics with \( K^2 < 1\% \) (even \( K^2 < 1\% \)). The \( V_{ph0} \) calculation can be useful for finding new shear-horizontal (SH) surface waves.
Unusual modes of LTW7-waves. The unusual LTW7 modes were also found and discussed in Ref. [1], which can be met as a crystal defect, like dispersive Bleustein-Gulyaev type waves [2].

The non-dispersive Zakharenko type waves. It is thought that the non-dispersive Zakharenko type waves representing extreme points of the phase velocity \( V_{ph}(kd) \), where \( k \) is the wavenumber and \( d \) is the plate thickness, can be found in the lowest-order modes of Lamb type waves, when the waves are studied in commonly used [100] and [110] propagation directions in crystals, for instance, non-piezoelectric cubic crystals (metals). It is also thought that the non-dispersive Zakharenko type waves cannot exist in the modes of Lamb waves propagating in isotropic plates that must be verified in experiments. Figure 1 shows the fundamental modes of Lamb type waves propagating in a silicon plate. Figure 2 shows the existence possibility of several non-dispersive Zakharenko waves in such the plates, taking into account anisotropy for the Si cubic crystal. Note that the non-dispersive Zakharenko waves split a dispersive mode into several sub-modes. The dispersive ultrasonic Lamb type waves are suitable for the wireless non-destructive inspection of aircraft Al-wings [4], where used LbTW-modes must be as weakly-dispersive as possible, because dispersion often leads to wave packet shape change and spreading that usually complicate the signal interpretation. Ref. [5] discusses the non-dispersive Zakharenko waves. It was also studied the non-dispersive Zakharenko waves in quantum systems, such as both the bulk and surface elementary excitations (BEEs and SEEs) in the liquid helium-II at low temperatures [6]. It is thought that each BEEs’ energy branch possesses one corresponding non-dispersive Zakharenko wave. The experimental work of Ref. [7] discusses existence of the Cooper pairing phenomenon in the positive roton energy branch of the BEEs’ energy spectra, see also Ref. [8].

The wave phenomenon called the non-dispersive ZTW-waves can exist in many structures [5], where dispersive waves can propagate, and can be mathematically defined by the following formulas using \( (k – k_0) \) or \( k \) and \( \omega_0 = \text{const} \) in the phase velocity \( V_{ph} = (\omega – \omega_0)/(k – k_0) \) (it is possible to take \( \omega_0 = 0 \) for simplicity meaning zero potential energy, for example, \( E_p = \hbar \omega_0 \) with \( \hbar \) being the Planck’s constant):

\[
\frac{dV_{ph}}{dk} = V_g \left( \frac{dV_{ph}}{d\omega} \right) = 0; \quad (1)
\frac{dV_{ph}}{kd} = (V_g – V_{ph})/(k – k_0); \quad (2)
\frac{dV_{ph}}{d\omega} = V_{ph}(1 – V_{ph}/V_g)/\omega. \quad (3)
\]

It is noted that \( \omega_0 \) position on the energy scale must be determined from an experiment. The first relationship between the derivatives of the phase velocity in equation (1) shows that there is independence of the phase velocity \( V_{ph} \) on both the angular frequency \( \omega \) and the wavenumber \( k \). It is noted that dispersive waves are defined as dependence of the \( V_{ph} \) on both the frequency \( \omega \) and wavenumber \( k \). The formulae in equations (2) and (3) clarify that the equality of equation (1) occurs when the phase and group velocities are equal in dispersion relations for the wavenumber \( k \neq 0 \) (> 0) and \( k < \infty \). It is noted that the group velocity \( V_g \) cannot be equal to zero, except the situation when there is the following straight line behavior of the group and

![Figure 1](image1.png)

**Figure 1.** The phase velocity \( V_{ph}(kd) \) for fundamental modes of Lamb (type) waves.

![Figure 2](image2.png)

**Figure 2.** The non-dispersive Zakharenko waves representing extreme points of the phase velocity \( V_{ph}(\omega d) \) (a) and \( V_{ph}(kd) \) (b) shown by the big points for a fundamental mode of Lamb type waves ([110] propagation direction for Si).
phase velocities: \( V_p(k) = 2V_{ph}(k) \) at \( k \to 0 \) or \((k - k_0) \to 0\) corresponding to a free quasi-particle existing in vacuum (or even forming “quasi-vacuum” that is possible to suggest), where \( k_0 \) is a non-zero wavenumber for zero kinetic energy \( E_k = \hbar^2(k - k_0)^2/2\mu \) that frequently occurs in quantum systems.

**Existence possibility of new supersonic surface wave.** It was analytically shown the possibility to find in crystals the other type of surface waves [9], representing a new supersonic surface wave polarized in the sagittal plane, like the surface Rayleigh waves are polarized, with the \( V_{ph} \) being higher than the speed \( V_L \) of the bulk longitudinal wave. Also, the anisotropy coefficient \( C^2 = [(C_{11} - C_{33})(C_{33} - C_{55}) - (C_{13} + C_{55})]/[(C_{33}C_{55})] \) [5, 9] was introduced representing an universal characteristics for in-plane polarized waves propagating in monocrystals and layered systems. It was also found that the surface Rayleigh type waves can exist, if there is the following condition for negative \( C^2 > -1 \) \( C_{11}/C_{33} - 2(C_{11}/C_{33})^{1/2} \). Ref. [10] further develops theory of wave propagation using the anisotropy term and studies wave propagation with the in-plane polarization in layered structures.

**Dispersive leaky Zakharenko waves.** It was numerically discovered the existence possibility of a new type of dispersive leaky waves [11] in layered systems with polarization, like the Love wave polarization. These new leaky type waves were called dispersive leaky Zakharenko type waves. It is noted that dispersive leaky Sezawa type waves with the Rayleigh-wave polarization, as well as surface Rayleigh type waves, are readily observed with the same experimental techniques, for example, see in Ref. [12].

**Supersonic surface Love type waves.** It was found that supersonic surface Love type waves [13] can exist in layered systems with the anisotropy factor \( \alpha = (C_{44}C_{66} - C_{46}^2)^{1/2}/C_{44} \), in which such anisotropic materials for substrates as Muscovite, Phlogopite, and Biotite (common micas) are used. Such shear-horizontal supersonic surface LTW-waves can propagate with the \( V_{ph} \) being higher than the speed \( V_L \) of the bulk longitudinal wave for Diamond, \( V_L \approx 17500 \) m/s representing the fastest known velocity in Acoustoelectronics. Also, the paper [13] offers a method for all-round automation of filter and sensor characterization as well as production.

**Slow surface Zakharenko waves.** It was discovered two new types of surface waves: slow surface Zakharenko modes [13, 14] possessing slow speeds, single modes, and the so-called anti-plane polarization with the \( V_{ph} \) being lower than the LTW phase velocity. One new type of surface waves can also exist when the LTW-waves can not propagate. The slow surface Zakharenko waves can be used for sensor and filter applications, like asymmetric Lamb waves (flexural plate waves) with the in-plane polarization are used generally operating in the 1-10 MHz frequency range. In general, technical devices are based on waves with a velocity lower than that of sound in liquids. Currently, there is a great interest in Capacitive Micromachined Ultrasonic Transducers (CMUTs) of microelectromechanical system (MEMS) structures on the Lamb type waves in integrated circuit (IC) technology. It is thought that the CMUTs can be also done on the slow surface Zakharenko waves that can be even technologically preferable.

**CONCLUSIONS**

It is thought that the discovered new waves have a great potential to be used in technical devices such as filters and sensors on (non)-dispersive surface acoustic waves (SAWs), because they supplement already existing well-known and commonly used SAWs and leaky waves. Also, it is welcomed additional discovery of several new acoustic (and not only acoustic) waves for the International Institute of Zakharenko waves, in which a great attention will be paid to existence of the non-dispersive Zakharenko waves in complex structures such as layered and quantum systems.

**ACKNOWLEDGEMENTS**

I would like to thank to the Board of the International Commission for Acoustics (ICA) for approving award of the ICA Young Scientist Conference Attendance Grant (ICA-ASA Grant for young acousticians) for me. Also, I thank to all researchers for their great interest in my research.

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