INVESTIGATION ON HIGHER ORDERS OF SPHERICAL HARMONICS EQUATIONS FOR EFFICIENT ROOM-AcouSTIC PREDICTIONS

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ABSTRACT

This paper presents an improved boundary condition and a higher order model (SP 3) which are both derived by a simplified spherical harmonics method (SPN) to predict reverberation time and sound pressure distributions in enclosures. Systematic comparisons of room-acoustic prediction among the diffusion model with the original [Valeau et al., JASA, 119, 1504-1513 (2006)] and improved boundary conditions, the higher order model, a geometrical-acoustics model, and several well-established theories, suggest that the modified boundary condition significantly improves the accuracy of the room-acoustic prediction. This paper also discusses a compromise between prediction accuracy and computational expense when ignoring higher orders.

INTRODUCTION

Recently, a diffusion equation model has drawn attention in room-acoustic prediction. Ollendorff [1] first proposed the diffusion equation model to describe diffuse sound fields in enclosures. More recently, Picaut [2] and his co-workers [3-5] have extended the application of the diffusion equation model to a variety of space types, including elongated space, such as street canyons, single-space enclosures. Among those work, Ref. [3] proposes a generalization of the diffusion model including two boundary conditions: homogenous Neumann and mixed boundary condition. Meanwhile, the diffusion model is found to be only suitable for low absorption coefficients [3-5]. The subject of this work is to present an improved boundary condition, a higher order model as well as the relationship between the diffusion model and the higher order model. The higher order model is derived from a transport equation using a simplified spherical harmonics (SPN) method [6, 7] and can be easily reduced to the diffusion model. The improved boundary condition is also derived from a boundary condition of the transport equation using the SPN method to incorporate with the diffusion model and the higher order model. Since the diffusion model is the first order approximation of the transport equation [8], the higher order model is expected to be able to better predict the sound fields of enclosures.

Predicted reverberation times and sound pressure levels (SPLs) are compared among the diffusion model with both the original boundary condition and the improved boundary condition, the higher order model with improved boundary condition, and other classical theories for cubic rooms with uniformly distributed absorbing surfaces of varied absorption coefficients. For a long room, a geometrical-acoustics model is used for the comparison of the SPL distribution. The results show that the diffusion model and the higher order model with the improved boundary condition improve the room-acoustic prediction given by the diffusion model with the original boundary condition [1-5].

MODEL FOR ROOM-AcouSTIC PREDICTION

A previous work [1, 2] relies heavily on the diffusion theory [8]: a first-order approximation of the integral equation (transport equation) which describes the diffusion of light in scattered media. Inspired by the work from [6, 7] our work describes the approximation of the transport equation
and retains only up to the third-order approximation by a simplified spherical harmonics method, i.e., SP$_3$.

To derive the improved boundary condition, all of the absorption will be first considered in the boundary condition, thus only scattering terms left in the local equation. We imagine that there are numerous scattering objects attached very near the boundaries uniformly scattering sound particles. In this procedure, up to now no energy is lost in the space under investigation, since no absorption is included in the local equation. Next we assume that the sound particles will be specularly reflected by the walls with a probability $1-\alpha$ while $\alpha$ of the sound particles will be absorbed by the boundaries, with $\alpha$ being the absorption coefficient.

This boundary condition along with a compatible local equation describes the sound field formed by uniformly scattering boundaries. The boundary condition for diffusion equation and SP$_3$ equations are also derived using the simplified spherical harmonics method.

For the third order approximation (SP$_3$), the local equation is a set of diffusion-like equations with Laplacian operators expressed as

$$
\frac{1}{c} \frac{\partial \phi_0 (r,t)}{\partial t} - \nabla \cdot \frac{\lambda}{3} \nabla \phi_0 (r,t) = \frac{1}{3} Q(r,t),
$$

$$
\frac{1}{c} \frac{\partial \phi_1 (r,t)}{\partial t} - \nabla \cdot \frac{\lambda}{7} \nabla \phi_1 (r,t) + \frac{5}{9} \phi_2 (r,t) = -\frac{2}{3} Q(r,t),
$$
in $V$ (Eq. 1)

where $\lambda=4V/S$ is the mean free path length of the room, $c$ is the sound speed, $Q$ is the sound energy density of the source, $V$ is the volume of the room, $S$ is the surface area of the room, $\phi_1$ and $\phi_2$ are composite moments[6].

$$
\phi_0 (r,t) = \phi_1 (r,t) - \frac{2}{3} \phi_2 (r,t),
$$

$$
\phi_2 (r,t) = \frac{1}{3} \phi_2 (r,t),
$$

(Eq. 2)

where $\phi_0 (r,t)$ is the sound intensity in the room.

The improved third order boundary condition can be expressed as

$$
\left(\frac{1}{2} - \alpha\right) \phi_0 (r,t) + \frac{2}{3} - \alpha \nabla \phi_0 (r,t) = \frac{1}{8} \alpha \phi_2 (r,t),
$$

$$
\frac{7}{24} \alpha \phi_2 (r,t) + \frac{2}{7} - \alpha \nabla \phi_2 (r,t) = \frac{1}{8} \alpha \phi_1 (r,t),
$$
on $\partial V$ (Eq. 3)

where $\partial V$ is the surface of the room.

To obtain the first order approximation equations (diffusion equation), one just simply reduce eqs. (2) and (3) by setting $\phi_2$ to be zero, leading to the local equation and the improved mixed (first order) boundary condition, respectively

$$
\frac{1}{c} \frac{\partial \phi_0 (r,t)}{\partial t} - \nabla \cdot \frac{\lambda}{3} \nabla \phi_0 (r,t) = \frac{1}{3} Q(r,t),
$$
in $V$ (Eq. 4)

$$
\left(\frac{1}{2} - \alpha\right) \phi_0 (r,t) + \frac{2}{3} - \alpha \nabla \phi_0 (r,t) = 0,
$$
on $\partial V$. (Eq. 5)

**SIMULATION RESULTS**

This Section investigates two basics room shape variations with varying absorption coefficients on interior wall surfaces.
**Cubic room**

A cubic room \((x = 5 \text{ m}, y = 5 \text{ m}, z = 5 \text{ m})\) is modelled with uniform absorption coefficient ranging from 0.05 to 0.5. The source is in the middle of the room.

![Figure 1](image1)

*Figure 1.* Deviations of the reverberation times for a 5m*5m*5m cubic room calculated by Kuttruff’s formula and other methods. (1) stands for the Neumann boundary condition. (2) stands for the original mixed boundary condition (3) stands for the improved mixed boundary condition.

**Long room**

For a long room, the dimensions are 4m*4m*40m, the source is at (3, 2, 3) m, the sound power level is 100dB. A sketch of the configuration is shown in Fig. 2. Distribution of sound pressure level is calculated and is plotted along the line \(y=0.5\) (\(x\) is from 0 to 40m) and at the height of 1m, as shown in Fig.3. The absorption coefficient of one wall is 0.4, other walls are 0.9, highly absorptive. The modified model more agrees with the geometrical-acoustic model (ray-tracing).

![Figure 2](image2)

*Figure 2.* A long room with dimensions 4m*4m*40m, the source of sound is at (3,2,3), the sound pressure level is plotted along the line \(y=0.5\) m

![Figure 3](image3)

*Figure 3.* Comparison of sound pressure level distributions along \(y=0.5\) m by four different models in a long room with one wall surface of two different absorption coefficients 0.9 and 0.4

**CONCLUSION**

This work introduces extensions of the diffusion equation model recently applied in room-acoustic predictions. In approximating a transport equation, this work describes the simplified spherical harmonics method which generates less coupling equations compare with regular...
spherical harmonics equations. An improved mixed boundary condition is applied in room-acoustics, resulting in more accurate solutions. In compromising the model accuracy and computational expense, a third order model (SP3) is considered in order to better predict the sound field than the diffusion model (SP1). Through simulations, the diffusion model and the SP3 with new boundary condition yield more similar results to those estimated by the geometrical-acoustics method. The simulation results also indicate that SP3 model provide comparable accuracy with those of the diffusion equation but requires almost twice longer calculation time.

References: