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MEASUREMENTS OF THE ELASTIC MODULI OF LAMINATED COMPOSITES USING ULTRASONIC TECHNIQUE

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ABSTRACT

In the present paper, a non-destructive technique (pulse echo) was used to determine the elastic constants of orthotropic composite materials through the measuring of the ultrasonic velocities. Measurements were carried out on woven fabric (weave glass cloth / polyester) composite material. From the measured velocity data, the elastic constants were determined through a numerical inversion. Ultrasonic tests were achieved also on polyester and aluminium specimens as a representative of isotropic material. The results were verified using the numerical solution and the data provided by the manufacturers. It is found that the applicability of the ultrasonic pulse echo technique is in a good agreement for the isotropic material rather than the orthotropic material.

INTRODUCTION

Fiber reinforced composites are being used increasingly as primary structural components in different modern applications. Knowledge of the elastic constants is fundamental for the complete characterization of the engineering material. Usually, elastic characterization is carried out by quasi-static test methods, but in recent time the use of alternative non-destructive test methods has increased. The disadvantages of the conventionally used techniques are; a) some engineering constants of anisotropic materials are difficult to measure, b) they are destructive in nature, c) high costs involved in producing samples of desired shape and size, d) in situ measurements are difficult. Ultrasonic methods, through transmission and pulse echo based on the longitudinal wave, have been effective in detecting the elastic constants and also relatively characterizing the other common defects. Pulse techniques are based on measuring the time of flight, that is, the time spent by the ultrasonic pulse to travel through the specimen from the transmitting to the receiving transducer. In the ultrasonic method the Young and shear moduli of the material can be calculated from knowledge of the dimension and density of the samples, and the transit time for longitudinal and transversal ultrasonic waves. Elastic moduli are determined by velocity measurements. Material microstructure can be characterized by velocity and attenuation measurements.

Recently, mixed numerical and experimental techniques for the characterization of both isotropic and anisotropic materials have received a lot of attention [1].The behaviour of the elastic ultrasonic waves within an anisotropic structure can be predicted if the stiffness constants of the material is known. Reconstruction of material properties of fiber reinforced composites especially elastic constants from experimentally measured ultrasonic velocities measured at different orientation of propagation are mathematically related to elastic constants and density ,through the Christoffel's equation [1,2]. The values of the parameters used in the last computation are the results of the identification procedure and yield the elastic properties. In principle, the approach makes it possible to identify all the elastic constants simultaneously from a single experiment without damaging the specimen. Theoretical contributions to the elastic properties of unidirectional fiber composite have been made before by Whitney [3], and Hashin [4]. The authors have developed approximate equations for longitudinal, transverse and shear moduli from constituent material properties for both hexagonal and random fiber arrays.

In the present study, pulse echo technique has been used to measure the ultrasonic velocities of the orthotropic composite materials. From the measured velocity data, the elastic constants are determined through a simple numerical inversion. The inversion method has been verified by the data predicted using the theoretical solution through the material properties and the data of the manufacturer.

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THEORETICAL ANALYSIS

Composites are described by means of the following stress-strain relations involving nine material constants:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix} \quad (1)$$

Where $[\,Q_{ij}\,]$ is the stiffness matrix and 1,2,3 denote the principal material axis as shown in Fig. 1.

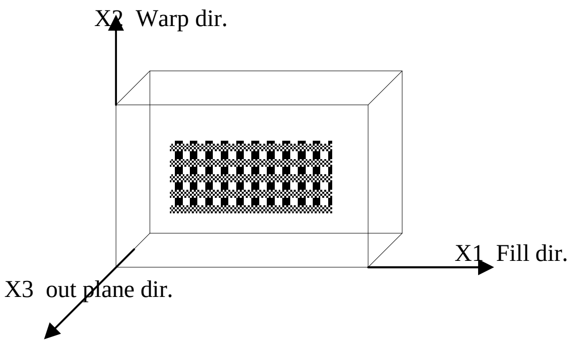


Fig . 1 Schematic diagram of tested specimen ,showing the principal axis

Based on their method of fabrication, it is reasonable to assume that the composites are macroscopically orthotropic, the planes of elastic symmetry being perpendicular and parallel to the fabric's reinforcing fibers. The stiffness form for an orthotropic material is explained in terms of the ply engineering constants as;

$$Q_{11} = Q_{22} = \frac{(1 - \nu_{13}\,\nu_{31})}{E_2\,E_3\,\Delta} \quad (2)$$

$$Q_{33} = \frac{(1 - \nu_{12}\,\nu_{21})}{E_1\,E_2\,\Delta} \quad (3)$$

$$Q_{44} = Q_{55} = G_{23} \quad (4)$$

$$Q_{66} = G_{12} \tag{5}$$

$$Q_{12} = \frac{(\nu_{12} + \nu_{23} \ \nu_{13})}{E_2 \ E_3 \ \Delta} \tag{6}$$

$$Q_{13} = Q_{23} = \frac{(\nu_{13} + \nu_{21} \ \nu_{13})}{E_2 \ E_3 \ \Delta} \tag{7}$$

$$\Delta = \frac{(1 - \nu_{12} \ \nu_{21} - \nu_{13} \ \nu_{31} - \nu_{23} \ \nu_{32} - 2 \nu_{21} \ \nu_{32} \ \nu_{13})}{E_1 \ E_2 \ E_3} \tag{8}$$

Ultrasonic data like phase or group velocities are related to the elastic constants of material as well as to the density. The general equation for wave propagation is obtained by solving the equations of motion and takes the form of the following determinant equation [1,2]:

$$\begin{bmatrix} A_{11} - \rho C^2 & A_{12} & A_{13} \\ A_{12} & A_{22} - \rho C^2 & A_{23} \\ A_{13} & A_{23} & A_{33} - \rho C^2 \end{bmatrix} = 0 \tag{9}$$

Where

$$A_{11} = l^2 Q_{11} + m^2 Q_{66} + n^2 Q_{55} \tag{10}$$

$$A_{22} = l^2 Q_{66} + m^2 Q_{22} + n^2 Q_{44} \tag{11}$$

$$A_{33} = l^2 Q_{55} + m^2 Q_{44} + n^2 Q_{33} \tag{12}$$

$$A_{23} = m \ n \ (Q_{23} + Q_{44}) \tag{13}$$

$$A_{13} = l \ n \ (Q_{13} + Q_{55}) \tag{14}$$

$$A_{12} = l \ n \ (Q_{12} + Q_{66}) \tag{15}$$

ρ is the density , C is the wave propagation velocity, *l,m,n* are the direction cosines w.r.t the axis X1,X2and X3

For a wave motion in the fill direction (*l* =1 ,*m*=*n*=0) there are three different waves travelling with the following velocities:

$$C_{11L} = \left(\frac{Q_{11}}{\rho} \right)^{\frac{1}{2}} \tag{16}$$

$$C_{12T} = \left(\frac{Q_{66}}{\rho} \right)^{\frac{1}{2}} \tag{17}$$

$$C_{13T} = \left(\frac{Q_{55}}{\rho} \right)^{\frac{1}{2}} \tag{18}$$

C_{ij}, where i,j numerical subscripts denote wave propagation and particle motion directions, respectively. The first wave is longitudinal, and the second and third waves are transverse. Wave velocities in other directions are obtained by substituting the appropriate values of the direction cosines. The longitudinal and transverse wave velocities are illustrated by the following relations.

$$C_{22L} = C_{11L} = \left(\frac{Q_{22}}{\rho}\right)^{\frac{1}{2}} \tag{19}$$

$$C_{33L} = \left(\frac{Q_{33}}{\rho}\right)^{\frac{1}{2}} \tag{20}$$

$$C_{23T} = C_{13T} = \left(\frac{Q_{44}}{\rho}\right)^{\frac{1}{2}} \tag{21}$$

In the direction at 45 deg. with the in plane X1-X2 waves velocities are obtained from the following equation :

$$\left(A'_{11} - \rho C^2\right)\left(A'_{22} - \rho C^2\right) - A'^2_{12} = 0 \tag{22}$$

Where

$$A'_{11} = \frac{\left(Q_{11} + Q_{66}\right)}{2} \tag{23}$$

$$A'_{22} = \frac{\left(Q_{22} + Q_{66}\right)}{2} \tag{24}$$

$$A'_{12} = \frac{\left(Q_{12} + Q_{66}\right)}{2} \tag{25}$$

The flexural wave associated with particle motion in the direction 3 (normal to the plane 1-2) travels with a velocity

$$C_{45/3T} = \left(\frac{\left(Q_{44} + Q_{55}\right)}{2\rho}\right)^{\frac{1}{2}} = \left(\frac{Q_{44}}{\rho}\right)^{\frac{1}{2}} \tag{26}$$

In the same manner, for the direction of 45 deg. with the plane X1-X3, the flexural wave in the direction 2 travels with velocity

$$C_{45/2T} = \left(\frac{\left(Q_{44} + Q_{66}\right)}{2\rho}\right)^{\frac{1}{2}} \tag{27}$$

EXPERIMENTAL PROCEDURE

The selected material for this study is woven fabric laminates of the type commonly used for thermal insulation. Each laminate consists of parallel plies of a woven fabric (glass clothe) impregnated with polyester resin. The tested specimens have nominal 8 mm thickness with 48 layers. The warp (lengthwise; direction – 2) and fill (crosswise; direction -1) direction of the reinforcing fabric are the same for every ply. The selected sample was of square section 12.5 x 12.5 mm. Nominal values of certain mechanical properties of the fabric and resin are listed in table 1. An electronic balance was used to measure the weight of the three small samples of the tested woven fabric composite plates. Averaged weight of weave glass cloth was measured as 21.34 gm. Average density of the samples is found to be 1780 Kg/m^3.The fiber volume fraction was calculated using the rule of mixtures as shown below, knowing the densities of the fiber and the matrix separately as given in table 1. The fiber volume fraction (Vf) is found of 50% and the void content (V_v) is equal to 0.1%. So, using the rule of mixture, density of the tested woven fabric composite plates is given by

$$\rho = V_f \rho_f + (1 - V_f)\rho_m \qquad (28)$$

Where the V_f is the fiber volume fraction

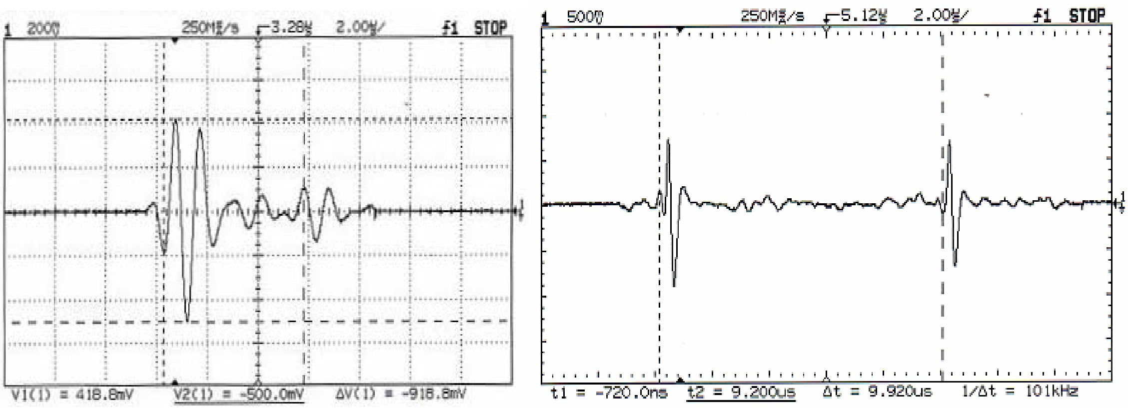
$$V_f = \frac{(\rho - \rho_m)}{(\rho_f - \rho_m)} \qquad (29)$$

The experimental material stiffness properties, Qij are calculated from the equations listed before where the wave velocity is determined experimentally. Karl Deutsch transducer S12W4 was used to propagate longitudinal and shear waves at a frequency of 1 MHz by using a pulse generator USIB 20. A fused quartz delay block was bonded to the specimen using phenyl salicylate. The non-homogeneous composite may be considered as an effective homogeneous medium with low frequency ultrasonic wave [11]. The received signal was displayed on an oscilloscope HP5461B, one of these signals is shown in Fig 2. The separation distance between the transmitted and reflected pulse is a measure of the time required by the waves to go and return through the sample. If the elastic constants of the tested specimens are known, it is possible by solving the equation of motion (Christoffel equation) to obtain the velocities in particular propagation directions. For any given direction of propagation in the present orthotropic fabric specimen, there exist three different modes of wave namely, longitudinal, shear, and pure shear. Elastic constants were estimated from the experimentally measured ultrasonic data in three different steps.

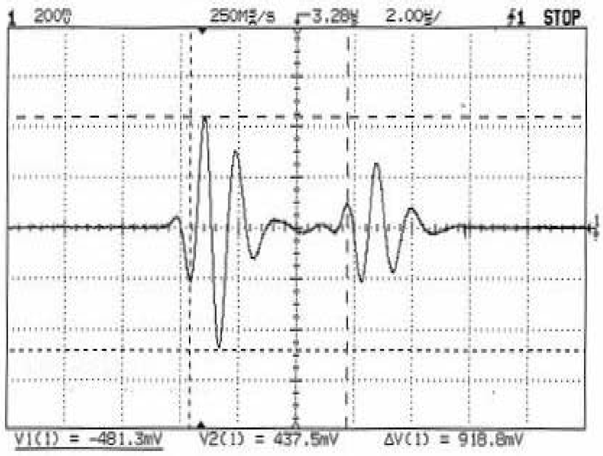
1. Q11 , Q22 and Q33 are determined from the normal incident velocity in the fill and warp directions and in the out plane direction, respectively .
2. Q44 is determined from the shear velocity measured in the plane X1-X2.
3. Q66 is determined from the measured shear velocity in plane X1- X3.

Table1 Mechanical properties of the tested specimen constituents

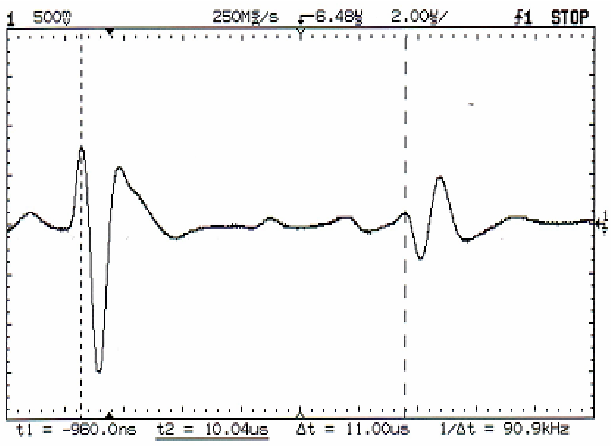
Property	Units	Polyester[13]	Fabric[13] *(weave glass cloth)*	Aluminium [14] *6061*
Density	Kg/m^3	2540	1200	2700
Young's modulus	GPa	3.5 ~ 4.2	70	69
Poisson's ratio	-----	0.37	0.22	0.33



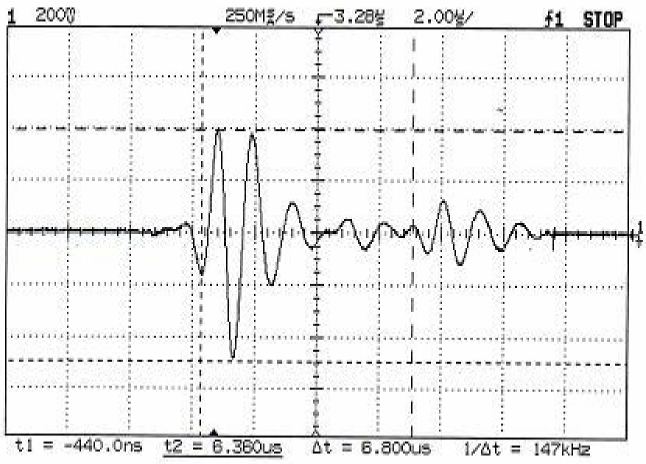
(a) longitudinal wave (a) Shear wave



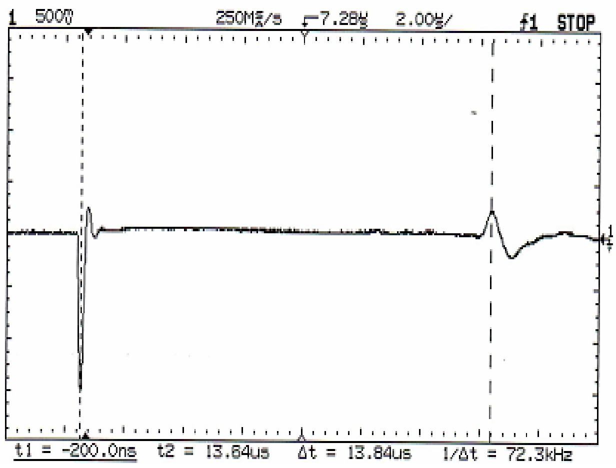
(b) longitudinal wave



(b) Shear wave



(c) longitudinal wave



(c) Shear wave

Fig.2 Typical received signals for (a) aluminium, (b) polyester resin and (c)fabric composites in fill direction, respectively

RESULTS

- In order to validate the measurement well characterized polyester resin (which are used as a matrix of the tested composite materials) and aluminium 6061 samples of 8 mm thickness were used as representative of isotropic material. For all tested samples the ultrasonic time domain signals have been stored. Three trial sets of data were taken for each sample.

- The schematic for contact testing is shown in Fig. 3. The elastic constant Q_{11}, $Q_{22,}$ and Q_{33} are calculated using equations (16, 19, and 20) by measuring the longitudinal velocity C_{11L} , C_{22L} , and C_{33L} ,respectively. The shear velocities were measured at normal incidence, for two polarizations. It means the velocity C_{31T} of wave propagating along axis -3 (out of plane)and polarized along axis-1(fill direction) and velocity C_{32T} of wave propagating along axis-3 and polarized along axis 2 (warp direction). The elastic constants C_{44} or C_{55} were calculating using the equation (26). While the elastic constant Q_{66} was predicted through the equation (27).

- Firstly, the engineering constants of the aluminium plate and polyester resin are measured using the contact testing, and compared with the manufacturer data as given in Table 2. The elastic constants of the woven fabric composite material are calculated by simple numerical inversion of the velocity measured by contact testing. Table 3 shows the elastic constants obtained by the contact testing compared with the results of the numerical solution [12] which is based on the composite constituent properties.

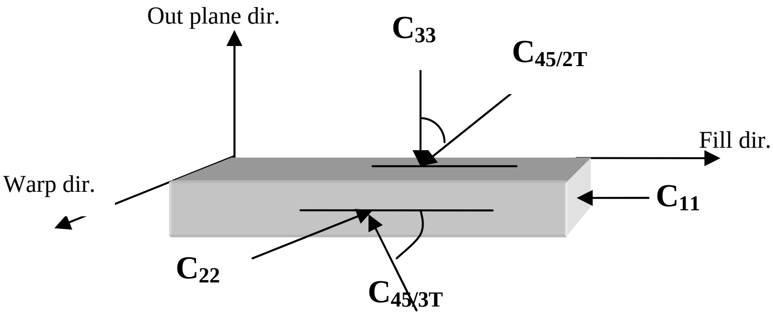


Fig. 3 Schematic of contact testing technique showing the measured velocity directions

The elastic constants of the tested woven fabric composite material are numerically estimated using the following relations which are based on their constituent properties. The Young's modulus and the Poisson's ratio of the fill and warp directions are calculated and taken as an average of the longitudinal and transverse values of the corresponding unidirectional layer. The other relations used are presented here as follow.

$$E_1 = \frac{\left[E_1^2\left(E_1 + 2E_2\right) + \left(1 + 2\upsilon_{12}^2\right)E_2^2\right]}{\left[2E_1\left(E_1 + \left(1 - \upsilon_{12}^2\right)E_2\right) - \upsilon_{12}^2 E_2\right]} \tag{30}$$

$$E_3 = \left(1 - \sqrt{V_f}\right)E_m + \frac{\sqrt{V_f}\,E_m}{1 - \sqrt{V_f}\left(1 - \dfrac{E_m}{E_f}\right)} \tag{31}$$

$$\nu_{23} = \frac{\nu_m}{1 - V_f \nu_m} + V_f \frac{\nu_f - \left(V_m \nu_m\right)}{1 - V_f \nu_m} \tag{32}$$

$$G_{12} = \frac{E_1}{2\left(1 + \nu_{12}\right)} \tag{33}$$

$$G_{23} = \frac{E_2}{2\left(1 + \nu_{23}\right)} = \frac{1}{\dfrac{1 + \upsilon_{23}}{E_2} + \dfrac{1}{2G_{12}}} \tag{34}$$

Table 2 Engineering constants of aluminium plate and polyester resin, (MPa).

Engineering constants, GPa	Aluminium material		Polyester resin	
	Contact testing	Manufacturing data[13]	Contact testing	Manufacturing data[14]
E_{11}	69952	69000	3514	3500 ~ 4200
G_{12}	24740	25940	1457	1277 ~ 1533

Table 3 Elastic constants of woven fabric composites

Elastic constants ,	Contact testing			Numerical solution
	Av. distance, mm	Av. of pulse time, µS	Inversion values GPa	GPa
Q_{11}	12.61	6.30	28.417	34.31
Q_{22}	12.45	6.14	29.268	34.31
Q_{33}	8.06	4.924	18.789	17.36
Q_{44}	12.62	14.06	5.735	7.48
Q_{55}	12.45	16.33	5.735	7.48
Q_{66}	8.38	9.786	5.217	9.36

Table 2 shows that the engineering constants of the aluminium material are slightly higher than the manufacturer data within 1.5 %. For the polyester resin, good agreement is obtained between the measured and manufacturer data of Young's modulus while the engineering modulus of rigidity is slightly lower than the manufacturer one. From the previous results the Poisson's ratio of polyester material is 0.23. Table 3 compares the measured and theoretical elastic constants of the tested woven fabric composite material. It is noted that the measured elastic constants in the fill and warp directions also the shear constants are less than the corresponding theoretical values. While the measured value of the elastic constant in the out plane direction, Q_{33}, is higher than the corresponding theoretical value. This difference may be due to the existence of void and may be due to some misalignment of the stacking sequence of the composite plates.

CONCLUSIONS

The elastic constants of the laminated woven fabric composite materials were determined by the non-destructive ultrasonic technique (pulse echo). Experiments, were also conducted on aluminum (6061) and polyester samples as a representative isotropic materials. Based on the results obtained, the following conclusions can be made.

a) The elastic constants which is obtained from the measuring wave velocity of the isotropic materials are in agreement with the manufacturer data.
b) The longitudinal elastic constants of the orthotropic composite material in the fill and warp directions are lower than the corresponding theoretical values, except the elastic value of the out-plane.
c) The measured values of the shear constants are also lower than the theoretical one.

It is believed that the tools of the non-destructive testing way may be play important role in future material development programs; as they provide a means to characterize the material and better understand of the mechanical properties.

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