Introduction and Applications of Phased Beam Tracing Method: Can We Interpret Low Frequency Response by the Particle Property?

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ABSTRACT
Everybody knows that there is the wave-particle duality in the characteristics of a sound field. Two different viewpoints on the sound propagation character evoked various analysis techniques which faithfully followed one of two principles. However, the phased beam tracing method (PBTM), originated from the particle approach, can be extended to the low-to-mid frequency analysis by considering the wave nature of sound as well. The diffraction can be dealt with by the uniform theory of diffraction and the interference can be accounted for by retaining the phase information in the beam tracing. The PBTM has many direct and potential applications. Calculations of a transfer function or an impulse response between source and receiver and the acoustic parameters in an enclosed 3-D space are typical application examples for the analysis, design, diagnosis, and refinement in acoustic field. Binaural simulation is possible for the virtual prototyping and design of an acoustic space. Identification of the source can be carried out by an inverse approach from the simulated acoustic transfer function, which can be called the far field acoustical holography. The PBTM has some beneficial points over the conventional wave-based methods and geometrical methods. The calculation time can be less than the wave-based methods. Besides, precision of the simulation can be far better than the conventional geometrical methods. So far, the method has demonstrated a good possibility to achieve a unified method to analyze a whole audible frequency range from very low to very high frequency ranges. Even if this method cannot fully cover the whole audible frequency range as a unified method, we believe that, at least, it can be used as a bridge to connect a mid-frequency gap between low-frequency modal methods and high-frequency geometrical acoustic methods in the room acoustics field.

A SUMMARY OF THE PRINCIPLE OF PBTM
The phased beam tracing method (PBTM) [1-3] is one of modified versions of the beam tracing technique by retaining the phase information during the sound propagation. Conventional geometrical acoustic methods cannot account for the wave nature of the sound at low frequencies. Phase information in the propagation model enables the inclusion of the interference phenomenon of sound in the analysis. The diffraction phenomenon which is dominant at low frequencies can be dealt with by the uniform theory of diffraction [4]. By tracing the beams emitted from a source, the sound field imposed to the target receiver can be detected in an enclosed room. Contributions from all beams are saved for each frequency. The scanning of a sound field is terminated after the assigned number of reflections is reached or the sound pressure is converged to a certain value. From the calculated pressure reflectogram, the steady-state response can be obtained simply by the summation of direct sound and successive transient components. If the steady-state pressures are computed for all frequencies within the frequency range of interest, a steady-state transfer function can be obtained. By taking the inverse Fourier transform to the transfer function, one can get an impulse response. In the simulation, boundary conditions should be prepared with a high precision.

APPLICATION EXAMPLES OF PBTM
A simple rectangular room and the calculated transfer functions are shown in Fig. 1. The source and receiver were located at (0.3, 0.3, 0.3) and (1.2, 1.2, 1.2), respectively, and the surface impedance of the walls was assumed constant as $Z_w = 4150 + 4150j$ rayl. The result in Fig. 1b shows a perfect agreement with that from the BEM. Through this example, we may say that,
with known wall reflection data, the PBTM can yield the same result with the wave based method, on condition that there is no geometrical characteristic which seriously makes the diffraction.

![Diagram of a rectangular room and the transfer function.](image)

Fig. 1. A rectangular room and the transfer function. (a) Room model, (b) magnitude of TF, |TF|.

Figure 2 shows an example of the impulse response (IR) calculation. A real room was chosen and a comparison was made between measured and simulated impulse responses [3]. Figure 2(b) and (c) show the impulse responses at 250 Hz and 500 Hz octave band, respectively. The simulated result agrees well with the measurement in the early part. Because the Schroeder cutoff frequency of this room was 180 Hz, the analysis at mid frequency was validated.

![Diagram of room model and impulse response at mid frequency.](image)

Fig. 2. Room model and impulse response at mid frequency. (a) Room model, (b) IR at 250 Hz band, (c) IR at 500 Hz band: ---, Measurement; ----, PBTM.

Figure 3 shows the energy IR at 250 Hz octave band for the model in Fig. 2(a). Improvement by the PBTM over the conventional beam tracing method (BTM) can be clearly seen in Fig. 3(b).

![Diagram of energy IR comparison.](image)

Fig. 3. A comparison of energy IR: (a) measurement, (b) calculated by PBTM, (c) calculated by BTM.

Figure 4 shows the calculated acoustic parameters from the impulse responses in Fig. 2. Calculated early decay time (EDT), definition, and center time agree reasonably with the measured values.
Fig. 4. Comparison of measured and calculated acoustic measures. (a) EDT, (b) definition, (c) center time: , measured; , PBTM; , BTM.

Figure 5 shows the binaural simulation at 500 Hz octave band in a room. Similar to the previous results, the binaural simulation shows a good correspondence with the measurement.

Identification of the noise source existing in a room can be performed from the measured sound pressures. When the transfer functions between the source surface and the field points are calculated by the PBTM, the distribution of source strengths can be inversely reconstructed. Practically, the widely known BEM based NAH [5] cannot be applied to the source locating in a room due to heavy computation of transfer functions. The relation between the $j$th source surface and $n$th field points are given by

$$
p_n = p_{o,j} e^{i(k+j0.5m)\theta_1} + p_{o,j} e^{i(k+j0.5m)\theta_2} \cdots + p_{o,j} e^{i(k+j0.5m)\theta_{nr}} a_{nr} \prod_{i=1}^{nr-1} r(\theta),
$$

(Eq. 1a,b)

Here, $p_n$ is the $n$th field point pressure, $p_{o,j}$ the pressure on $j$th source surface, $nr$ the total number of reflections. The matrix formulation for all source and field points is expressed as

$$
\begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_n
\end{bmatrix} =
\begin{bmatrix}
H_{1o1} & H_{1o2} & \cdots & H_{1om} \\
H_{2o1} & H_{2o2} & \cdots & H_{2om} \\
\vdots & \vdots & \ddots & \vdots \\
H_{no1} & H_{no2} & \cdots & H_{nom}
\end{bmatrix}
\begin{bmatrix}
p_{o,1} \\
p_{o,2} \\
\vdots \\
p_{o,m}
\end{bmatrix}.
$$

(Eq. 2)

For $n$ field points and $j$ source points, the source strength is inversely reconstructed by the following equation,

$$
P_{o,j} = H_{o,j} P_{o}, \quad \Rightarrow \quad P_{o,j} = H_{o,j}^I P_{o,j}.
$$

(Eq. 3)

In estimating the source pressures, great care should be taken to select the optimum field points. From the equally spaced candidates, the field points were determined by the effective independence technique (EfI) [6]. Figure 6 shows the simulation example and the result of the inverse calculation. After reconstructing the pressure distributions on the source surface existing in a room, the SPL at the same field points were recalculated as if room boundaries are invisible. This technique is very useful to estimate the noise power level when one cannot move the noise...
source to an anechoic chamber, e.g. heavy diesel engines or generators. In this example, any regulation technique during the inverse process was not applied. By adopting an effective regularization technique, the reconstruction error can be significantly reduced.

\[ j : \text{No. of source surfaces} \]
\[ n : \text{No. of measurements} \]

Fig. 6. Holography by PBTM: (a) model, (b) comparison of measured and reconstructed source intensity.

Diffraction can be included in the analysis by employing the uniform theory of diffraction (UTD). Figure 7 shows comparisons between simulated and measured responses.

\[ (-1.79, 1.35) \]
\[ (0.1, 1.13) \]

Fig. 7. Simulation of diffraction: (a) edge model, (b) PBTM combined with UTD, (c) ordinary PBTM. Measurement; ---, PBTM.

CONCLUDING REMARKS
In this paper, the capability of PBTM in calculating room transfer function, impulse response, and acoustic measures, and conducting source identification were demonstrated. It is thought that the application target can include small spaces such as vehicle cabin can be also dealt with by the present method as well as large enclosed spaces depending on the frequency of interest. We have seen that the PBTM can be a useful mid-frequency method bridging a gap between modal methods and geometrical acoustics methods in the acoustic simulation of an enclosure or, with a further study, it can be extended to be a unified technique covering the whole audio frequency range.

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References: