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Analysis of the ASTM standards for impulse excitation of vibration and acoustic resonance techniques for rectangular parallelepipeds

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Abstract

Impulse excitation of vibration and acoustic resonance are techniques for determining the elastic constants of solid materials from the lowest resonance frequencies, excited by means of an impulse or by a periodic stimulus. Several geometries can be used for this purpose: parallelepipeds, disks, cylinders, etc. The details of the technique are the subject of many ASTM standards, whose suitability and precision are analyzed in this work in the case of parallelepipeds. The results show that errors of the order of 1% can be expected even if the samples dimensions and resonance frequencies are measured to a much higher precision (better than 0.1%).

Resumen

Las técnicas de resonancia acústica y de excitación de vibración por impacto consisten en excitar la muestra de un material sólido mediante un estímulo periódico o un golpe suave, determinando sus frecuencias de vibración más bajas y calculando a partir de ellas las constantes elásticas del material. Las técnicas se han desarrollado para varias geometrías, paralelepípedos, discos, cilindros, etc. Los detalles de aplicación se encuentran condensados en varias normas de la ASTM, y constituyen el tema de este trabajo para el caso de paralelepípedos. Los resultados muestran que deberían esperarse errores del orden del 1% en las constantes determinadas mediante las fórmulas estándar, aún si las dimensiones y las frecuencias se miden con una precisión mucho mayor (mejor que el 0.1%).

1 Introduction

The sonic resonance and impulse excitation of vibration techniques are well established and widely used techniques for the determination of the dynamic elastic properties of a large diversity of materials (steel, concrete, glass, ceramic, graphite, etc). They are, in particular, covered by several ASTM standards (see for instance E1875-00e1, E1876-07, C1548-02 (2007) and references therein). The basic technique consists in vibrating a sample by either applying a harmonic perturbation or exciting it with an impulse, and inferring the elastic moduli from the measured lowest resonance frequencies. They are simple to apply, and require only some basic hardware: a PC with an appropriate sound card and an inexpensive microphone.

These techniques have been developed for bars of circular, square or rectangular section, for spheres, disk shaped samples, etc. Prismatic bars are simple to prepare, and relevant modes are easily excited. In contrast, the excitation of torsional modes in cylindrical samples is much more complex. As compared to other techniques to determine the elastic properties, both impact excitation of vibration and sonic resonance are highly repetitive, and provide accurate results (Radovic, 2004). This constitutes a strong motivation to improve the currently used analytical formulae in such a way that the full precision allowed by the technique can be achieved, and to assess the error involved in the several analytical expressions recommended by the standards.

The method in its simplest form is only adequate for linear elastic isotropic and homogeneous materials; this is the level of complexity handled by the referenced ASTM standards and taken into consideration in this work. From the early references there was interest in piezoelectrics, crystals and rocks, which require more general elastic constitutive relations. The approach to these problems is mainly numeric, which limits the applicability of the results (Holland 1968, Demarest 1971, Ohno 1976).

On the simpler isotropic case there are several approximate theories that provide analytical formulae to relate the lowest flexural, longitudinal and torsional resonance frequencies to the elastic moduli (Pickett). These formulae have been tested against experimental data by several authors (Spinner & Valore 1958, Spinner, Reichard & Tefft 1960, Spinner & Tefft 1961), leading in some cases to the proposal of empirical correction factors as in Spinner & Valore 1958 for the fundamental torsional resonance that are still commonly used (see for instance ASTM standards E1876-01, C1548-02 (2007), E1875-00e1), although in the latest revision of ASTM standard E1876-07 they were replaced by theoretical approximate expressions from Spinner & Tefft 1961.

Current laboratory equipment allows the measurement of the sample dimensions and resonance frequencies with very high precision, typically better than 0.1 %. The question arises then if the available analytical expressions allow translating this precision to the determination of the elastic moduli and sound speeds. In order to answer this basic question a means to solve the linear elasticity equations to much better precision is needed. In the early references (Holland 1968, Demarest 1971, Ohno 1976) there are quite successful attempts to solve the 3-D elastic problem with zero stress (Neumann) conditions, by means of a variational formulation. They did not, however, systematically explore the differences between the analytical results and the numerical solutions. To be fully reported elsewhere, we developed a numerical approach following the same ideas, that allowed us to compute the resonance frequencies to a very high precision (much better than 0.01%), and made an extensive computational analysis of the available formulae (Etcheverry & Sánchez, 2008). The results of this analysis applied to ASTM standards are summarized below.

2 Results

In the following we consider a prismatic sample of density ρ and dimensions L_x, L_y, L_z, with $L_x \geq L_y \geq L_z$.

2.1 Fundamental flexural mode

The standard expression used in the mechanical resonance techniques to determine the Young modulus E from the measured frequency $F_{f,1}$ is (ASTM Standards E1875-00e1, E1876-07, C1548-02 (2007)):

$$E = 0.9465\,\rho\,F_{f,1}^2\,L_x^2\,\frac{L_x^2}{L_z^2}\,T_1\left(\frac{L_z^2}{L_x^2},\nu\right) \tag{1}$$

where T_1 is a correction factor that takes into account the finite thickness of the bar and the Poisson ratio ν:

$$T_1 = 1 + 6.585(1 + 0.0752\nu + 0.8109\nu^2)\frac{L_z^2}{L_x^2} - 0.868\frac{L_z^4}{L_x^4} - \frac{8.340(1 + 0.2023\nu + 2.173\nu^2)}{1 + 6.338(1 + 0.1408\nu + 1.536\nu^2)\dfrac{L_z^2}{L_x^2}}\frac{L_z^4}{L_x^4} \tag{2}$$

For long and thin samples (small L_z/L_x ratio) the correction is small, and can be well approximated by

$$T_1 = 1 + 6.585\frac{L_z^2}{L_x^2} \tag{3}$$

The numerical analysis shows that the error of formulas (1, 2) is smaller than 1% if the Poisson ratio is smaller than 0.35, and the length-to-width ratio of the sample is larger than about 2. For long and thin samples (small L_z/L_x) the simplified expression (3) is not only much simpler to use because no iteration is required, but can also be more accurate than the full correction (2).

2.2 Fundamental torsional mode

The torsional vibration mode is used to determine the shear modulus G. The expression recommended in ASTM E1876-01, E1875-00e1, C1548-02 (2007), is based on Spinner & Valore 1958:

$$G = 4\rho L_x^2 F_{t,1}^2 \frac{B}{1+A} \tag{4}$$

where B is a theoretical correction factor given by:

$$B = \frac{\dfrac{L_y}{L_z} + \dfrac{L_z}{L_y}}{4\dfrac{L_z}{L_y} - 2.52\left(\dfrac{L_z}{L_y}\right)^2 + 0.21\left(\dfrac{L_z}{L_y}\right)^6} \tag{5}$$

and A is an empirical correction term based on the experimental determination of the resonance frequencies of steel samples by Spinner & Valore 1958:

$$A = \frac{0.5062 - 0.8776\left(\frac{L_y}{L_z}\right) + 0.3504\left(\frac{L_y}{L_z}\right)^2 - 0.0078\left(\frac{L_y}{L_z}\right)^3}{12.03\left(\frac{L_y}{L_z}\right) + 9.892\left(\frac{L_y}{L_z}\right)^2} \tag{6}$$

In the current revision of E1876, ASTM E1876-2007, the correction factor was changed to favor the analytical expressions from Spinner & Tefft 1961:

$$G = 4\rho L_x^2 F_{t,1}^2 R \tag{7}$$

where the correction factor R is given by:

$$R = \frac{1 + \left(\frac{L_y}{L_z}\right)^2}{4 - 2.521\frac{L_z}{L_y}\left(1 - \frac{1.991}{e^{\pi\frac{L_y}{L_z}} + 1}\right)}\left(1 + \frac{0.00851 L_y^2}{L_x^2}\right) - 0.060\left(\frac{L_y}{L_x}\right)^{\frac{3}{2}}\left(\frac{L_y}{L_z} - 1\right)^2 \tag{8}$$

No reason is provided for the change. There is a claim that it should be accurate to within 0.2% for length-to-width ratio larger than 3.3 and width-to-thickness ratio smaller than 10. Otherwise, the stated error is about 1%.

The relative error in G obtained using expressions (4-6) has a complex structure, and can attain a magnitude of several percents. Choosing a length-to-width ratio of about 5 allows to achieve errors smaller than 1% for all Poisson ratios. Formulas (7-8) included in the latest revision of ASTM E1876 (2007) standard (but not so in the still current ASTM E1875-00e1, C1548-02 (2007) standards) have a much better overall accuracy, and can be used with errors smaller than 0.5% in the whole range of Poisson ratios and width-to-thickness ratios, provided the length-to-width ratio is larger than 3.

2.3 Fundamental longitudinal mode

The expressions for the longitudinal vibration mode reported in the referenced ASTM standards are misleading. The correct version should be taken from the Spinner & Tefft 1961 paper:

$$E = 4\rho L_x^2 F_{l,1}^2 \frac{1}{K} \tag{9}$$

K is a correction factor given by:

$$K = 1 - \frac{\pi^2 \nu^2 D^2}{8 L_x^2} \tag{10}$$

where D is an effective sample diameter computed as:

$$D^2 = \frac{2}{3}(L_y^2 + L_z^2) \tag{11}$$

The overall quality of the approximation is quite good, with errors smaller that 0.1% for length/width ratios larger than 3. Errors increase as the length/width ratio diminishes, and also when the Poisson ratio increases.

3 Conclusions

The accuracy of the expressions for impact excitation and resonance techniques included in ASTM standards is analyzed. The results show that provided the sample dimensions are properly chosen, the limit to the accuracy introduced by their use is smaller than about 0.5%. Currently E1876-07 revised formulas for the torsional mode should be preferred.

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