

THE MODELLING OF FREQUENCY DEPENDENT BOUNDARY CONDITIONS IN FDTD SIMULATION OF CONCERT HALL ACOUSTICS

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ABSTRACT

The finite difference time domain (FDTD) method is a powerful method for the direct simulation of room acoustics impulse response. One feature of the method is that it allows visual inspection of the propagation of the sound field over time, which is particularly useful in the study of surface reflection, scattering and diffusion in the time domain. However, such a study will require the incorporation of frequency dependent surface boundary conditions in the time domain method. This is not straightforward, especially when one considers the implications of multiple reflections in a room acoustics simulation. This paper investigates various possibilities of implementing frequency dependent boundary conditions in FDTD simulations. This includes the direct convolution method, a mass-damper-spring approximation, and the Z-Transform method. The implication on accuracy and computation time will be discussed. The paper will also demonstrate the use of a filter design approach to represent typical frequency dependent absorption coefficients of wall surfaces, and the implementation of this procedure in the FDTD simulation of the acoustics of a simple concert hall.

1. INTRODUCTION

The finite-difference time-domain (FDTD) method is a simple numerical method that replaces differential equations by finite differences and has been successfully used to predict the changes in electric and magnetic fields in Electrodynamics. Since Maloney and Cummings [1] presented the parallelism between the fundamental EM equations (Maxwell's equations) and Acoustics (continuity and Euler's equations), many acousticians are trying to use and develop the FDTD for room acoustic prediction.[2-5] In room acoustic simulation, the FDTD equations are directly derived from the wave equations. Acoustic phenomenon such as diffraction, diffusion and interference effects (such as room modes) are all formulated naturally. FDTD requires the whole acoustic field to be discretised into an interconnection of elements, unlike the Boundary Element Method. For large spaces, a large number of elements would be needed to represent the field accurately, which requires a considerable amount of computational power. However, calculating the pressure and velocity at the nodes for each point in time and space allows the whole acoustic field to be animated. Propagation of sound waves through the field can be shown visually. The visual representation can help easily identify the acoustic flaws in the space which would help the user determine ways to improve the performance of the space. FDTD allows the current design to be assessed and provides Architects and Architectural acousticians with information which may allow them to further improve the design. This makes it a potentially very useful technique for designing acoustic spaces. Although the FDTD has many advantages, many aspects of the FDTD for room acoustics still need to be developed and improved. One of the obstacles is the modelling of frequency dependent boundary conditions. In this paper, three different boundary conditions modelling approaches will be investigated and discussed.

2. FINITE DIFFERENCE TIME DOMAIN

2.1 FDTD equations

In room acoustic simulation, FDTD models the sound propagation using a finite difference scheme in time and space. Since it is generally better to operate a finite difference scheme on

lower order derivatives, the first order Euler and Continuity differential equations are used.

Euler Equation:
$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_0} \nabla p$$
 Continuity equation: $\nabla \cdot \vec{u} = -\frac{1}{\rho_o c^2} \frac{\partial p}{\partial t}$ (Eq. 1)

where p is the sound pressure, \vec{u} is the particle velocity vector. Because of their interdependence, the sound pressure and particle velocities need to be calculated alternatively in a staggered grid. The 3D FDTD equations can be written as follows.

$$p(x_{i}, y_{j}, z_{k}, t_{n+1/2}) = p(x_{i}, y_{j}, z_{k}, t_{n-1/2}) - \rho_{0}c^{2} \frac{\Delta t}{\Delta x} [u_{x}(x_{i+1/2}, y_{j}, z_{k}, t_{n}) - u_{x}(x_{i-1/2}, y_{j}, z_{k}, t_{n})] - \rho_{0}c^{2} \frac{\Delta t}{\Delta y} [u_{y}(x_{i}, y_{j+1/2}, z_{k}, t_{n}) - u_{y}(x_{i}, y_{j-1/2}, z_{k}, t_{n})] - \rho_{0}c^{2} \frac{\Delta t}{\Delta z} [u_{z}(x_{i}, y_{j}, z_{k+1/2}, t_{n}) - u_{z}(x_{i}, y_{j}, z_{k-1/2}, t_{n})] u_{x}(x_{i+1/2}, y_{j}, z_{k}, t_{n}) = u_{x}(x_{i+1/2}, y_{j}, z_{k}, t_{n-1}) - \frac{1}{\rho_{0}} \frac{\Delta t}{\Delta x} [p(x_{i+1}, y_{j}, z_{k}, t_{n-1/2}) - p(x_{i}, y_{j}, z_{k}, t_{n-1/2})] u_{y}(x_{i}, y_{j+1/2}, z_{k}, t_{n}) = u_{y}(x_{i}, y_{j+1/2}, z_{k}, t_{n-1}) - \frac{1}{\rho_{0}} \frac{\Delta t}{\Delta y} [p(x_{i}, y_{j+1}, z_{k}, t_{n-1/2}) - p(x_{i}, y_{j}, z_{k}, t_{n-1/2})] u_{z}(x_{i}, y_{j}, z_{k+1/2}, t_{n}) = u_{z}(x_{i}, y_{j}, z_{k+1/2}, t_{n-1}) - \frac{1}{\rho_{0}} \frac{\Delta t}{\Delta z} [p(x_{i}, y_{j}, z_{k+1/2}, t_{n-1}) - \frac{1}{\rho_{0}} \frac{\Delta t}{\Delta z} [p(x_{i}, y_{j}, z_{k+1/2}, t_{n-1})] (Eq. 3)$$

2. 3 Courant limit

When using the FDTD equations it must be ensured that the space step size does not exceed the speed of sound for a given time step size. This is referred to as the Courant limit [6]. In a 3D situation, waves can propagate horizontally, vertically and diagonally. For a square grid it must be ensured that the waves propagating diagonally do not exceed the speed of sound. The Courant limit is given by [6]: $\Delta t \leq \frac{\Delta x}{c\sqrt{n}}$, where n is the number of dimensions, and

 $\Delta x = \Delta y = \Delta z$ is assumed.

2. 4 Implementing sources

In order to generate sound propagation in the FDTD grid a source needs to be placed into the grid. The source signal that was implemented is a Gaussian pulse $S(t) = Ae^{(-\sigma \cdot t)^2}$.

3. MODELLING FREQUENCY DEPENDANT BOUNDARIES

Since room boundary conditions are generally frequency dependent, it is essential to model them correctly in a FDTD grid simulation. Botteldooren [7] demonstrated the use of a mass-damper-spring system to approximate boundary conditions in a FDTD model. Rienstra [8] showed how analytical models of simple boundary such as mass-damper-spring systems or Helmholtz resonators can be expressed in the time domain by using a Padé approximation or a Z-Transform. These approximations can then be implemented into time domain acoustic simulation programs. The Z-Transform was implemented into the FDTD to model acoustic impedance by Özyörük and Long earlier in 1996 [9]. Generally there is limited literature in this area of FDTD development for room acoustics. The purpose of this paper is to evaluate the efficiency and accuracy of different boundary condition modelling methods in FDTD. A simple locally reacting mass-damper-spring boundary will be used as an example. Three methods will be explored and evaluated:

- 1. A mass-damper-spring specific method using numerical differentiation and integration;
- 2. Direct convolution of the boundary reflection impulse response;

3. Z-Transform based IIR filter.

The methods will be compared to the analytical solution of the mass-damper-spring system in a 1D simulation, and then applied to a 3D space. In the 3D application, single reflected pulses calculated from the 3 boundary modelling methods were compared to observe the most efficient form of model that produces least memory, highest accuracy, and efficient calculation time.

3. 1 The discrete integration-differentiation method

This equation is derived from the mass-damper-spring equation by applying discrete approximations to the differentiation and integration of the velocity in the equation.

$$p = -m_b \frac{\partial u}{\partial t} - r_b u - k_b \int u \partial t \quad \Rightarrow \quad \frac{\partial u}{\partial t} = -\frac{1}{m_b} \left(p + R_b u + k_b \int u \partial t \right) \quad (\text{Eq. 4})$$

where m_b is the mass per unit area of boundary kgm⁻², R_b is the specific acoustic resistance of the boundary Nsm⁻³, and k_b is the bulk modulus of the boundary Nm⁻³. For this application the velocity of the boundary needs to be found from an incident pressure. Discretising the formula and approximating the differentiation and integration by finite difference and discret summation, the FDTD update equation can be re-arranged to:

$$u(x_{b},t_{n}) = \frac{u(x_{b},t_{n-1}) - \frac{\Delta t}{m_{b}} \left[p(x_{b+1/2},t_{n-1/2}) + k_{b} \Delta t \sum_{i=2}^{n-1} u(x_{b},t_{i}) \right]}{\left[1 + \frac{\Delta t \cdot R_{b}}{m_{b}} + \frac{k_{b} \Delta t^{2}}{2m_{b}} \right]}$$
(Eq. 5)

3.2 Convolution

Any boundary reflection can be thought of as a impulse response filter. The incident wave interacts with the boundary and a filtered version is reflected back into the space. In DSP, a filter can be implemented by convoluting the impulse response of the filter with the input signal. This method is expected to be the most robust and should not be limited to low frequency. However, convolution is usually a computationally expensive process, especially if a high level of accuracy is required. The impulse response can be found by performing an Inverse Fourier Transform of the frequency domain transfer function. This impulse is currently non-causal. If this impulse response was used to model a reflection from the boundary a second delayed reflection would be generated. In order to avoid this second reflection, only the first half of the impulse is used. To maintain the same energy in this impulse, the rejected second half is then flipped over back and added on to the impulse from sample 2 onwards. The impulse has been truncated to a maximum of 200 samples to reduce computation.

3. 3 Z-plane filter design

The convolution method discussed above is a Finite Impulse Response Filter. A more efficient way to implement a time domain filter is to use an Infinite Impulse Response filter. An IIR filter usually requires fewer coefficients to represent the same filter and hence should be less computationally expensive. An IIR filter will now be designed based upon the reflection coefficient.

$$R(\omega) = \frac{j\omega m + r + \frac{k}{j\omega} - \rho_0 c}{j\omega m + r + \frac{k}{j\omega} + \rho_0 c} \implies R(\omega) = \frac{(j\omega)^2 + j\omega \left(\frac{r - \rho_0 c}{m}\right) + \omega_{res}^2}{(j\omega)^2 + j\omega \left(\frac{r + \rho_0 c}{m}\right) + \omega_{res}^2}$$
(Eq. 6)

The mass and stiffness of the boundary determines the resonant frequency $\omega_{res} = \sqrt{\frac{k}{m}}$. The

pole and zero locations are found from the roots of the numerator and denominator. Assuming that $\omega_{res} > (r + \rho_0 c)/2m$, the poles and zeros become:

$$0_{a1}, 0_{a1}^{*} = -\left(\frac{r-\rho_{0}c}{2m}\right) \pm j\sqrt{\omega_{res}^{2} - \left(\frac{r-\rho_{0}c}{2m}\right)^{2}}$$
(Eq. 7)

$$P_{a1}, P_{a1}^{*} = -\left(\frac{r+\rho_0 c}{2m}\right) \pm j \sqrt{\omega_{res}^{2} - \left(\frac{r+\rho_0 c}{2m}\right)^{2}}$$
(Eq. 8)

Using a standard Bilinear Transform, the Z-Transformed reflection coefficient becomes:

$$R(Z) = \frac{U_{REF}(Z)}{U_{INC}(Z)} = gain \cdot \frac{\left(1 - 2\operatorname{Re}(0_{d1}) \cdot Z^{-1} + |0_{d1}|^2 \cdot Z^{-2}\right)}{\left(1 - 2\operatorname{Re}(P_{d1}) \cdot Z^{-1} + |P_{d1}|^2 \cdot Z^{-2}\right)}$$
(Eq. 9)

where,

$$0_{d1} = \frac{2f_s + 0_{a1}}{2f_s - 0_{a1}}, \quad 0_{d1}^* = \frac{2f_s + 0_{a1}^*}{2f_s - 0_{a1}^*}, \quad P_{d1} = \frac{2f_s + P_{a1}}{2f_s - P_{a1}}, \quad P_{d1}^* = \frac{2f_s + P_{a1}^*}{2f_s - P_{a1}^*}, \quad gain = \frac{(2f_s - 0_{a1})(2f_s - 0_{a1}^*)}{(2f_s - P_{a1})(2f_s - P_{a1}^*)}$$

The difference equation of the filter is obtained by rearranging the equation to:

$$u_{ref,n} = gain \cdot (u_{inc,n} - 2\operatorname{Re}(0_{d1}) \cdot u_{inc,n-1} + |0_{d1}|^2 \cdot u_{inc,n-2}) + 2\operatorname{Re}(P_{d1}) \cdot u_{ref,n-1} - |P_{d1}|^2 \cdot u_{ref,n-2}$$
(Eq. 10)

The Bilinear Transform compresses a continuous transfer function with a frequency range of $\pm \infty$ to a discrete transfer function with a frequency range of $\pm fs/2$. In order to match the continuous frequency to the discrete frequency, the filter has to be pre-warped, $\omega_{d,res} = 2f_s \tan\left(\frac{\omega_{res}}{2f_s}\right)$

replaces ω_{res} in the equations for the zeros and poles.

For the difference equation expressed by Equation 10, the incident velocity is the input and the reflected velocity is the output. It should be noted that the FDTD node contains information about both the incident and reflected velocities. For the filter to work properly, the incident velocity needs to be extracted and entered into the equation.

4. RESULTS AND DISCUSSION

4.1 Visual comparison of the 3 methods in 1D FDTD

The 3 methods were compared to the theoretical model in order to identify their accuracy.



Figure 1.- Comparison of 3 the methods with the theoretical model

From Fig.1, the error in the Integration-differentiation method increases much faster than the other two as frequency increases. This is caused by the additional numerical differentiation and integration needed in the method. The results show that this method is limited to systems with a resonant frequency less than f res = $0.05 \times f$ s. The convolution method, using 200 samples, provides the best results out of the 3 methods. At low frequency the results seem almost identical to the theoretical model. It is only when the resonant frequency is increased to around $\frac{1}{4}f$ s that the accuracy is compromised. This is however more likely a limit due to the FDTD time and step sizes rather than the convolution itself. The Z-Transform technique also produces results very similar to the theoretical response, and the accuracy is generally similar to that of the direct convolution method, with good accuracy up to f res = $0.2 \times f$ s. However, it should be

noted that the accuracy is additionally dependent on the inherent limitations of the Z-Transform method itself. For example, errors can be caused by a low resonant frequency with a low Q-factor, i.e. $\omega_{res} < (r + \rho_0 c)/2m$. This makes the pole locations completely real and hence they are situated on the real axis. In order to accurately represent the required frequency response, a higher order filter will be required in such cases.

0.04 Integration-differentiation Convolution 0.03 Z-plane 0.03 0.02 ра 0.02 Pressure, 1m 0.015 0.0 0.00 1m -0.005 150 100 200 250 300 1m Time (sample point) Figure 2.- 3D FDTD grid Figure 3.- Reflection from the 3 boundaries (computational array 169*169*169) (Direct pulse + Reflected pulse) Integration-differentiation Convolution Z-plane ра Sound level [dB] Pressure, 80 100 120 140 Frequency [Hz]

4. 2 Results in 3D FDTD

Time (sample point) Figure 4.- Expansion of reflected part in Fig.3

The 3D results are shown in Figures 2-5. In order to quantify which is the quickest technique, the runtimes were measured. All measurements were conducted in the same grid so that run times could be directly compared. The quickest technique used to model the mass-damper-spring is the Integration-Differentiation method. The convolution method is by far the most computationally expensive (in both runtime and memory requirements), and is over 5 times more expensive that the Integration-Differentiation technique in 3D FDTD. The Z-Transform method is slightly more expensive than the Integration-Differentiation method in 1D FDTD. In 3D FDTD the Z-Transform method is about as efficient as the Integration-Differentiation method. The results clearly show that, with a proper filter design, the Z-Transform is the most efficient and accurate approach to model frequency dependent boundary conditions in room acoustics.

Figure 5.- FFT of reflected part



Figure 6.- Calculation time comparison in 3D FDTD, 1 reflection only.

4. 3 Modelling Room Surface Absorption Coefficient in FDTD

Although the above results are for a mass-damper-spring hypothetical system, the Z-Transform method can be readily applied to other types of frequency dependent boundary conditions. The advantage of using the reflection coefficient as the basis of the Z-Transform formulation is that the magnitude of the reflection coefficient can be easily related to the usual absorption coefficient by the simple approximation $R = \sqrt{1-\alpha}$. Since phase information is not available from the usual absorption coefficient data. the frequency



spectrum of the amplitude alone can *Figure 7. An example of Z-Transform filter designs* be used to generate a suitable filter design using techniques such as the bilinear transformation of standard *constrainted in this example.*

analogue prototypes or modern optimisation methods such as Least p-th norm, and linear phase or minimum phase constraints can be applied if desired.

5. CONCLUSIONS

For interior acoustic problems, the boundary conditions must be modelled as accurately as possible. Errors in the modelling of the boundaries will affect the measured performance, and such errors will also increase with reflection order. It is therefore important to find an accurate and efficient method to model boundary conditions in FDTD simulations. In this paper, 3 different techniques were tested, using a hypothetical mass-damper-spring system as an example boundary. It was found that the most accurate method is the convolution method. However, this is extremely computationally expensive and requires around 5 times more processing time than the Integration-differentiation method to create accurate results in 3D FDTD. The Integration-differentiation method is the most efficient method. However, its accuracy deteriorates much quicker at higher frequencies, and the method cannot be readily used to model real walls using existing empirical absorption data. The Z-Transform method is computationally as efficient as the Integration-differentiation method but is far more versatile and accurate, and is considered the best overall. The formulation of the boundary condition presented in this paper is based on the reflection coefficient. This provides a convenient way of translating the absorption coefficient into a suitable filter design. Obviously the lack of phase information in the absorption coefficient data is a limitation, but until such information is available it is an efficient and practical way of modelling room acoustics boundary conditions in FDTD.

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