EFFICIENT REPRESENTATION OF EDGE DIFFRACTION IMPULSE RESPONSES

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ABSTRACT
Edge diffraction impulse responses can be used together with image source models in modeling of room acoustics, to offer improved accuracy. Such impulse responses may have a long temporal decay, and an efficient representation of them would be beneficial to avoid computationally demanding convolution in auralization. A diffraction impulse response can be modeled roughly by using a simple low-order IIR filter. For more accurate modeling, it is possible to take advantage of the distinct temporal structure of such impulse responses. The response includes an initial decay part, a knee point where the response drops by a factor of 0.5, a second decay part, and an end point, where the response stops. We present a method which uses three simple IIR filters to model the impulse response efficiently. The first filter implements the first decay part, the second filter has a delayed output, which implements the first knee point. The summed response of the two filters implement the second decay part, and a third filter cancels the output of the system with a delay that corresponds to the end point. Edge impulse responses can be modeled accurately with this method, and the benefit of the method is discussed for different cases.

INTRODUCTION
A common method to model the acoustics of a space bounded by planar surfaces is the image source model, where the reflections from the boundaries are represented by secondary sound sources [1]. However, the method does not take into account diffraction, where the corners between the planar surfaces, or edges of the surfaces, also act as secondary sound sources [2]. The diffraction can be modeled accurately [3], and the secondary sound sources produced by diffraction have also been combined with the image source model [4].

The image source model can be implemented computationally effectively by modeling each reflection by a DSP structure, where the sound propagation path, attenuation, and frequency-dependent filtering is implemented by simple delays and low-order digital filters. However, the modeling of secondary sources produced by edge diffraction, or edge image sources, is not as straightforward. Typically the temporal response of edge image sources are relatively long, which leads to computationally demanding implementations. The impulse responses of edge image sources have a temporal response which has a smoothly fading slope, however, with one or two discontinuities. Such systematic behavior suggests that the edge impulse responses could be modeled with simplified computation. De Rycker modeled them with low-order low-pass FIR filters [5], and Lokki with warped third-order IIR filters [6]. However, as will be shown later, these solutions are appropriate only in cases where the discontinuities in the responses occur in a part of the response which has relatively low energy.

In this paper, we are examining a digital filter configuration which models the responses accurately both in the temporal and spectral domains. Each temporal discontinuity is modeled with a separate, delayed low-order IIR filter. The paper is organized as follows: The edge diffraction is presented first as a physical phenomenon, after which the common methods to model impulse responses with low-order filters are shown. After this, the technique developed in this work is presented for the responses generated by finite edges.
Figure 1: The geometry of a wedge, with external wedge angle $\theta_W$. The source and receiver have cylindrical coordinates $(r, \theta, z)$ as indicated. Two virtual half planes, $P_S$ and $P_R$ are used for illustrating the distances $m$ and $l$.

**EDGE DIFFRACTION**

An edge diffraction IR on discrete-time form, $h(n)$ can be written such that each sample value is given as an integral over two short segments of the edge, ranging from $z_{n-}$ to $z_{n+}$,

$$h(n) = -\frac{\nu}{4\pi} \sum_{i=1}^{4} \int_{z_{n-}}^{z_{n+}} \frac{\beta_i}{m l} dz,$$

(1)

where $\nu$ is the wedge index $= \pi/\theta_W$, $m, l$ are the two distances, from-source-to-edge-point, and edge-point-to-receiver, respectively, see Fig. 1. The $\beta_i$ terms are directivity functions, and further details on these expressions can be found elsewhere [3]. For each sample value, the numerical integration is carried out over the segment of the edge which gives a total path length, via an edge point, that corresponds to the IR time for that specific sample value. This implies that there will always be two segments that contribute to the sample value, one segment on each side of the apex point of the edge (the point with the shortest travel path). The summation of two segments is the cause of the knee point referred to above. For a finite edge, there will for a time range be edge segments only on one side of the apex point. Interestingly, the two contributions are exactly equally strong, so that the IR value steps down to half its value after this knee point. Then, at some later time instant, also the further edge end will be reached and the IR amplitude falls to zero. In case of symmetry such that the two edge end points give exactly equal path lengths, there will be a single step down to zero. These knee points are illustrated in Fig. 2.

**MODELING OF IMPULSE RESPONSES WITH LOW-ORDER FILTERS**

The simplest way to model a given impulse response is to realize it directly as a finite impulse response (FIR) filter, corresponding to linear convolution as the time-domain input-output signal mapping. The obvious disadvantage in modeling typical audio related responses is then that high FIR filter orders are required to capture the essential characteristics of the system. More profoundly, there are usually strong reasons to presume that the system to be modeled is inherently recursive, implying that an infinite impulse response (IIR) filter structure would be preferable. An edge diffraction impulse response can in general be characterized as a monotonically smoothly decaying function, implying that a very simple pole-zero IIR filter will suffice as its model. The corresponding prototype response would in fact be the response of a first-order all-pole filter; somewhat higher numerator and denominator orders are in practice required to match the phase of the response and to account for the particular decay profile. However, a typical edge diffraction impulse response exhibits multi-stage decay with essential discontinuities, which are inefficiently modeled by a single pole-zero model.

Many methods and software tools are available, e.g., in MATLAB, for IIR or pole-zero filter design with respect to a determined target impulse response. In the case of edge diffraction impulse responses, and in particular from the perspective of our proposed modeling configuration, it seems natural to choose a time-domain approach. The modeling can then be seen as a “curve fitting” problem, which is very intuitive from the design point of view. It is however also important to choose as a starting point a method that provides
the output-error between target and model response that is of interest. From a more practical point of view, Prony's method misplaces modeling capacity by exactly reproducing the first samples of the response at the least-squares optimization problem based on an equation-error configuration [7]. However, the essential the data, resulting in an iterative procedure that circumvents the non-linear optimization task inherent to output-error optimization. This is also why Steiglitz-McBride is valued over Prony in our case: it is really efficient parametrization in the least-square error sense. The well-known Steiglitz-McBride method [7] for pole-zero filter design was chosen, because it was considered to be a good alternative for accurate low-order approximations. This can be characterized by comparing it to a another classical approach, the so called Prony's method [8]. Both are “covariance methods” in the sense that no assumptions on the data outside the time-window of interest is presumed, which is advantageous for our particular application. In fact, also the parameter estimation algorithm is basically the same for the two methods: a linear and quadratic least-squares optimization problem based on an equation-error configuration [7]. However, the essential difference is that the Steiglitz-McBride method tries to mimic an output-error setup by sequentially filtering the data, resulting in an iterative procedure that circumvents the non-linear optimization task inherent to output-error optimization. This is also why Steiglitz-McBride is valued over Prony in our case: it is really the output-error between target and model response that is of interest. From a more practical point of view, Prony’s method misplaces modeling capacity by exactly reproducing the first samples of the response at the expense of poorer modeling of the tail of the response. Both methods are potentially unstable due to the covariance type of pole estimation, with an obvious accumulation of the probability of explosion in the case of the Steiglitz-McBride method, which is why Prony’s method is more frequently used for modeling audio related responses. Another characterization of the applicability of these methods is given in [9]. In our cases the Steiglitz-McBride performed clearly better and we did not encounter any stability or numerical inaccuracy problems.

**MODELING IMPULSE RESPONSES FROM A SINGLE FINITE EDGE**

As explained earlier, the ends of a diffractive edge produce discontinuities to the response, which include a knee point, where the response drops by a factor of 0.5, and an end point, where the response stops. The modeling of responses related to different geometries is considered in the following.

**Responses with an end point and knee points**

Let us consider first a case, where there is a knee point between start and end points of the response to be modeled, shown in Fig. 2. This kind of responses occur typically if the sound wave reaches points on both sides of the apex point of the edge including the apex point. If such a response is modeled using a single low-order IIR, the modeling of the knee points and end point is not possible, if the time difference between start and knee points is large enough.

In this paper, we have taken a new method to overcome this specific problem. Three low-order IIRs are used in the modeling, as is shown in Fig. 3 a. The filter IIR1 is designed to fit the smooth decay of the edge response, and IIR2 is used to produce the knee point. Correspondingly, IIR3 is used to produce the abrupt end point. The delays $t_{1,2,3}$ are used to temporally position the responses of IIRs to match with the original response. In practice, IIR1 is designed to model the response between the start and knee points. The infinite response of IIR1 is computed, and the response of IIR2 is designed to produce a response which will follow the original response between the knee point and the end point when summed with the IIR1 response. In

![Figure 2: A diffraction impulse response for a finite wedge. The first part (i) corresponds to when both sides of a wedge, around the apex point, contribute to the impulse response. In part (ii), only one side of the finite wedge is contributing, and in the last part, (iii), the furthest end of the wedge has been reached and the diffraction impulse response is zero.](image)
Figure 3: a) DSP structure used to model edge diffraction impulse responses. b) The responses of different parts of the structure when modeling an edge diffraction impulse response.

Figure 4: Edge diffraction impulse response modeled with single IIR filter of order (5,6) and with a three-filter DSP structure with filter orders of (2,3), (2,2) and (1,1). a) Temporal domain results. b) Difference between modeled responses and original in spectral domain.
similar fashion, the response of IIR\textsubscript{3} is designed to fit the phase reversed response IIR\textsubscript{1} + IIR\textsubscript{2} starting from end point. The output of the filter structure models the temporal responses accurately, as shown in Fig. 3 b.

Impulse responses for different edge geometries were computed using Svensson’s MATLAB toolbox \cite{10}, and modeled with the presented method. The current solution was compared with the response from a conventional IIR filter having the same order, which was typically of order (6,6). It was found that the current solution gives more accurate results than single-IIR solution, however, in many cases the effect was negligible. When the first knee point was relatively far in time from the start point of the response, also the single-IIR solution was accurate, since the level of the response at the knee point was low. If the knee point was situated just after the start point in time, its effect on the total response was strong. However, in such cases also the single-IIR model could model quite precisely the response, due to relatively high order of the filter. An example of such modeling is given in Fig. 4. It can be seen, that the three-IIR structure is more accurate in temporal and spectral domain, however, the effect is quite small.

**Responses with an end point and without a knee point**

When an edge impulse response contains only a start point and an end point, the structure presented in Fig. 3 can be used, however, filter IIR\textsubscript{3} is not needed. In practice, IIR\textsubscript{1} is designed to fit the response between the start and end points, and IIR\textsubscript{2} is designed to cancel the output of IIR\textsubscript{1} after the end point.

Some geometries were tested, and it was found that with this kind of responses the proposed modeling method provided prominently better results than single-IIR modeling in many cases. Especially when the response resembled a ‘trapezoid’ function, i.e., the response decayed slowly, the single-IIR modeling provided faulty results. This happened since the end point occurred in a part of the response which had high level, however, relatively far in time after the beginning point. In contrast, the two-IIR modeling approach provided accurate results with the same computational complexity. This is illustrated with one example shown in Fig. 5. It can be seen that the single-IIR model fails to reproduce the edge diffraction impulse response both in temporal and spectral domain, however, the response obtained with the two-IIR method is accurate in both temporal and spectral domains.

**FUTURE WORK**

In this paper, only the edge diffraction from a single edge was considered. At least in principle it is possible to use the presented method to responses containing responses from multiple reflection and diffraction components. Each discontinuity followed by a smooth section in the response would then be modeled with a delay and IIR filter. As an encouraging example of such an approach, measured room responses have been modeled based on a similar sequential partitioning of the response and a corresponding delayed parallel IIR.
Another task for future work in this scope would be to try to find a method to compute the parameters for three-IIR realization of edge diffraction from the geometrical parameters of source, receiver and edge shown in Fig. 1. The $\tau_{1,2,3}$ parameters can be found by simple geometrical computation. Finding the parameters of IIR filters would be a more demanding task.

CONCLUSIONS

A novel method to model edge diffraction impulse responses was presented in this work. An edge diffraction impulse response consists of onset, decay, knee point and end points. At the knee point the response is scaled abruptly by a factor of 0.5. The knee and end points are problematic with traditional filter designing methods. In the proposed method, a structure of delay and low-order IIR filter is used to model each discontinuity in the response. With this system, such responses can be modeled accurately both in the temporal and spectral domains. The method provides prominent enhancement in modeling accuracy when the response has a relatively high level when a discontinuity occurs, as with responses which have temporally slow decay.

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References


