ABSTRACT
This paper illustrates the effect of diffraction of an oblique incident plane wave at the edges of a flat impedance strip, on the scattered sound pressure and absorbed sound intensity along the surface of the strip for various incident angles. The strip is flush-mounted on an infinite rigid baffle. When the ratio of strip length to wavelength is much lower than one, regardless of the incident angle, the scattered surface pressure is large and almost uniform, and the scattered and incident surface waves always superimpose in phase, which produces a large and nearly uniform absorbed intensity. When the ratio is close to or much higher than one, the scattered pressure and intensity distributions become oscillatory with maxima and minima where diffraction bands are formed. As the incident angle increases, the oscillation around the baffle-strip (BS) edge increases where the bands become narrower, and the scattered and incident waves superimpose increasingly destructive that the intensity drops. Meanwhile, the oscillation around the strip-baffle (SB) edge decreases where the bands become wider, and the scattered and incident waves superimpose increasingly constructive that the intensity increases. As a result, the intensity rises across the strip more with the incident angle of the plane wave.

INTRODUCTION
The significance of sound diffraction due to discontinuities in acoustical impedance at the edges of a finite material to the sound absorption of the material has been well known [1]. Interestingly, it can be examined from the diffraction pattern that the measured sound pressure around the two opposite edges of a material oscillated similarly for a normal incident plane wave, but the oscillation was less around one edge and more around the other edge for an oblique incident plane wave [2]. However, the physical picture of an oblique plane wave diffraction at the edges of a given material is obscure, and it is unclear how the diffraction affects the distributed absorption and thus absorption coefficient of the material at different incident angles. In this paper, the effect of the diffraction on the scattered surface pressure of a finite impedance strip as well as on the superposition between the scattered and incident surface waves, is illustrated for various incident angles of a plane wave. The distributed absorption of the strip determined by the wave superposition and subsequently, the variation of its absorption coefficient with the incident angle, are explained.

SOLUTIONS TO SCATTERED SURFACE PRESSURE AND ABSORBED INTENSITY

Figure 1.- Schematic illustration of the finite impedance strip excited by the oblique plane wave.
For the strip of acoustical impedance, $\zeta$, shown in Fig. 1, the total sound pressure for $-\infty < x < \infty$ at $z = 0$ (i.e., on the surfaces of the baffle and the strip) can be written as

$$p_{to}(x) = p_{inc}(x) + p_{bl}(x).$$ (1)

$p_{sc}$ is the scattered pressure which implicitly consists of the specularly reflected pressure of the baffle and the strip, and the diffracted pressure of the two ends of the strip. By discretizing the strip into $M$ small elements and considering the sound pressure of the strip at the $n$th element (i.e., $x = x_n^{(s)}$), Eq. (2) can be expressed as

$$\tilde{p}_{to}(x_n^{(s)}) + \frac{k}{2 \zeta L} \sum_{m=1}^{M} \tilde{p}_{to}(x_m^{(s)}) \hat{H}_{nm}^{(1,0)} = p_{bl}(x_n^{(s)}),$$ (3)

where $p_{bl}(x_n^{(s)}) = 2p_{inc} e^{-jkx_n^{(s)} \sin \theta}$. The tildes refer to approximated pressures due to the discretization and $\hat{H}_{nm}^{(1,0)}$ is the mean Hankel function for the radiation contribution of the $m$th element to the total pressure at the $n$th element:

$$\hat{H}_{nm}^{(1,0)} = \frac{H^{(1,0)}(k | x_n^{(s)} - x_m^{(s1)}|) + H^{(1,0)}(k | x_n^{(s)} - x_{m+1}^{(s1)}|)}{2}.$$ (4)

$x_n^{(s)}$ is the mid-point of the $n$th element, while $x_m^{(s1)}$ and $x_{m+1}^{(s1)}$ are the end points. Since $n = 1, \ldots, M$, there are $M$ equations of the form of Eq. (3), which can be written in matrix form as

$$[P_{TO}] = [R][P_{BL}],$$ (5)

where $[P_{TO}] = [\tilde{p}_{to}(x_1^{(s)}) \ldots \tilde{p}_{to}(x_M^{(s)})]^T$, $[P_{BL}] = [p_{bl}(x_1^{(s)}) \ldots p_{bl}(x_M^{(s)})]^T$, and

$$[R] = \begin{bmatrix}
1 + k \hat{H}_{1,1}^{(1,0)} / 2 \zeta & \cdots & k \hat{H}_{1,M}^{(1,0)} / 2 \zeta \\
\vdots & \ddots & \vdots \\
k \hat{H}_{M,1}^{(1,0)} / 2 \zeta & \cdots & 1 + k \hat{H}_{M,M}^{(1,0)} / 2 \zeta
\end{bmatrix}^{-1}. $$ (6)

$[\cdot]^T$ and $[\cdot]^{-1}$ respectively denote the transpose and the inverse of the matrix. $R_{nm}$ in $[R]$ describes the radiativity contribution of the $m$th element to the total pressure at the $n$th element due to the blocked pressure exerted on the former element. By using Eqs. (1) and (5),

$$[P_{SC}] = [R][P_{BL}]-[P_{IN}],$$ (7)

where $[P_{SC}] = [\tilde{p}_{sc}(x_1^{(s)}) \ldots \tilde{p}_{sc}(x_M^{(s)})]^T$ and $[P_{IN}] = [p_{in}(x_1^{(s)}) \ldots p_{in}(x_M^{(s)})]^T$. The absorbed intensity at the $n$th element in the $z$ direction is given by

$$\text{Re}[I_{I(n,sc)}^{(2)}(x_n^{(s)})] = - \text{Re}[\tilde{p}_{sc}(x_n^{(s)})^2 / 2 \rho_0 c_0 \zeta_n^2].$$ (8)

where $\text{Re}[\cdot]$ denotes the real part of the complex quantity and $*$ denotes the complex conjugate. The sound absorption coefficient of the strip can then be obtained as

$$\alpha_{(sc)} = \langle \text{Re}[I_{I(n,sc)}^{(2)}] \rangle / \text{Re}[I_{I(n)}^{(2)}] = - \frac{1}{M} \sum_{n=1}^{M} \text{Re}[I_{I(n,sc)}^{(2)}(x_n^{(s)})] / P_{inc} \frac{\zeta_n^2}{\rho_0 c_0}.$$ (9)

In Eq. (9), the numerator is the spatial average absorbed sound power per unit length of the strip (\langle \cdot \rangle denotes spatial average), and the denominator is the normal intensity of the incident plane wave.
RESULTS AND DISCUSSION

For illustrations, characteristics of \( \alpha_{(sp)} \), \( \bar{p}_n(x_n^{(s)}) \), and \( \text{Re}[I_{T_{(sc)}}^{(z)}(x_n^{(s)})] \) of a strip with \( L=1.8 \, \text{m} \) and \( \zeta_l=3+j \) are presented for different values of \( \theta \). The absorption coefficient \( \alpha_{(sp)} \), specularly reflected pressure \( p_{sp}(x_1^{(s)}) \), and absorbed intensity \( \text{Re}[I_{T_{(sp)}}^{(z)}] \) of an infinite strip of the same \( \zeta_l \), are provided for comparison. \( P_{\text{in}}=1 \, \text{Pa} \) and \( L/\lambda=0.2, 2, \) and \( 20 \), are used (\( \lambda \) is wavelength).

Figure 2(a) shows that when \( L/\lambda \) is lower, the deviation of \( \alpha_{(sc)} \) from \( \alpha_{(sp)} \) is larger, and \( \alpha_{(sc)} \) separates from \( \alpha_{(sp)} \) at smaller \( \theta \). Also, \( \alpha_{(sc)} \) is close to \( \alpha_{(sp)} \) for small \( \theta \) but deviates significantly from \( \alpha_{(sp)} \) for large \( \theta \), and it always rises with \( \theta \) toward infinity at \( \theta=90^\circ \). These two features of \( \alpha_{(sc)} \) are related to the diffraction at the edges of the finite strip, whose effect can be quantified by the difference between the magnitudes of the spatial average absorbed intensities of the infinite and finite strips, \( \text{Re}[I_{\text{di}}^{(z)}] = \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \| - \|\text{Re}[I_{T_{(sp)}}^{(z)}]\| \); note: \( \| \) is not used for \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \)

The latter case occurs only around some minima of \( \|\text{Re}[I_{\text{di}}^{(z)}]\| \) at small \( \theta \). It is obvious that when \( L/\lambda \) is lower, \( \|\text{Re}[I_{\text{di}}^{(z)}]\| \) is larger, and reaches \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) at small \( \theta \) increases. So, \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) deviates more from \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) at small \( \theta \) and is greater than \( \|\text{Re}[I_{T_{(sp)}}^{(z)}]\| \) quicker. This explains the first feature of \( \alpha_{(sc)} \). Since both \( \|\text{Re}[I_{\text{di}}^{(z)}]\| \) and \( \|\text{Re}[I_{T_{(sp)}}^{(z)}]\| \) drop with \( \theta \) to zero while \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) rises with \( \theta \), \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) is always finite and non-zero. These lead to the second feature of \( \alpha_{(sc)} \). The variation of \( \|\text{Re}[I_{T_{(sc)}}^{(z)}]\| \) with \( \theta \) and \( L/\lambda \) can be explained by the change in the diffraction with \( \theta \) and \( L/\lambda \).

The behavior of the diffraction can be viewed from the distribution of \( \|\bar{p}_n(x_n^{(s)})\| \), and it is only determined by the combined behavior of \( R_{n,m} \) and \( p_{bl} \) because \( |p_{bl}|=P_{\text{in}} \) is a constant [see Eq. (7)]. The behavior of \( R_{n,m} (=e^{j\phi_{n,m}^{(R)}}) \) is controlled by \( T_{n,m}^{(1,0)} \) that is independent of \( \theta \) but depends on \( k \) and \( x \) in the form of \( k \|x_n^{(s)}-x_m^{(s)}\| \) [see Eqs. (4) and (6)]. As \( x_m^{(s)} \) increases or decreases away from a given \( x_n^{(s)} \) for a fixed \( k \) (i.e., the argument of the Hankel function increases), \( \phi_{n,m}^{(R)} \) rises (i.e., \( \phi_{n,m}^{(R)} \) is symmetrical about the \( n^{th} \) point) due to the properties of Hankel function. Also, the change of \( \phi_{n,m}^{(R)} \) with \( x_m^{(s)} \) are slower for smaller \( k \) (i.e., lower \( L/\lambda \)) because \( x_m^{(s)} \) has to increase more in order to attain the same change in \( \phi_{n,m}^{(R)} \) as for a larger \( k \). As examples, Fig. 3...
Due to the behaviors of $\phi_{n,m}$ and $\phi_{\text{pbl}}$, $R_{n,m,\text{pbl}} (=|R_{n,m,\text{pbl}}|e^{j\phi_{n,m,\text{pbl}}})$ exhibits an interesting feature of phase. Since $\phi_{n,m}$ has the same trend as $\phi_{\text{pbl}}$ when $x_n^{(s)} < x_n^{(s)}$, the addition of $\phi_{\text{pbl}}$ to $\phi_{n,m}$ increases the rate of phase change with $x_n^{(s)}$, where
\( \phi_{n,m}^{(R)} \) drops faster than \( \phi_{n,m}^{(Rpbl)} \) (i.e., \( \phi_{n,m}^{(Rpbl)} = \phi_{n,m}^{(R)} + \phi_{n,m}^{(pbl)} \)). Also, as the trends of \( \phi_{n,m}^{(R)} \) and \( \phi_{n,m}^{(pbl)} \) are opposite when \( x_m^{(s)} > x_n^{(s)} \), the addition of \( \phi_{n,m}^{(pbl)} \) to \( \phi_{n,m}^{(R)} \) reduces the rate of phase change that \( \phi_{n,m}^{(Rpbl)} \) rises slower than \( \phi_{n,m}^{(R)} \). Furthermore, when the variations of \( \phi_{n,m}^{(R)} \) and \( \phi_{n,m}^{(pbl)} \) with \( x_m^{(s)} \) are closer to each other, \( \phi_{n,m}^{(Rpbl)} \) changes faster if the trends of \( \phi_{n,m}^{(R)} \) and \( \phi_{n,m}^{(pbl)} \) are the same but slower if the trends are opposite. The variations are closer for larger \( \theta \) because \( \phi_{n,m}^{(R)} \) and \( \phi_{n,m}^{(pbl)} \) are functions of the forms of \( kx \) and \( kx \sin \theta \) respectively. So, when \( \theta \) is larger, \( \phi_{n,m}^{(Rpbl)} \) decreases faster for \( x_m^{(s)} < x_n^{(s)} \) and increases slower for \( x_m^{(s)} > x_n^{(s)} \). Some examples are shown in Fig. 4.

It can be seen that \( \phi_{120,m}^{(Rpbl)} \) varies rapidly when \( x_m^{(s)} < x_{120}^{(s)} \) but slowly when \( x_m^{(s)} > x_{120}^{(s)} \).

\[ \phi_{120,m}^{(Rpbl)} \text{ varies slower for } \theta = 45^\circ \text{ than for } \theta = 80^\circ \text{ when } x_m^{(s)} < x_{120}^{(s)}, \text{ but the variation is faster for the smaller } \theta \text{ when } x_m^{(s)} > x_{120}^{(s)}. \]

The change of \( \phi_{120,m}^{(Rpbl)} \) is also more rapid for higher \( L/\lambda \).

\[ \text{Figure 5.- Distributions of specularly reflected and scattered surface SPLs for } L/\lambda = 3 + j. \]

From the above behavior of \( \phi_{n,m}^{(Rpbl)} \), the \( M \) values of \( R_{n,m}^{pbl} \) (i.e., the radiated waves from all points) for a given \( n \), are either in phase, out of phase, or neither when \( \phi_{n,m}^{(Rpbl)} \) has many jumps that the number of in-phase and out-of-phase \( R_{n,m}^{pbl} \)'s is comparable. Thus, the sum of \( R_{n,m}^{pbl} \)'s that controls the behavior of \( |\tilde{p}_{sc}(x_m^{(s)})| \), alternates with \( n \) between constructive and destructive, and the alternation decays with distance from the edges. So, \( |\tilde{p}_{sc}(x_m^{(s)})| \) oscillates with maxima and minima, where diffraction bands are formed if the spacing between two adjacent minima is defined as a diffraction band. The oscillation decays with distance from the edges where the bands gradually disappear. When \( L/\lambda = 1 \) or \( L/\lambda > 1 \), and \( \theta \) is small, the fast and slow variations of \( \phi_{n,m}^{(Rpbl)} \) for \( x_m^{(s)} < x_n^{(s)} \) and \( x_m^{(s)} > x_n^{(s)} \), imply that \( |\tilde{p}_{sc}(x_m^{(s)})| \) oscillates rapidly near the baffle-strip (BS) edge that the bands are narrow but slowly near the strip-baffle (SB) edge that the bands are wide. As \( \theta \) increases, the faster and slower variations of \( \phi_{n,m}^{(Rpbl)} \) for \( x_m^{(s)} < x_n^{(s)} \) and \( x_m^{(s)} > x_n^{(s)} \), cause the oscillation of \( |\tilde{p}_{sc}(x_m^{(s)})| \) and its decay to be faster near the BS edge that the bands become narrower and disappear quicker, but slower near the SB edge that the bands become wider and disappear slower. This phenomenon is evident in Fig. 5 for \( L/\lambda = 2 \) and 20, where the bands are narrower near the BS edge and wider near the SB edge for larger \( \theta \). In other words, as \( \theta \) increases, the diffraction effect of the BS edge decreases, while the diffraction effect of the SB edge
increases, spreads closer to the BS edge, and dominates the behavior of \(|\overline{p}_{sc}(x_n^{(s)})|\) across the strip when \(\theta\) is large. For \(L/\lambda<1\), regardless of \(\theta\), \(a_{n,m}^{(phb)}\) almost does not vary and has no jumps, where \(R_{n,m}p_{bo}/s\) are always in phase that the alternation of the constructive sum of \(R_{n,m}p_{bo}/s\) is very slow. In this case, \(|\overline{p}_{sc}(x_n^{(s)})|\) is almost uniform, and the diffraction effects of both edges are always comparable but larger than those for \(L/\lambda=1\) or \(L/\lambda>>1\) (e.g., see Fig. 5 for \(L/\lambda=0.2\)).

CONCLUSIONS

The diffraction has an effect of shifting the phase of \(\overline{p}_{sc}(x_n^{(s)})\) away from the phase of \(p_{in}(x_n^{(s)})\), and the extent of the effect is similar to that for \(|\overline{p}_{sc}(x_n^{(s)})|\). So, for \(L/\lambda=1\) or \(L/\lambda>>1\), as \(\theta\) increases, the shift around the BS edge does not change much that \(\overline{p}_{sc}(x_n^{(s)})\) is increasingly out of phase with \(p_{in}(x_n^{(s)})\), while the shift near the SB edge increases that \(\overline{p}_{sc}(x_n^{(s)})\) and \(p_{in}(x_n^{(s)})\) are still in phase. Thus, the superposition of the scattered and incident waves is increasingly destructive and constructive around the respective edges, where \(|\text{Re}[I_{sc}^{(s)}]|\) drops near the BS edge but rises near the SB edge more with \(\theta\) (see Fig. 6 for \(L/\lambda=2\) and 20). Also, for \(L/\lambda<<1\), the phase shift is substantial that \(\overline{p}_{sc}(x_n^{(s)})\) is always in phase with \(p_{in}(x_n^{(s)})\) over the strip regardless of \(\theta\). Hence, the scattered and incident waves always superimpose constructively along the strip that \(|\text{Re}[I_{sc}^{(s)}]|\) is nearly uniform and always higher than \(|\text{Re}[I_{sc}^{(sp)}]|\) across the strip (see Fig. 6 for \(L/\lambda=0.2\)). As a result, \(|\text{Re}[I_{di}^{(s)}]|\) and \(\alpha_{sc}\) are greater for larger \(\theta\) and lower \(L/\lambda\) as in Fig. 2.

REFERENCES