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Reverberation time, mean free path and sound absorption in concert halls

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Hidaka Takayuki and Nishihara Noriko
Takenaka R&D Institute, 1-5-1, Otsuka, Inzai, Chiba 270-1395, Japan
hidaka.takayuki@takenaka.co.jp

ABSTRACT

Sabine and Eyring equations are commonly used to estimate the reverberation time (RT) in concert halls. Derivations of the two equations are based on well-defined and different physical assumptions, and they are both slightly different from the realistic condition in actual halls. In many researches so far, comparatively simple room shapes have been utilized to study the question on which equation is preferable. In this study, the sound fields in rooms with more complex or wide-ranging shape are numerically analyzed by computer simulation techniques and CAD models for architecture. Focusing on the fundamental relation between the mean free path as a measure of the room shape and the RT, validity of the two equations in actual halls is discussed. Next, the influence of the room shape and/or the choice of the RT equation on the effective sound absorption is examined.

INTRODUCTION

Sabine and Eyring equations are commonly used to predict the reverberation time of concert halls. Both are based on simplified mathematical models so that the physical assumptions for the both equations deviate slightly from actual situation. The basic premises of Sabine and Eyring equations are summarized in Table I. Concerning to the sound particle's behavior in a hall, the Sabine equation assumes each particle hits the wall at random timing, while the Eyring assumes each hits the wall simultaneously [1]. Regarding to the surface diffusivity, the Eyring requires all the surfaces are diffusive reflecting. On the other hand, for sphere, when 78% of the surfaces are specular reflecting, that is, 22% walls reflect the sound randomly, the Sabine coincides with the exact value [2].

When we focus on the free path length (FPL) of sound particles, the FPL should be deterministic value for the Eyring and all the free path length equal 4*V/S*, that is a value in the classical theory [3]. Here, *V* means the room volume, and *S* the total surface area. If FPL is treated as stochastic variable and it obeys the exponential distribution, the Sabine equation is obtained [4]. In this case, the average of the FPL equals 4*V/S*. Similarly, when we focus on the total reflection number *N* at time *t*, *N* is also deterministic value for Eyring situation. There is a famous relation by Schroeder [4] that when *N* obeys Poisson distribution the Sabine equation is derived.

Generally, the absorption coefficient α = 1 means 100% absorption, which when used in the Eyring equation yields RT = 0. In the Sabine equation RT approaches zero only when this coefficient is very large. For the same reverberation time, the following equality is derived by equating the two equations, $\alpha_{sab} = -2.3 \log (1 - \alpha_{ey})$, and for each sub-surface S_i in a room, the associated absorption coefficient is $\alpha_{eyi} = (\alpha_{ey} / \alpha_{sab})*\alpha_{sabi}$ [5]. In actual concert halls the Sabine absorption coefficient α_{sab} is less than 0.4 [6], which means that it can be used without experiencing large values, and we sometimes choose it here because the Sabine RT equation is simpler and in any particular case, the Eyring absorption coefficient can be obtained easily from the Sabine coefficient.

The most important parameter of the statistical model of the reverberation process, probably, is either FPL or the total reflection number. Unfortunately, we can't measure these quantities directly in actual sound field. Only the decay rate of sound pressure is measurable by direct method. But, by means of the computer simulation, the sound field in the "modeled" halls can be analyzed so that the statistical distribution of FPL is found. This will give us more precise clue to understand the sound field in actual halls.

Table I Sabine vs. Eyring equations, $\gamma^2=(\langle FPL^2\rangle-\langle FPL\rangle^2)/\langle FPL\rangle^2$.

	Reflection of each sound occurs	Degree of random reflection, 1-*s*	Free Path length, FPL	Limit condition at α=0 or 1
Ref.	Müller / Cremer (1982)	Joyce (1978) & this study	Kuttruff (2000)	
Eyring	Side by side, at same timing with constant period	*s* =0, random reflection occurs at every surface	FP=4V/S, deterministic value, $\gamma^2=0$	RT=0 for α=1
Sabine	One after another, at random manner.	*s* =0.78 for sphere *s* ≈0.5 for rectangle *s* ≈0.3 for SB hall	FP obeys exp. distribution, random value, MFP=4V/S, $\gamma^2=1$	RT=0 for α=∞. But α<0.4 for real hall for music

BASIC DATA

Table II overviews 10 halls studied: 5 shoebox halls (SB) and 5 non-shoebox halls (non-SB). The first column is a theoretical MFP, 4*V/S*, when perfect diffusion is assumed. The 2nd column is a MFP obtained by numerical method in this study. It is noted that the ratio (Column 3) between these two MFP becomes 1.05 and 1.00 for two categories. One another parameter, expected value of FPL^2 over MFP^2 equals 1.65 for SB and 1.85 for non-SB, which will be cited later. After Joyce [2], (ensemble average of) MFP for imperfect diffusion is larger than or equal to the MFP (=4*V/S*) for perfect diffusive condition. Comparing the result in column 3 to his result, we find that for non-SB halls perfect diffusion is indicated while for SB halls the deviation is about 5% different from perfect diffusion.

At the numerical simulation, three source points were set on the stage, from each of which 100,000 sound rays were generated and traced them for 1 second. The number of surfaces of the CAD models was from 800 to 1,600.

Table II List of simulated halls.

		MFP [T]	MFP [N]	[N]/[T]	$\langle \ell^2\rangle/\langle \ell\rangle^2$
		4V/S	Ray Tracing		
SB Hall	Amsterdam Concertgebouw	12.8	13.4	1.05	1.63
	Basel Stadt Casino	9.4	9.9	1.05	1.63
	Boston Symphony Hall	11.9	12.5	1.05	1.65
	Vienna Musikvereinssaal	11.0	10.8	0.98	1.62
	Zurich G. Tonehalle	8.5	9.6	1.13	1.73
	Average	**10.7**	**11.4**	**1.05**	**1.65**
non-SB Hall	Buffalo KH Music Hall	11.0	10.6	0.96	1.92
	Carnegie Hall	11.2	12.2	1.09	1.81
	Cleaveland Severance Hall	10.0	9.5	0.95	1.91
	Salle Pleyel	11.3	11.4	1.01	1.74
	Tokyo Bunka-Kaikan	11.2	10.5	0.94	1.86
	Average	**10.9**	**10.9**	**1.00**	**1.85**

MEAN FREE PATH AND ABSORPTION IN CONCERT HALLS

After derivation process of Eyring equation, the decaying sound energy in a room at time t is,

$$E(t) = E(0)(1-\alpha)^{ct/\mathrm{MFP}} \quad (1)$$

from which following relation is obtained.

$$RT = \frac{-6\ln 10}{c}\frac{\mathrm{MFP}}{\ln(1-\alpha)} = 0.04\frac{\mathrm{MFP}}{\alpha} \quad (2)$$

Here, absorption coefficient α is material value of itself, that is independent from sound field, but MFP should depend on hall shapes even if V/S is the same.

By applying numerical values in Table II, we get following result as an average.

$$\mathrm{MFP} = \begin{Bmatrix} 1.05 \\ 1.00 \end{Bmatrix} \times \frac{4V}{S} = k \times \frac{4V}{S} \quad \text{for} \begin{cases} \mathrm{SB} \\ \mathrm{non-SB} \end{cases} \quad (3)$$

Then, we can rewrite Eq. (2),

$$RT = 0.04\frac{V}{(\alpha/k)S}, \quad k = \begin{cases} 1.05 & \text{for SB} \\ 1.00 & \text{for non-SB} \end{cases} \quad (4)$$

This means, if α and V/S remained constant, we can expect that RT for SB is longer that RT for non-SB, where k is a parameter that depends on the hall shape. Writing absorption coefficient calculated from RT measured in SB hall by α_{SB} and that in non-SB by α_{NSB}, we obtain,

$$\alpha_{NSB} = 1.05\alpha_{SB} \quad (5)$$

that is, the effective absorption coefficient in non-SB hall is 5% larger than that in SB hall.

Figure 1 shows Beranek's result [5] on the absorptivity of audience area in halls. From measurements of RT's in actual halls, using Sabine equation, he found that the absorption coefficient α_{NSB} is about 6% larger than α_{SB} on average. Obviously, this study supports his conclusion from different aspect, and demonstrates that the effective absorptivity in a hall varies depending on the room shape, that is, the statistical distribution of FPL,

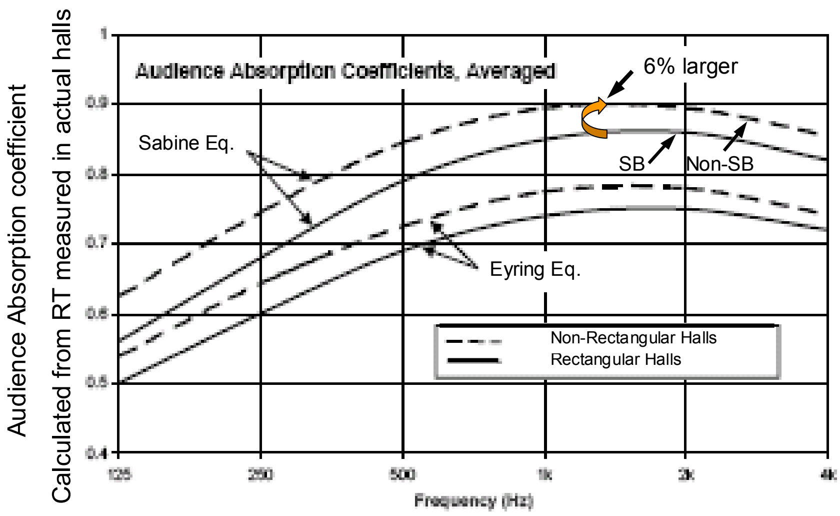


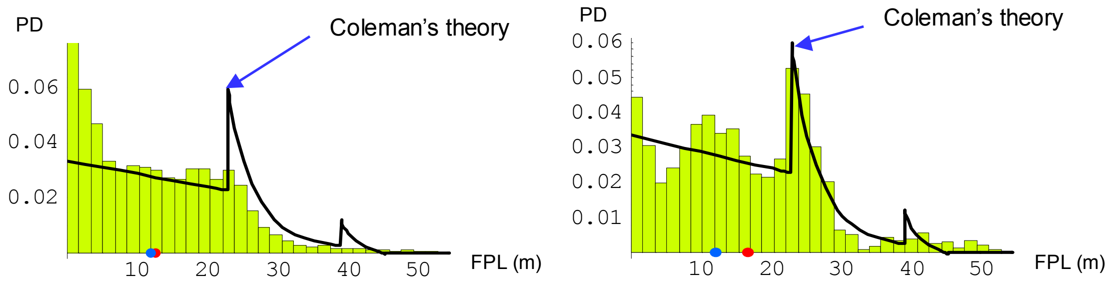
Fig.1 Audience absorption coefficient After Beranek (2006).

NUMERICAL ANALYSIS

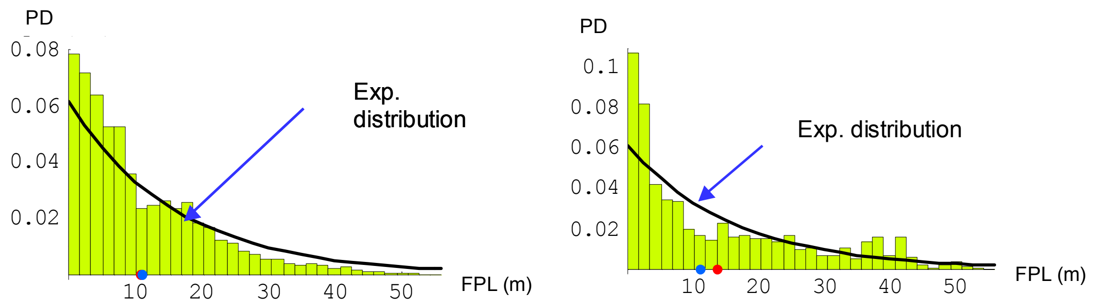
The distributions of the free path lengths FPL are compared in Fig. 2. The values were estimated by computer simulation. The vertical axes give the probability density (PD) of the free path lengths and the horizontal axes show the FPL's in m. The upper two graphs are for a SB hall and the lower two for a non-SB hall. The left graphs correspond to the condition that all surfaces have the same absorptivity, i.e., approximately like that for an unoccupied hall. The right graphs assume that the audience area is completely absorbing, i.e., like that for an occupied hall. The red circles indicate the MPF's and the blue circles indicate the theoretical values, $4V/S$, for the halls.

For SB hall, the solid line plots the Coleman's theoretical distribution function of FPL for pure rectangles assuming perfect diffusion [7]. This curve is for the pure rectangle corresponding to the Boston Symphony's proportion, that is, Width, Height and Depth are 22.9, 18.6, and 39 meters. We see that, under unoccupied condition, the FPL distribution is fairly distorted from the theoretical curve when the FPL is large, while under occupied condition, the distribution is close to that of the pure rectangle. The peaks in the distribution are influenced by the sound propagation in the upper space in SB hall. We may say that the reverberant sound fields in SB for occupied and unoccupied condition possess different distribution from each other.

For non-SB hall, the solid line means the exponential distribution with the mean value $4V/S$ [3]. The difference between unoccupied and occupied is not obvious, and the both are close to the theoretical curves. As shown in Table I, when the FPL obeys exponential distribution, the Sabine equation holds with $4V/S$.



Shoe-Box (Boston SH), equal absorptivity α=0.2(Left), Audience Area: α=1(Right)



Non-SB (Buffalo KMH), equal absorptivity α=0.2(Left), Audience Area: α=1(Right)

Fig. 2 Free path length distribution

SOUND ABSORPTION IN SHOEBOX HALL

Joyce [2] derived an exact integration equation that the energy density in a room satisfies, from which he obtained a solution of the effective absorption coefficient α_{eff} in a room by power series expansion.

$$\alpha_{eff} = \alpha + \frac{\alpha^2}{2(1-s)}\left\{2-(1+s)\frac{<\ell^2>}{<\ell>^2}\right\}+... \qquad (6)$$

This equation shows his conclusion up to the second term (note α<0.4 in real halls), where the first term is equal to the Sabine absorption coefficient and the second term depends on the second moment of FPL, ℓ. The required parameter is $<\ell^2>/<\ell>^2$ (see Table I), and all the surfaces was assumed to have same absorption in this study. Another parameter in Eq. (6) is the degree of random reflecting surface 1-*s*, where for all surfaces are specular reflecting, *s*=1, and for all surfaces are diffusive reflecting, *s*=0.

At present stage, there is no objective method to estimate *s* in actual halls, so Rayleigh Criteria [8] is utilized as an alternative. This is an indicator of the roughness of irregular surface and one can judge which surface reflection is more dominant diffusive or specular,

$$\sigma < \lambda/8\cos\theta_i : \text{ specular reflection}$$
$$\sigma > \lambda/8\cos\theta_i : \text{ diffusive reflection} \quad (7)$$

here, the wavelength is λ; θ_i is the incident angle of a sound wave at the surface; and σ is the standard deviation of the height of the irregularities on a surface in a hall - for example, in Boston Symphony Hall, σ for the sidewalls and ceiling ranges from 0.13 to 0.33 m as estimated from drawings and photographs.

Figure 3 is a plot of σ versus frequency where three areas are shown, that for rough surface scattering, that for specular reflection and an intermediate zone. The two curves for θ_i = 45°and 60° are calculated from Eq.(7), i.e., are Rayleigh criteria. For the Boston Hall, at 500 Hz, the value of σ lies mostly in the intermediate zone of Fig. 3, indicating that the average θ_i is between about 45°and, perhaps, 80°, and at 1000 Hz it lies entirely in the rough surface area.

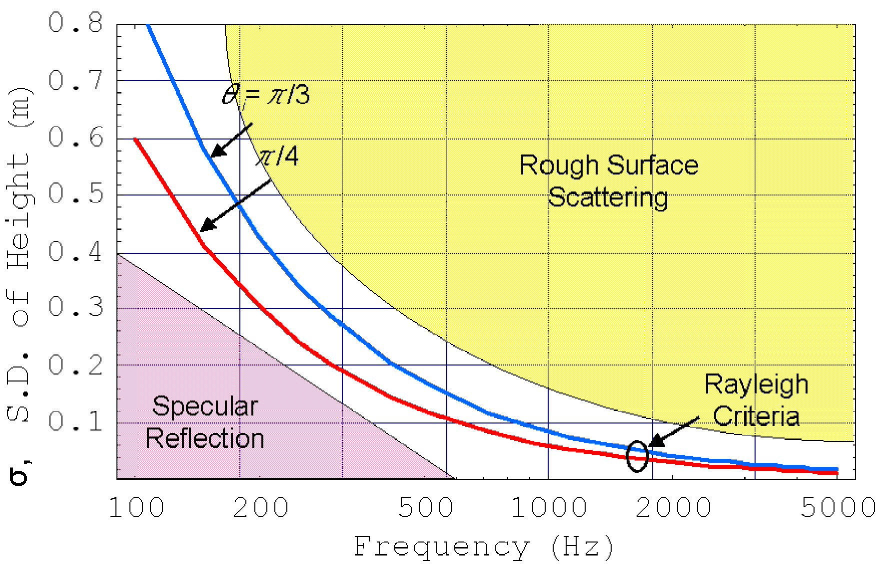


Fig. 3 Criteria of rough surface scattering.

Figure 4 is a numerical comparison of RT equations for a SB hall. The vertical axis is effective absorption coefficients and the horizontal axis is the Sabine absorption coefficient α, where the parameter $<\ell^2>/<\ell>^2 = 1.65$ is taken from numerical analysis. Two broken lines are the Eyring and Sabine equations. The red line means *s*=0 in Eq. (2), that is, all surfaces are diffusive reflecting, and the green one means *s*=0.25. It is seen that the Sabine equation coincides with the Joyce's exact theory when 75% wall is diffusive reflecting. This is plausible result in real halls, while the Eyring equation does not fit the Joyce's theory even when 100% surface is diffusive reflecting, i.e., *s*=0

CONCLUSION

Based on numerical analysis, following conclusions are indicated, (a) effective absorption in non-Shoebox hall is 5% larger than that in Shoebox hall; (b) sound diffusivity in non-SB hall satisfies necessary condition of perfect diffusion and the MFP is approximately equal to the ideal value 4*V/S*; (c) sound diffusivity in SB hall deviates from 4*V/S*, and the sound diffusivity under unoccupied is different from that under occupied condition; and (d) for SB halls, Sabine equation is equivalent to the Joyce's exact theory in the range of $\alpha < 0.35$, when about 70% of the wall is diffusive reflecting. Finally, there exists no single reliable RT equation because the random surface ratio varies hall to hall. But if we properly categorize the actual halls, we can obtain plausible equation by combining the Sabine and Joyce equation.

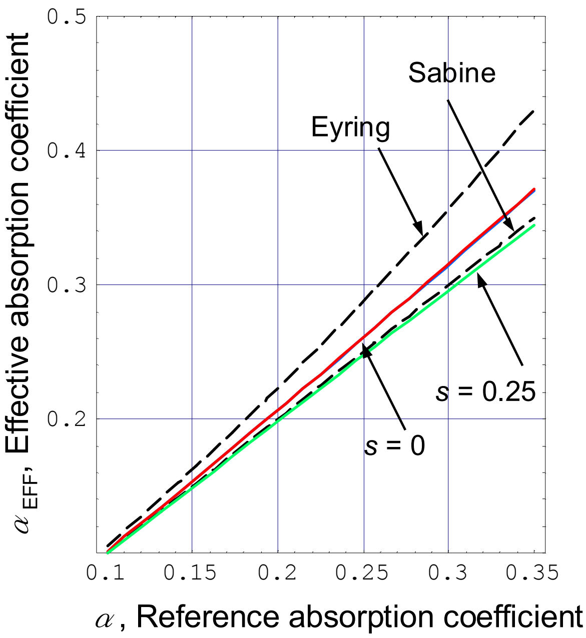


Fig. 4 Joyce's theory for shoebox hall.

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