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Investigating the Scattering Behavior of Incident Plane Waves using BEM

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ABSTRACT

In order to effectively simulate the sound field at low frequencies, where the dimensions of the walls are comparable to the wavelength, one needs to clearly know the reflection and diffraction properties of walls. In this paper, we present a numerical technique involving solution of boundary integral equations based on spatial discretization in order to investigate the behavior of incident plane waves at arbitrarily shaped wall surfaces. For comparison purposes we also introduce a simple point-source based model to calculate scattered wave fronts. The incident plane waves are considered at various angles and scattered waves computed in both models are then compared with the measured data. It is found that while the point-source model can give reasonable asymptotic results, the advanced numerical model matches with the measurement data significantly better in quantity and quality.

1. INTRODUCTION

Nowadays with the development of computational facilities, use of simulation softwares for room acoustic modeling has been widely increased. Particle-based approaches like ray tracing and image source methods are generally being used in order to calculate the impulse response of a room. These approaches, however, are not fully complete without taking into account the wave nature of the sound. For instance, scattering behavior of incident plane wave on arbitrary surface cannot be explained only by these approaches. Hence, in order to study the behavior of sound field effectively, need of incorporating the scattering behavior into existing particle model arises.

There are many researches concerning scattering behavior and many scattering indices have been proposed [1,2,3]. As one of the indices, the scattering coefficient is defined as the ratio of the non-specularly reflected sound energy to the total reflected energy [1]. Due to measurement difficulties, the available literature for scattering coefficient values is very limited [4,5]. Hence the need of a computational tool in order to predict the scattering coefficient arises.

In this paper we investigate the use of BEM in order to calculate the Mommertz [1] scattering coefficient of any arbitrary surface and then we compare it with the available measured data [5]. We also introduce a point source model in order to verify our approach asymptotically. As part of the research a new software tool “EASE Scatterer” was developed and is presented here.

2. THEORY

Boundary element method (BEM) is a numerical tool to approximate the solution of Boundary value problems. One of the main advantages of BEM is that only the boundary of the domain under consideration needs to be discretized. First we will describe how to obtain a boundary value problem from the Helmholtz equation and then we will illustrate the application of BEM.

2.1. Mathematical Formulation

The first step in the mathematical formulation of the BEM is to convert the differential equation governing the problem into a boundary integral equation/value problem [6]. The Helmholtz equation in the frequency domain is:

$$\nabla^2 p(r) + k^2 p(r) = 0 \qquad \text{(Eq. 1)}$$

Before trying to solve any problem using BEM, one needs to have the fundamental solution *G* (also called the freespace Green's function) of the problem. Next, we form an integral from the Helmholtz equation by using a weighted residual method as follows:

$$\int_{\Omega} G(r;r_0)(\nabla^2 p(r) + k^2 p(r))\ d\Omega = 0 \qquad \text{(Eq. 2)}$$

The fundamental solution of a particular equation is the weighting function that is used in the boundary element formulation of that equation. It is therefore important to be able to find the fundamental solution for a particular equation.

Applying the Green-Gauss divergence theorem, equation (2) becomes:

$$c(r_0)p(r_0) + \int_{\Gamma} \frac{\partial G(r;r_0)}{\partial n} p(r) d\Gamma(r) - \int_{\Gamma} G(r;r_0)\frac{\partial p(r)}{\partial n} d\Gamma(r) + p_{in}(r_0) = 0 \qquad \text{(Eq. 3)}$$

where

$$c(r_0) = \begin{cases} 1, r_0 \in \Omega \\ 0.5, r_0 \in \Gamma \end{cases}$$

Next we divide the boundary into boundary elements, where each element has a center node representing a field quantity (here pressure) of the element. So the equation (3) is transformed into algebraic system of equations with relation to the center node quantity *p(r)* and *∂p(r)/∂n.*

$$c(r_i)p(r_i) + p_{in}(r_i) + \sum \left\{ \int_{\Gamma} \frac{\partial G(r;r_i)}{\partial n} d\Gamma(r) \right\} p(r_j) = \sum \left\{ \int_{\Gamma} G(r;r_i) d\Gamma(r) \right\} \frac{\partial p(r_j)}{\partial n} \qquad \text{(Eq. 4)}$$

where, p_{in} represents the pressure of the incident wave.

It is difficult to compute this incident wave pressure at boundary element points, which are shadowed or cannot be seen directly by the incident wave. As an approximation in this model, we have calculated the incident pressure at such points assuming them to be in direct sound field.

The algebraic system of equations (eqn. 4) can be solved to obtain reflected pressure at boundaries using effective solvers. In the next step, pressure at desired receiver locations can be obtained using equation (3). For scattering coefficient calculations, the reflected pressure at some far field distance can be calculated (~1000 m).

2.2. Point Source Model

In addition to the BEM model, the complex reflection properties are also calculated utilizing an elementary wave method [7] based on the geometrical parameters of the surface.

In short, a fairly simple approach is used to compute the reflected wave front. Radiating point sources are placed along the boundary of the structure utilizing a density sufficient for the considered wavelength. Based on the source locations the complex pressures of the individual elementary waves are summed in the far field to obtain the polar response. Naturally, this response depends on the angle of incidence and on the frequency. It is noteworthy that with this model neither shadowing nor diffraction effects are taken into account explicitly.

3. EXAMPLES AND RESULTS

Next we apply our methodology of both approaches to different surfaces to investigate the scattering behavior of incident plane waves.

3.1. Semi Ellipses (10 cm deep)

A model was created in EASE Scatterer as shown in fig. 1a and the scattering coefficients for 1/3 octave center frequencies were calculated using BEM and point source model for different incident angles. Fig. 1b illustrates the scattering behavior at –56.9° using the point source model at 1 KHz.

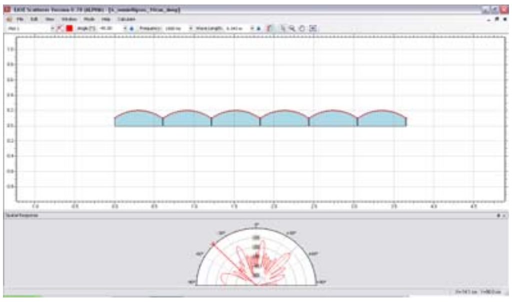

Figure 1a.-Semi Ellipses Model

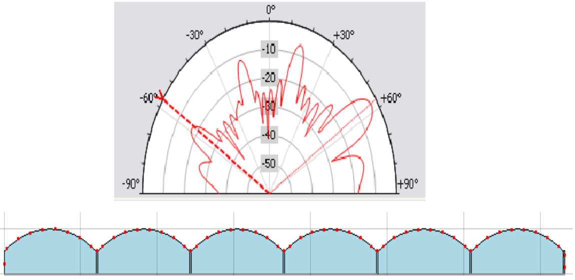

Figure 1b.-Scat. Behavior for Incident Angle: –56.9°

3.1.1. Qualitative Analysis

Figures 2a and 2b show the simulated results at 1/3-octave center frequencies. One can observe that for normal angle of incidence (0°), point source model and BEM matches fairly well but for oblique incidence (-56.9°), the BEM gives more accurate results if one compares them with the measured data. This may be due to the fact that in this case, for oblique incidence, diffraction behavior becomes more prominent. Moreover the interaction between the neighboring elements is taken care of by the BEM approach more effectively. For instance, there are first and second order reflections taking place within the structure and the point source approach does not take this into account.

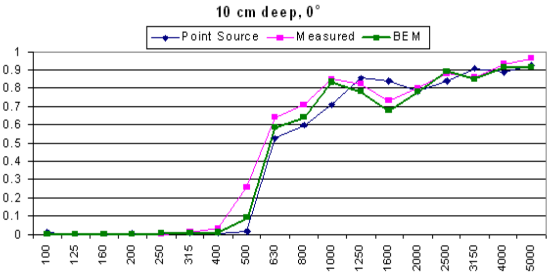

Figure 2a.-Incident Angle 0°

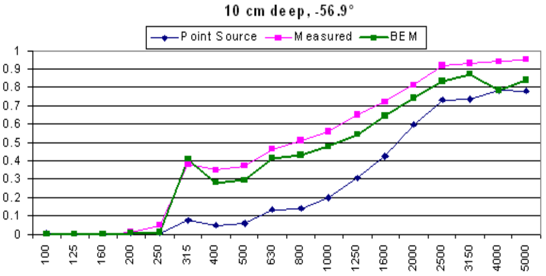

Figure 2b.-Incident Angle -56.9°

3.1.2. Quantitative Analysis

Fig. (3a, 3b) shows the differences between measured and simulated data over frequencies using both approaches.

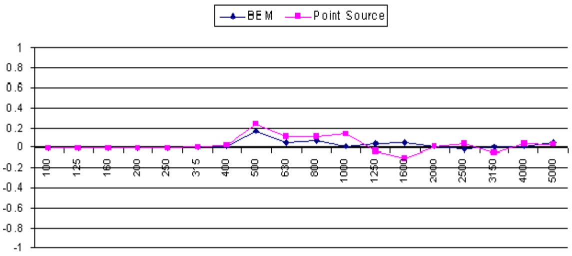

Figure 3a.-Incident Angle 0°

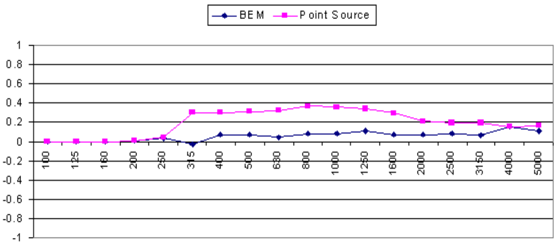

Figure 3b.-Incident Angle -56.9°

Now to compare the point source model and BEM quantitatively, we calculated the root mean square errors (Table 1) for both the cases.

Table 1.- Root Mean square errors

Incident Angle	BEM	Point Source
0°	0.0479	0.08366
-56.9°	0.0734	0.2387

From the data (Table 1), it is quite clear that the point source model can provide a good estimation of scattering coefficients but BEM is more accurate.

3.2. Schroeder Diffusers

A Schroeder diffuser model was created as shown in Figure 4:

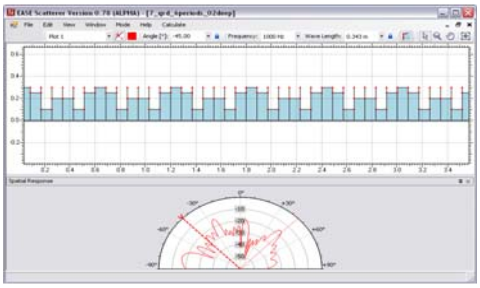

Figure 4.-Scattering behavior using BEM

3.2.1. Qualitative Analysis

Fig. 5a shows the scattering coefficients for Schroeder diffuser for normal angle of incidence. As there is not much interaction between neighbouring elements, point source model and BEM both are effective in predicting the scattering behaviour. In fig 5a. at 315 Hz and in fig. 5b at 400 Hz there are some mismatches and the plot is slightly shifted towards higher frequencies, this could be due to not taking shadowing into account in our model. However, once again it can be observed from fig. 5b that for oblique angles, the point source model is not reliable for complicated surfaces whereas BEM is quite efficient considering the errors associated with the measurements as well.

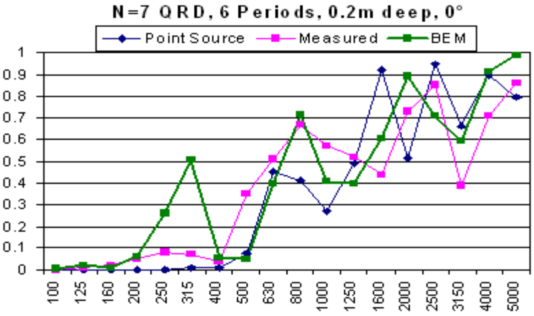

Figure 5a.-Incident Angle 0°

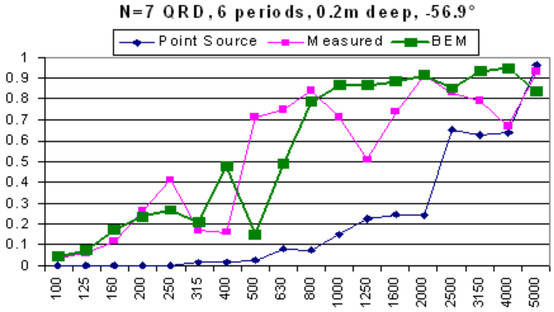

Figure 5b.-Incident Angle -56.9°

3.2.2. Quantitative Analysis

As before, plots (fig. 6a, 6b) show deviations from measured data for both cases:

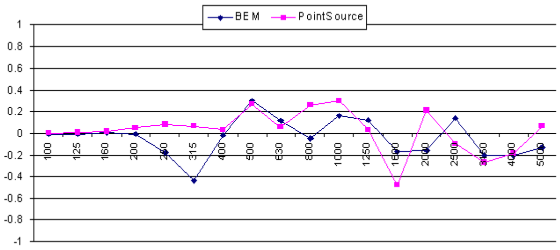

Figure 6a.-Incident Angle 0°

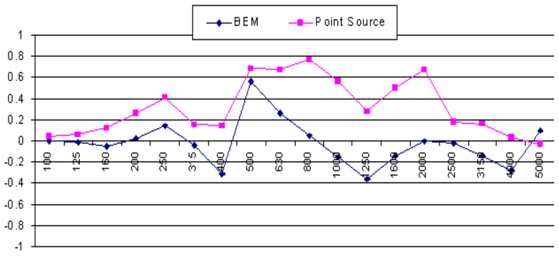

Figure 6b.-Incident Angle -56.9°

Table 2.- Root Mean Square errors

Incident Angle	BEM	Point Source
0°	0.1736	0.19
-56.9°	0.2088	0.4040

Table 2 further confirms that errors associated with the point source approach are comparable to the BEM for normal incident angle but almost double in case of oblique angle of incidence.

3.3. Semi-Cylinders (12 periods, 7.32 m wide, Incident Angle: -56.9°)

In this example, we just provide a qualitative overview of obtained simulation results. Looking at the figure 7b, and from the previous examples, it is quite clear that the advanced BEM approach matches significantly better with the measured data compared to the point source model and can be used effectively in predicting scattering coefficients for various surfaces.

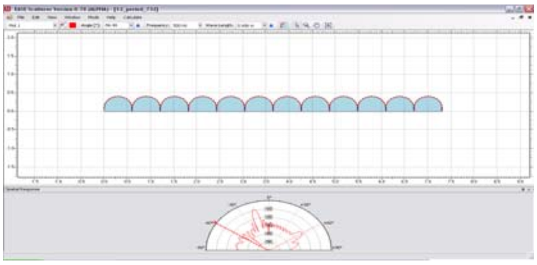

Figure 7a.-Scattering behavior at –56.9°

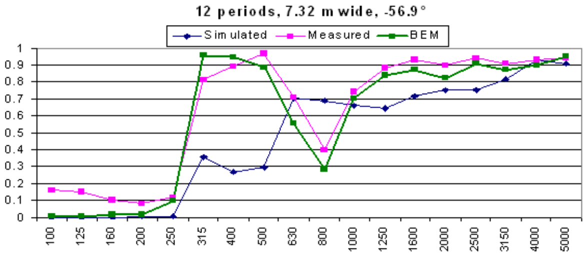

Figure 7b.-Comparison(BEM, Point Source)

4. RANDOM INCIDENT SCATTERING COEFFICIENT

Next we will discuss whether using a single random incident scattering coefficient for all incident angles is valid for room acoustic modelling. We will consider different geometries and examine the scattering coefficients for various incident angles.

4.1. Triangles (9 periods, 45°)

A model consisting of nine triangles each having 45° angle (fig. 8a) was created in EASE Scatterer and scattering coefficients were calculated from 90° to -90° at an interval of 5°.

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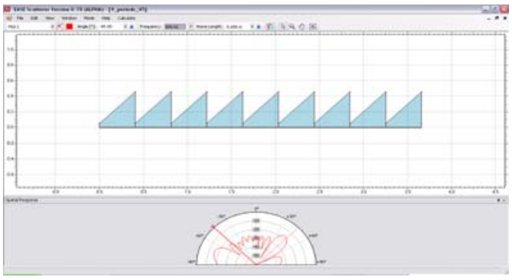

Figure 8a.-Triangle Model (9 periods)

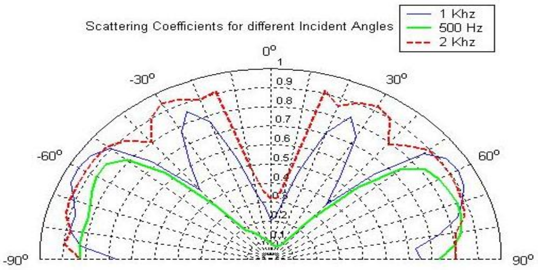

Figure 8b.-Scat. Coeff. for different incident angles

One can observe from the figure 8b, that the scattering coefficient values at 1 KHz are ranging from 0.223 to 0.958. Therefore, using only a single scattering coefficient for all incident angles and for a single reflection does not seem to be valid. However, if the order of reflections *n* is high (*n* > 20) in room acoustic simulations, then one can assume that the incident ray is coming at various angles and therefore in an average sense, a random incident scattering coefficient can be calculated using the Paris' formula [8]. If one considers the Semi Ellipses model (fig 1) at 1 KHz, the scattering coefficients (fig 10) are not varying much over various incident angles. However, for the Schroeder diffuser (fig. 9), it is again varying too much over various incident angles. This explains that depending upon the geometry as well; the values of scattering coefficients should be used with care.

4.2. Schroeder diffuser (N=7,QRD)

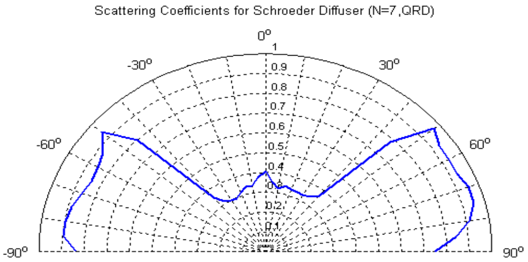

Figure 9.-Scat. Coeff**.,** Schroeder

4.3. Semi-Ellipse –10 cm deep

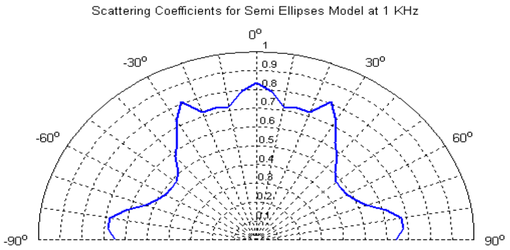

Figure 10.-Scat. Coeff., Semi Ellipses

5. CONCLUSION

The scattering behavior of incident plane waves at arbitrary surfaces using BEM and a point source model has been investigated. It has been shown that while the point source model gives reasonable good results, the BEM approach is more effective in terms of quality and quantity. Moreover, the variations of scattering coefficients as the incident angle changes have been shown and the resulting limitations for use in room-acoustic modeling have been discussed.

6. ACKNOWLEDGEMENTS

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