Basic study for the miniaturization of thermoacoustic cooler
~ Determination of an insertion position and channel radius of the stack ~

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ABSTRACT
To establish the design method for the miniaturization of the thermoacoustic cooler, the experimental discussions are performed. The straight acoustic-tube is used then the acoustic power is supplied from the end of the tube. The ceramic stack which consists of a lot of narrow channels is inserted in the tube. By changing the insertion position and channel radius of the stack, the decrease of temperature at the end of the stack is observed as the energy conversion parameter. A non-dimensional parameter $\omega \tau$ is determined by the resonant frequency and the channel radius of the stack. The cooling performance is discussed several values of $\omega \tau$. It is found that the insertion position, at which the best cooling performance is obtained, is depended on the value of $\omega \tau$. The value of $\omega \tau$ indicates the ratio of the standing wave mode and the traveling wave mode in the tube that contribute heat exchange in the stack. Hence, it is confirmed that the best insertion position of the stack can be designed by the given frequency and channel radius of the stack, so that $\omega \tau$ is the important index for the miniaturization of the thermoacoustic cooler.

INTRODUCTION
A thermoacoustic cooler applying thermoacoustic phenomena has been proposed as a cooling system. A loop tube is one of the thermoacoustic coolers. The loop tube does not employ harmful refrigerants such as chlorofluorocarbons. Further, this system can utilize waste heat as an energy source. In addition to such advantages, the system works without any moving parts.

The loop tube consists of a prime mover to convert the thermal energy to the acoustic energy, and a heat pump to convert the acoustic energy to the thermal energy. A prime mover and a heat pump have a stack having very narrow channel. The heat exchange in the stacks is very important for the energy conversion between thermal energy and acoustic energy.

we have been studying about the heat exchange in the stack of heat pump using acoustic-tube connected compression driver, for example a loudspeaker etc.. From our past results, we obtained the results that the best insertion position of the stack varies by a non-dimensional parameter $\omega \tau$.

When we select the insertion position of the stack, the phase difference $\phi$ (between the sound pressure and the particle velocity in the acoustic-tube) in the acoustic-tube is determined by the insertion position. This phase difference $\phi$ determines sound field contributing to the heat exchange in the stack. Therefore, the insertion position of the stack is very important. When we downsize a thermoacoustic cooler, the determination of insertion position is difficult. Because the insertion position is more strict for the miniaturization of the thermoacoustic cooler. So this report describes basic study for the index of designing a thermoacoustic cooler to make it minimum size. We discuss relationship between $\omega \tau$, $\phi$, and the insertion position of the stack.

THEORY
In the heat pump, the heat exchange is performed between the working gas and the stack wall surface. Hence, for cooling performance, the efficient heat exchange between the working gas and the stack wall surface is very important. The non-dimensional parameter to describe $\omega \tau$ indicates efficiency of the heat exchange between the working gas and the stack wall surface. The parameter $\omega \tau$ are expressed by the following equation.
\[ \omega \tau = \frac{\omega r^2}{2\alpha} \]  
(Eq. 1)

Where \( r \) is channel radius of the stack, \( \alpha \) is the thermal conductivity, and \( \omega \) is the angular frequency of the sound wave. When the working gas and temperature is constant, \( \omega \tau \) depends on only the resonant frequency and the channel radius of the stack.

It is reported that, when the stack is inserted at the position where the phase difference \( \phi \) is a little larger than 0 \( ^\circ \), the heat exchange in the stack becomes efficient [1, 6, 7]. Because, both the modes of standing wave and traveling wave is used for the heat exchange. Thus we should select appropriately the phase difference. For the efficient heat exchange in the stack, the insertion position of the stack should be determined in consideration of \( \phi \) and \( \omega \tau \).

MEASUREMENT

The block diagram of measurement system is illustrated in Fig. 1. The driver side of acoustic-tube is defined as forward, and the end side is defined as backward. We moved gradually the insertion position of the stack from the middle of the acoustic-tube to backward, and measured a change of temperature decrease on the cooling point. The cooling point is back-end of the stack. This temperature decrease is measured using K-type thermocouples. The cooling point of the stack is defined as the insertion position of the stack. The phase difference between the sound pressure and the particle velocity is evaluated by two-sensor power measurements [8, 9].

The acoustic-tube used in this experiment is designed having a closed end. Inner diameter of the acoustic-tube is 0.042 m. The acoustic-tube was provided sound wave chosen frequency for the one-wavelength mode. The length of stack is set as the constant ratio to the one of the acoustic-tube. We choose the value of \( \omega \tau \) by adjusting the driving frequency and channel radius. So that we evaluate for several value of \( \omega \tau \), we measured a change of temperature decrease under the following three driving frequencies. For 42.5 Hz, the length of acoustic-tube and stack is set as 8.02 m and 50 mm, respectively. The value of \( \omega \tau \) is adjusted as 0.7 and 1.3 by changing channel radius. For 77.5 Hz, the length of acoustic-tube and stack is set as 4.50 m and 30 mm respectively. The value of \( \omega \tau \) is adjusted as 1.3 and 1.7 by changing channel radius. For 100 Hz, the length of acoustic-tube and stack is set as 3.30 m and 15 mm, respectively. The value of \( \omega \tau \) is adjusted as 1.7 by changing channel radius.

RESULTS

The several temperature decreases was measured by changing the insertion position and \( \omega \tau \). It is assumed that the peak-position of temperature decrease is the efficient position for the heat exchange. The peak-position was observed in each condition, and it is confirmed that the peak-position varies according to \( \omega \tau \). To make these results simply, the peak-position was normalized by the each length of acoustic-tube. The normalized insertion positions are shown in Table 1 with the value of \( \omega \tau \). The peak-position is discussed by using the normalized position. Fig.2 shows the distribution of phase differences by the measured value.
Table 1. Normalized insertion position of the stack.

<table>
<thead>
<tr>
<th>Length of tube [m]</th>
<th>$\omega \tau$</th>
<th>0.7</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.02 (42.5Hz)</td>
<td>Insertion position [m]</td>
<td>4.1</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>0.51</td>
<td>0.53</td>
</tr>
<tr>
<td>4.50 (77.5Hz)</td>
<td>$\omega \tau$</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>Insertion position [m]</td>
<td>2.39</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td>3.30 (100Hz)</td>
<td>$\omega \tau$</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Insertion position [m]</td>
<td>1.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Normalized</td>
<td>0.54</td>
<td></td>
</tr>
</tbody>
</table>

**DISCUSSION**

The normalized position is categorized by the value of $\omega \tau$. When $\omega \tau$ is 0.7, the normalized position of the temperature decrease exists on 0.51 for 42.5 Hz. When $\omega \tau$ is set as 1.3, two conditions are realized. The peak-position exists on 0.53 for 42.5 Hz and 0.53 for 77.5 Hz. For the case of $\omega \tau$ is 1.7, there are also two conditions are existed. The peak-position exists on 0.55 for 77.5 Hz and 0.54 for 100 Hz. As the value of $\omega \tau$ increases, the peak-position gradually moved from 0.51 to 0.55. It is found that when $\omega \tau$ is set the same value as 1.3 or 1.7, the peak-positions exist on the same normalized position, although the driving frequency is quite different. We considered that these results were caused by the phase difference $\phi$.

The phase difference $\phi$ is expressed on the normalized position as shown in Fig.2. The distribution of phase differences are overlapped and similar, although the driving frequencies are different. The normalized positions exist from 0.51 to 0.55. It is confirmed that when the normalized position moves from 0.51 to 0.55, the phase differences are changed from 0° to -90°. For example, when the value of $\omega \tau$ is chosen as 0.7, the phase difference of the peak-position is selected near 0°. Therefore, in this condition, the traveling wave mode is dominant. On the other hand, when the value of $\omega \tau$ is chosen as 1.7, the phase difference of the peak-position is selected near 90°. So in this case, the standing wave mode is dominant. As the value of $\omega \tau$ increases, the large value of the phase difference is selected. It is regarded that the peak-position is determined by $\omega \tau$. By these results, we can propose that $\omega \tau$ is the basic index for designing a thermoacoustic cooler.

**CONCLUSIONS**

For the efficient heat exchange in the stack of a heat pump, relationship between the phase difference $\phi$, a non-dimensional parameter $\omega \tau$, and the insertion position of the stack is
important. It is confirmed that insertion position should be determined in consideration of $\phi$ and $\omega \tau$. So we measured the several temperature decreases by changing the insertion position and the value of $\omega \tau$.

When $\omega \tau$ is set the same value, the peak positions of temperature decrease exist on the same normalized position although the driving frequency is quite different. The peak-positions exist on the position where the phase difference $\phi$ vary from $0^\circ$ to $-90^\circ$. As the value of $\omega \tau$ increases, the large value of the phase difference is selected. It is regarded that the peak-position is determined by $\omega \tau$. From these results, a non-dimensional parameter $\omega \tau$ can be used as basic index for determination of the insertion position for the thermoacoustic cooler.

References: