NUMERICAL STUDY ON SOURCE DISTANCE DEPENDENCY
OF HEAD-RELATED TRANSFER FUNCTIONS

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ABSTRACT
The source-distance dependency of head-related transfer functions (HRTFs) on a horizontal plane is investigated numerically, using the boundary element method. HRTFs for planar sources, which are independent of distance, and spherical sources, which are located at distances of up to 3.0 m at intervals of 1 cm, are simulated with a B&K 4128C scanned with laser and CT scanner. The HRTF spectrum is compared between source distances and directions using a head-centered coordinate system. The analysis reveals that the main features of the HRTF spectrum, such as spectral peaks and notches, show significant variations with source distance. Furthermore, an ear-centered coordinate system is also incorporated in order to clarify the effect of the angle of incidence with respect to the outer ear. The comparison between these two coordinate systems reveals that the source-distance dependency of the ipsilateral HRTFs can be mainly attributed to the outer ear angle of incidence, whereas that of the contralateral HRTFs is attributed to the presence of head. Results also show that, in an ear-centered coordinate system, the ipsilateral HRTFs do not depend strongly on source distance larger than 25 cm from the center of a head, whereas the contralateral HRTFs depend on source distances less than 1.8 m.

INTRODUCTION
Sound localization is an important function of human auditory system. With interaural cues, such as interaural level differences (ILD) and interaural time differences (ITD), as well as spectral cues, we can perceive the direction and distance of a sound source. These cues are involved in acoustic transfer functions from the sound source to each ear, i.e. the Head-Related Transfer Functions (HRTFs). HRTFs are dependent on listener's head and pinnae shapes.

The binaural technique is a promising approach to reproduce or synthesize a desired auditory space. This technique is achieved by controlling acoustic signals at a listener's ears, i.e. binaural signals [1]. Binaural signals can be synthesized by HRTFs, enabling development of a virtual auditory display (VAD) system (e.g., [2]). Generally, HRTFs are measured in an anechoic chamber for a finite number of source positions, yielding HRTF databases (e.g., [3]). Most HRTF measurements are performed for a variety of source directions, but for only a single distance to the head (e.g., 1 m). In a strict sense, VADs implementing such a limited set of HRTFs can present virtual sound sources only at fixed distance from the head. Assuming that HRTFs do not depend on source distance, one possible approach for presenting a virtual sound source of various distances is to give distance-level decay to HRTFs, i.e. distance interpolation.

However, as Morimoto et al. reported, an HRTF spectrum for sources within 1 m to the head shows significant variation with source distance [4]. The characteristics of the HRTFs for sources within 1 m of the head, however, have not been investigated thoroughly. With rigid sphere analysis, Brungart et al. showed that HRTFs within 1-m distance vary significantly with source distances, whereas HRTFs over 1-m distance are almost independent of source distance. ITDs are also independent of source distance, whereas ILDs depend significantly on source distance [5]. Duda et al. confirmed these findings by bowling ball measurements [6]. Brungart et al. reported that a nearer source yields a low-pass filtering effect; a nearer source increases an overall gain of ipsilateral HRTFs but decreases that of contralateral HRTFs; a frequency and azimuthal angle of spectral notches at higher frequencies vary with a source distance [7]. This phenomenon can be explained with “Auditory parallax” [8], i.e., the angle of incidence to the outer ear varies with source distance in a head-
centered coordinate system, even when the angle of incidence to the head center is held constant.

As described above, the source-distance dependency of the HRTF for source distances within 1 m is beginning to be revealed. The rigid sphere analysis, however, does not take outer ears into account. Furthermore, Brungart et al.’s works were conducted in sparse source-distance resolution, 0.12, 0.25, 0.5 and 1.0 m. In this paper, the source-distance dependency of the HRTFs is investigated in detail with numerically simulated HRTFs for spherical waves and planar waves using the Boundary Element Method (BEM).

**NUMERICAL SIMULATION OF THE HRTFS**

The numerical simulation method used in this work is based on the BEM, which enables a fast HRTF computation for various positions [9, 10] of point and planar source [11]. HRTFs for planar waves are approximately identical to HRTFs for spherical waves originating from infinite distance, and are therefore independent of source distance. Hence, the HRTFs for planar waves can be a benchmark for investigating the source-distance dependency of the HRTFs for spherical waves. HRTFs of Brüel & Kjær 4128C are simulated. A head model (Fig. 1a) was constructed from laser- and MicroCT-scanned surface geometrical data [10]. Both ear canals are blocked at their entrances. The head model consists of 28,000 triangular elements whose length is 5.64 mm in maximum. This element length corresponds to 1/4 wavelength of 15 kHz and 1/3 wavelength of 20 kHz.

HRTFs for spherical and planar waves originating from sound sources located in a horizontal plane are investigated. The head surface is assumed to be acoustically rigid. Transfer functions are calculated up to 20 kHz at every 86 Hz. Figure 1(b) illustrates the position of the point sources and planar source. The origin is set to the midpoint of both ears. \( \theta_h \) represents an azimuthal direction, in degrees. \( r_h \) represents a distance from origin, in meters. The right ear HRTFs are calculated for point sources located at \( \theta_h \) between -175° and 180° at increments of 5 degrees, and \( r_h \) between 0.1 m and 3.0 m at increments of 0.01 m. If \( \theta_h = 60, r_h = 0.1 \) corresponds to a point ~1 cm away from the surface of dummy head. Because the HRTFs for a planar source are independent of source distance, only azimuthal direction varies. The receiver is located adjacent to the entrance of the ear canal. All the calculated HRTFs are normalized by transfer functions at the origin in the absence of the head.

![Fig. 1 – Numerical condition](image)

**HRTF SPECTRUM VARIATION WITH SOURCE DISTANCE**

Figure 2(a) shows HRTFs for point sources (spherical HRTF) located at \( \theta_h = 60 \) and \( r_h = 0.13, 0.25, 0.5, 1.0 \) and 3.0 and HRTFs for planar sources (planar HRTF) located at the same azimuthal directions. Transfer functions up to 20 kHz are shown. As the source distance increases, the spectra of spherical HRTFs become closer to those of planar HRTFs. As the source distance decreases, the spherical HRTFs gain below 6 kHz. Above 6 kHz, the amplitude and frequency of their peaks and notches vary. Figure 2(b) shows the results for \( \theta_h = -60 \). As a source distance decreases, the spherical HRTFs attenuate at full bandwidth. Also, the frequency of notches around 2 kHz and 7–8 kHz vary with the source distance. Brungart et al.
also observed these features in measured HRTFs [7].

**Variation in spectral peaks and notches**

Figure 3 shows the spherical HRTFs at every 30-degree azimuthal direction. The abscissa represents frequency, and the ordinate represents point source distance. The angles written around the head denote azimuthal direction $\theta_h$. The gray shaded area indicates HRTFs that could not be calculated because the point source would have been located inside the head ($\theta_h = 0$, $r_h < 0.12$).

Results of $\theta_h = 30$–150 show that the HRTF spectrum gains at lower frequencies with decreasing source distance. The depth and frequency of the notches above 5 kHz changes with a source distance. Generally, except for a spectral notch at reference point b ($\theta_h = 90$, 10–18 kHz) and notches at $\theta_h = 150$, notches become deeper and peaks become higher with decreasing source distance, yielding HRTF spectra with significantly emphasized peaks and notches. The ipsilateral HRTFs of a rigid sphere do not have such notches at higher frequencies [7], indicating that these notches are produced by a pinna.

The spectral notch at reference point a ($\theta_h = 30$, 9 kHz) moves to lower frequency with becoming deeper, as the source distance decreases. Spectral notches at reference points a' ($\theta_h = 0$, 9 kHz) and a''($\theta_h = -30$, 9 kHz), as well as a notch at 20 kHz in the same figure, all have the same characteristics.

The notches at reference point a and at 20 kHz also appear at $\theta_h = 60$ for very small source distances ($r_h<0.02$).

A spectral notch at reference point b ($\theta_h = 90$, 10–18 kHz) is disappearing as a source distance decreases, its frequency, however, is almost constant unlike the notch at reference point a. This is because the incident angle to the outer ear ($\theta_e$) is constant regardless of a source distance as sources are located at just right ($\theta_h = 90$).

A spectral notch at reference point c demonstrates a typical example of a notch frequency that varies with a source distance. The notch appears at 5 kHz for $r_h = 3.0$, but moves to 6-7 kHz for $r_h = 0.1$. This phenomenon is due to a source-distance dependency of $\theta_e$. For $r_h = 3.0$, $\theta_h = 150$ corresponds to $\sim 151$ degrees of $\theta_e$, but for $r_h = 0.1$, the same $\theta_h$ corresponds to $\sim 193$ degrees of $\theta_e$, generating a 42-degree difference of $\theta_e$. In contrast, a spectral notch at around 19 kHz for the same $\theta_h$ does not vary its frequency for $r_h > 0.2$. 

Fig. 3 – Spherical HRTFs at every 30 deg. azimuth in a head-centered coordinate system.
Spectral notches in the contralateral HRTFs, such as those at reference points d, e and f, vary their depths and frequencies significantly with source distance. These notches are caused by wave diffraction and interference due to a head. Furthermore, a source-distance dependency of such notches is attributed to the presence of the head, as Brungart showed by a rigid sphere analysis [7]. Due to the presence of the head, a sound wave originating from a contralateral source does not reach directly to the receiver ear. Instead, it diffracts on the surface of the head and travels along the head surface to receiving the outer ear. Hence, the contralateral HRTFs spectrum does not depend on $\theta_e$, but does depend significantly on $\theta_h$.

Effect of an incident angle to the outer ear

The source-distance dependency of the HRTFs, especially in the ipsilateral HRTFs, is thought to be due to a variation of $\theta_e$. Figure 4 shows the spherical HRTFs calculated in the ear-centered coordinate system. The origin is located at the receiver point located at the entrance to the ear canal; $r_e$ is the distance between the source point and the receiver point. The gray shaded area indicates HRTFs that could not be calculated; this area is much larger than the corresponding area in Fig. 3. Note that, unlike Fig. 3, the ordinate ($r_e$) ranges from 0.01 m to 3.0 m.

When $r_e = 3.0$, the spherical HRTFs for both coordinate systems (Figs. 3 and 4) show almost the same characteristics of the planar HRTFs, indicating that these two coordinate systems do not yield any significant difference in this case. Unlike Fig. 3, spectral notches at $\theta_e = 0$ (reference point a) and 30 (reference point a’) do not depend significantly on source distance for $r_e > 0.1$; this is because $\theta_e$ does not vary with source distance. In contrast, the HRTFs gain prominently below 3 kHz for $r_e < 0.1$. Additionally, a spectral notch at reference point a moves slightly for $r_e < 0.1$. The former phenomenon is due to the presence of the head, as also shown for all ipsilateral HRTFs ($\theta_e = 0$–180). The latter phenomenon can be explained by a source-distance dependency for the outer ear. Brungart et al. reported that the outer ear’s frequency response varies with source distances within 4 cm to the outer ear [13]. Considering the dimensional ratio of the outer ear and the head, the outer ear would have only a slight influence on the source-distance dependency of the HRTFs, unless the source is located adjacent to the outer ear, in which case its influence would be significant.

The HRTF spectra for $\theta_e = 90$ are almost identical to those shown in Fig. 3. Note that the ordinate values are different between the two figures. A small spectral notch is seen at 10 kHz for a source located at 2 cm to the entrance to the ear canal ($r_e = 0.02$).

The spectral notch at reference point c for $\theta_e = 150$ case does not vary significantly in either depth or location for $r_e > 0.25$. Below 0.25 m, however, the notch becomes shallower, disappearing at $r_e = 0.05$. A notch at 19 kHz shows the same feature. Spectral notches appear at 4, 8, and 11–20 kHz for $\theta_e = 150$. A source location of $\theta_e = 150$ and $r_e = 0.03$ corresponds to the backside of the pinna. The HRTF for $\theta_e = 180$ has a similar spectral notch. Hence, these notches are produced if a source is located near the backside of the pinna.

Unlike in Fig. 3, deep spectral notches, shown for the cases of $\theta_e = -60$ (reference point e) and -120 (reference
point \( f \), move to lower frequency as source distance decreases, because these notches are due to the presence of the head. Not only source distance from the head but also the angle of incidence to the head center varies, as the origin is set to the outer ear. On the other hand, a result for \( \theta_e = -90 \) is similar to that shown in Fig. 3, as sources are located on the interaural axis.

As shown above, spectral notches at higher frequencies of the ipsilateral HRTFs spectra (\( \theta_e = 0–180 \)), which are produced due to the pinna, do not depend significantly on a source distance larger than tens of centimeters from the outer ear. This indicates that a source-distance dependency of higher-frequency spectral notches of the ipsilateral HRTFs is due to a changing incident angle to the outer ear (\( \theta_e \)) in a head-centered coordinate system. In contrast, the pinna increases its impact on the source-distance dependency of the HRTFs as the source becomes closer to it, generating spectral notches that are not seen for further sources.

**Objective evaluation**

Spectral distances (SDs) to the planar HRTFs are calculated for the spherical HRTFs in both coordinate systems. SDs are calculated as follows:

\[
SD = \frac{1}{K} \sum_{k=1}^{K} \left( \frac{20 \log_{10} \left( \frac{H_{\text{spherical}}(r, \theta, \omega)}{H_{\text{planar}}(\theta, \omega)} \right)}{2^\omega} \right),
\]

where \( K \) is calculated frequency; \( \omega \), angular frequency; \( H_{\text{spherical}}(r, \theta, \omega) \), an HRTF for a point source located at a distance of \( r \) and an azimuthal angle of \( \theta \); \( H_{\text{planar}}(\theta, \omega) \), an HRTF for a planar source located at an azimuthal angle of \( \theta \). \( \theta = \theta_h \) and \( r = r_h \) for a head-centered coordinate system, whereas \( \theta = \theta_e \) and \( r = r_e \) for an ear-centered one.

Results for each coordinate system are shown in Figs. 5(a) and (b), respectively. The abscissa represents the azimuthal angle, and the ordinate represents the distance from each origin point, i.e. the head center or the entrance to the ear canal. Colors closer to blue represent smaller SD values; dark blue part represents a SD of less than 1 dB. The gray shaded area indicates that a SD could not be calculated because the source is located inside the head. As shown in Fig. 5(a), SDs are larger for the contralateral HRTFs than for the ipsilateral HRTFs, revealing that the contralateral HRTFs depend more on source distance. This is because the source-distance dependency of the head has a relatively large effect, as the head is present between the source and the receiver ear. In contrast, SDs are generally smaller for the ipsilateral HRTFs as the presence of the head has a smaller effect. For the ipsilateral HRTFs, the largest distance when SDs are less than 1 dB is \( r_h = 0.38 \) at \( \theta_h = 170 \). For the contralateral HRTFs, it is \( r_h = 1.69 \) at \( \theta_h = -90 \).

In an ear-centered coordinate system (Fig. 5b), this distance is \( r_e = 1.79 \) at \( \theta_e = -110 \) for the contralateral HRTFs, indicating that the contralateral HRTFs have a significant dependency on source distance even in an ear-centered coordinate system. In contrast, it is \( r_e = 0.18 \) at \( \theta_e = 75 \) for the ipsilateral HRTFs. \( r_e = 0.18 \) at \( \theta_e = 75 \) corresponds to a source distance of about 0.25 m in a head-centered coordinate system. Hence, these results demonstrate that the ipsilateral HRTFs depend less on source distance, compared with a head-centered coordinate system.

Assuming that a source-distance dependency of the HRTFs is sufficiently small when SDs to
the planar HRTFs are less than 1 dB, source distance has negligible effects on the HRTFs for sources located further than 1.7 m in a head-centered coordinate system and 1.8 m in an ear-centered one. For the ipsilateral HRTFs, the criterion decreases to ~0.4 m in a head-centered coordinate system and ~0.2 m in an ear-centered coordinate system.

CONCLUSIONS
In this paper, the source-distance dependency of HRTFs is investigated in detail, with numerically simulated HRTFs both for a spherical and planar wave. In a head-centered coordinate system, HRTF spectra vary significantly with decreasing source distance. As the source distance decreases, the ipsilateral HRTFs gain at lower frequencies, and the contralateral HRTFs attenuate at full bandwidth. Spectral notches vary their depths, widths, and frequencies significantly. In an ear-centered coordinate system, higher-frequency spectral notches of the ipsilateral HRTFs, which are produced due to a pinna, do not vary significantly for sources located further than several tens of centimeters from the outer ear, and show a weak source-distance dependency. The comparison between both coordinate systems reveals that the source-distance dependency of higher-frequency spectral notches in the frontal and ipsilateral HRTFs can be attributed to changing angle of incidence with respect to the outer ear in a head-centered coordinate system. Furthermore, the source-distance dependency of a pinna is significant for a source adjacent to the pinna, producing spectral notches that are not seen for further sources.

SDs to the planar HRTFs are calculated in order to objectively evaluate the source-distance dependency of the HRTFs. Results show that SDs are less than 1 dB for a point source located further than 1.7 m and 1.8 m in a head-centered and ear-centered coordinate system, respectively. For the ipsilateral HRTFs, they are 0.4 m and 0.2 m, respectively.

From these results, we can conclude the following: The HRTFs for sources further than 2 m can be distance-interpolated from HRTFs in the same range based on a head-centered coordinate system. It is also reasonable that the ipsilateral HRTFs for sources located from tens of centimeters to 2 m can be distance-interpolated from those in the same range. For the contralateral HRTFs in the same range, however, careful handling is required, because their dependency on source distance is larger. When interpolating the HRTFs for nearby sources (within tens of centimeters), the source-distance dependency of the ipsilateral HRTFs can be reduced by using an ear-centered coordinate system. However for the contralateral HRTFs, the source-distance dependency is much larger due to the presence of the head, requiring more careful handling. Furthermore, the accuracy of distance interpolation decreases significantly because the HRTFs for sources adjacent to the pinna differ considerably from those for more distant sources.

References