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Atenuación de los modos propios en perforaciones y tuberías debida a la dispersión sobre la superficie rugosa interna

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Abstract

The attenuation data for Stoneley wave and higher order modes in a borehole (well) and in a pipe is considered as an important information about porosity and permeability of the surrounding walls and/or possible wall defects. In the case of the porous wall the attenuation mechanism is associated with the fluid flow through the interface between the wall and the permeable elastic medium surrounding the fluid. However, there is another mechanism which leads to the attenuation of wave in borehole and pipes. Additional attenuation can occur due to modal scattering if the inner wall surface is statistically rough. In this paper the problem of the wave propagation in a circular waveguide with rough, elastic walls is solved using a small perturbation limit and the mean field approach.

Resumen

La atenuación de la onda Stoneley y los modos de orden superior en una perforación y en una tubería se consideran como información importante sobre la porosidad y la permeabilidad de las paredes circunstantes y/o posibles defectos de las paredes. En el caso de paredes porosas el mecanismo de atenuación se asocia con el flujo de fluido a través de la interface entre la perforación y el medio elástico permeable que limita al fluido. No obstante, hay otro mecanismo el cual conduce a la atenuación de la onda en perforaciones y tuberías. La atenuación adicional puede ocurrir debida al scattering modal si la superficie de la pared interna es estadísticamente rugosa. En este informe se resuelve el problema de la propagación de la onda en una guía de ondas circular con paredes elásticas y rugosas usando el límite de pequeñas perturbaciones y la aproximación del campo medio.

1 Introduction

Pipelines are used widely to convey fluids and gases in petro-chemical, water and energy sectors. The quality of the inner pipe wall can deteriorate rapidly because of the chemical reactions, wall material erosion, thermal cracking and sedimentation processes. This leads to the increased hydrodynamic drag in the pipe, reduced hydraulic capacity and potential structural failures. In a majority of cases the direct visual quality inspection of pipes is difficult or impossible because of the operational, safety and access issues. As a result, there is a clear need for quick, inexpensive and accurate methods for the characterization of the boundary conditions in pipes. In this respect, the use of Stoneley and Lamb waves which can propagate long distances along the fluid-solid interface of a buried, fluid filled pipe appears a very attractive noninvasive boundary characterization technique. The frequency dependent phase velocity and attenuation (dispersion characteristics) of these modes are sensitive to the wall thickness and material properties and can be measured in-situ using remote sensors to provide a basis for the inversion problem.

The method to estimate the attenuation coefficient which we propose in this report is based on the long-wave approximation which assumes that the height of roughness is considerably smaller then the wavelength of sound in the filling fluid and in the material of the pipe. The basic method has been detail in the papers [1-3], where it was used to estimate the attenuation of eigen-mode scattering on rough walls of empty and fluid-filled boreholes.

2 The problem statement

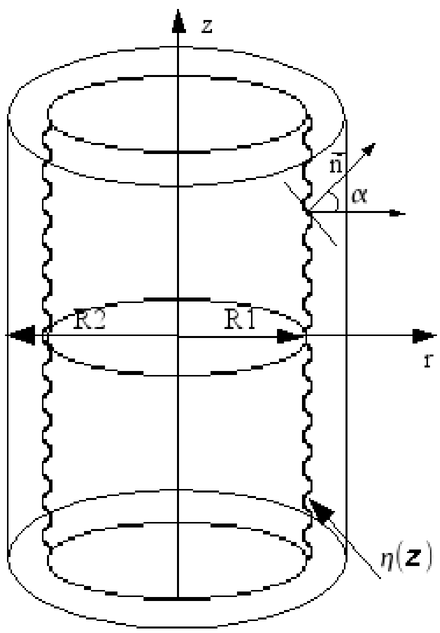

Figure 1. Sound propagation in a pipe with internally rough walls.

The problem is as following: there is infinite fluid-filled pipe with rough inner surface. The pipe is suspended in vacuum. The roughness is assumed cylindrically symmetrical and is given by the random function $\eta(z)$ which defines the spatial deviation of the inner radius, $r = R1 + \eta(z)$, from average inner tube radius, $R1$. The external surface of the pipe is smooth and has outer radius $R2$. The axis of the pipe runs along the z axis. There is a point monochromatic source on the pipe axis. The problem is schematically represented in figure 1. The wave field in the elastic wall is described fully by the scalar and vector potentials. The wave field in the pipe is described by scalar potential only. These potentials satisfy the corresponding wave equations.

The boundary conditions on the inner $r = R1 + \eta(z)$ and outer $r = R2$ surfaces consist of: 1) equal internal and external forces, applied to the inner surface of the pipe ; 2) the equal radial shears in the fluid and in the elastic wall of he pipe; 3) zero forces applied to the external wall of the pipe. Also it is necessary to take into account the singularity at the point $r \to 0$ associated with presence of the point source.

3 Theoretical approach

The boundary conditions can be expanded using the Taylor series in the vicinity of the average radius of the pipe $R1$ so that the matrix integral equation for the coefficients in eigen-functions derived by formulating the standard perturbation problem can be obtained. In this work the solution of this equation has been obtained using the mean field approach. According to this approach small-order complex corrections to the otherwise real eigen-values can be obtained to account for the presence of wall roughness. As result it has been shown that the modal attenuation coefficient is the imaginary part of the small-order correction to the eigen-number and is given by the following expression

$$\alpha_i(\omega) = \frac{\sigma^2 k_s^2}{2\pi(\partial \det L_0(x)/\partial x)|_{x_i^0}} \int_{+\infty}^{-\infty} dx' \, W(k') M\left(x_i^{\,0}, \tilde{x}\right) \big/ \det L_0(\tilde{x}), \tag{1}$$

where $M(x,\tilde{x}) = \sum_{i,j=1}^{5} L_{2ij}(x,\tilde{x}) \operatorname{minor} L_0(x)_{ij}$ is the scattering amplitude, $\operatorname{minor}(L_0(x)_{ij})$ is the matrix minor L_0, c is the roughness amplitude, k_s is the wave number of the shear wave, L_0, L_2 are some 5x5 matrixes, $x = x_i^{\,0} - x'$, $x_i^{\,0}$ are the dimensionless solutions of the dispersion equation for the smooth pipe, W(k') is the Fourier image of the correlation function describing the wall roughness.

These attenuation coefficients may be presented as a sum of partial attenuation factors caused by cross-scattering of eigen-modes. In the case when the pipe is suspended in vacuum, there is no scattering to bulk waves. In this case the attenuation factors may be written in the following analytical form

$$\alpha_{Ri}(\omega) = \frac{\sigma^2 k_s^4}{\left(\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_i^0}\right)} \left\{ \frac{W(k_i^0 - k_j) M(x_i^0, x_j) + W(k_i^0 + k_j) M(x_i^0, -x_j)}{\left.\frac{\partial \det L_0(x)}{\partial x}\right|_{x_j}} \right\}. \tag{2}$$

The first term in the brackets corresponds to the scattering of the i-th eigen-mode into other eigen modes propagating in the same direction (“forward” scattering). The second term corresponds to the scattering of this mode into modes propagating in the opposite direction (“backward” scattering).

Apparently from equation (2) in the case of scattering of one eigen-mode into an eigen mode of the same type, the argument of correlation function in the first term equals zero. Therefore, for large correlation lengths and decreasing spectrum of correlation function only the first term will survive and the forward scattering will give the main contribution to the attenuation coefficient for such process. In this case, the attenuation factor will not depend on the form of the correlation function. In other cases the attenuation factor will depend on the correlation function spectrum.

Apparently from expressions (1), (2) the frequency behavior of the partial attenuation factor is defined by the following factors: the behavior of the scattering amplitude, $\mathrm{M}(\mathrm{x},\tilde{\mathrm{x}})$, and derivatives of the determinant $\partial \det \mathrm{L}_0(\mathrm{x})/\partial \mathrm{x}$ taken along the phase curves and by the spectrum of the correlation function of the wall roughness. The spectrum of this correlation function depends on the parameter $ka = \omega a / c_S$. The scattering amplitude and determinant depend on the parameter $kR1 = \omega R1 / c_s$.

According to its behavior the attenuation factor can be defined at two scales. The forward scattering into the same mode is defined by the partial attenuation factor related to the parameter $kR1 = \omega R1 / c_s$. However, since the values of phase velocities for different modes become close with increasing frequency of sound, this behavior will be dominant for partial attenuation factors in the case of the forward scattering. Thus, it is convenient to analyze behavior of the partial attenuation coefficients for forward and backward scattering at different scales $kR1 = \omega R1 / c_s$ and $ka = \omega a / c_s$.

Besides it is convenient to normalize the attenuation factors so that in the dimensionless form their typical values will be of the order of unit. Taking into account that attenuation factors (2) are proportional to the multiplier $\sigma^2 k_s^4$ and the correlation function spectrum $\mathrm{W(k)}$ which is, in its turn, proportional to the correlation length a, it is possible to choose a normalized multiplier e.g. the magnitude $(\sigma^2 a / R1^4)$ with dimension m^{-1}. Such a normalization amplitude of the modal forward scattering is not dependent both on the form of the correlation function and on the correlation length. It is necessary to note that this choice is not universal and depends on the from form of the correlation function and on the type of modal scattering.

4 Numerical results

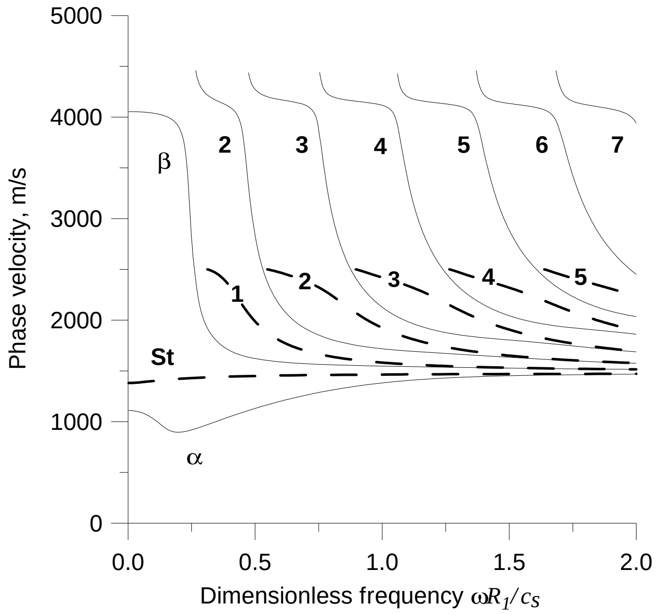

Figure 2. The frequency dependence of the phase velocity of the eigen-modes propagating in the fluid-filled pipe (solid line) and in the well (dashed line) with the same parameters

Figure 2 shows the frequency dependences of the phase velocity of the eigen modes propagating in the fluid-filled pipe with smooth walls. This figure illustrates strong interaction between individual modes leading to the energy exchanges between modes which dispersion curves are close to each other. That promotes more intense migration of energy between modes at presence of a roughness. Unlike the fluid filled borehole case [1-3] there are two main eigen-modes which don't have cutoff frequencies. One of them is so-called mode propagating in the fluid; the other is so-called mode propagating mainly in the elastic wall. The phase velocity of higher-order modes decreases as the frequency increases from the longitudinal wave velocity limit at the cutoff frequency of the pipe.

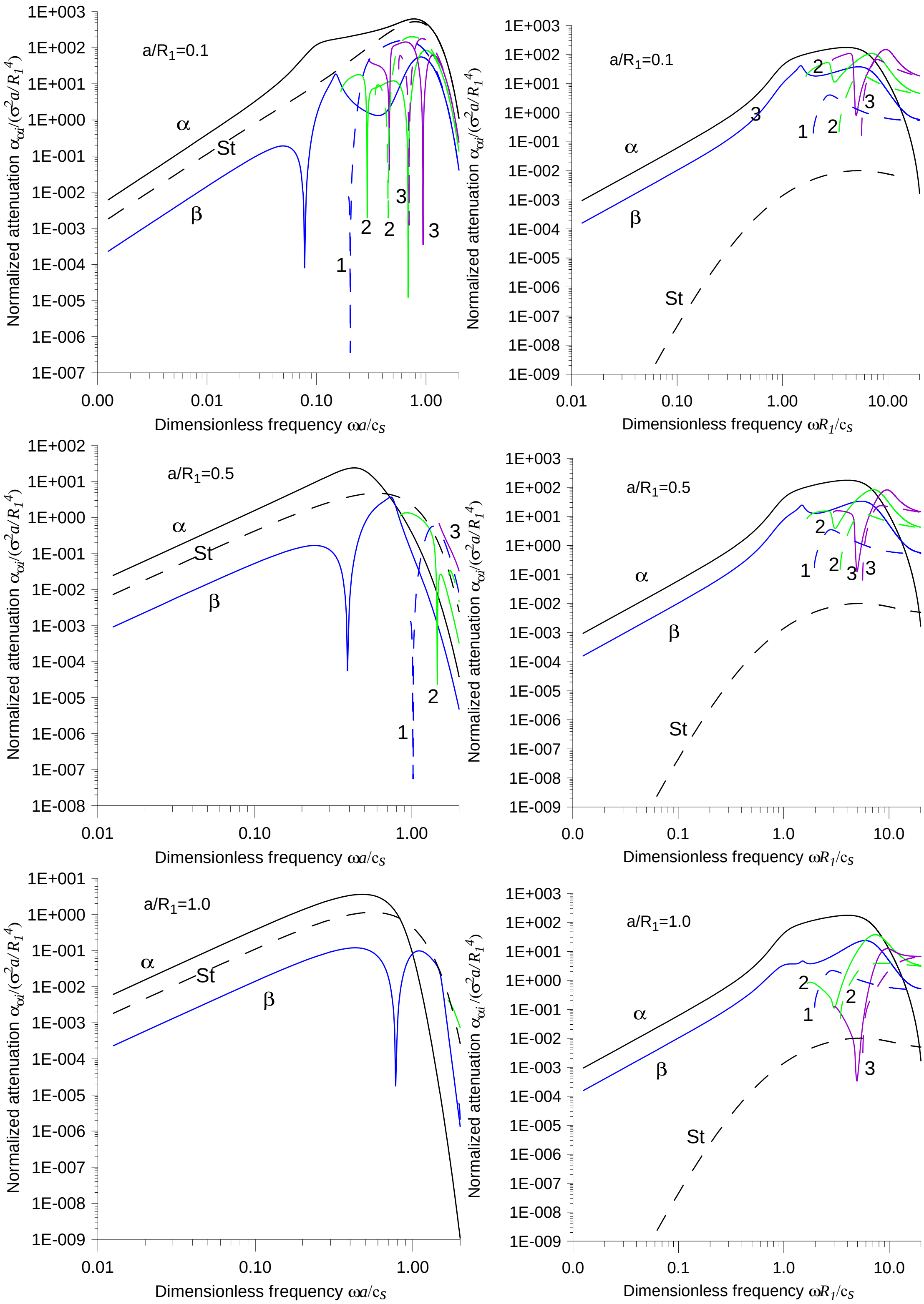

Figure 3. The frequency dependence of the partial attenuation factors of -mode. Forward scattering of eigen-modes is shown on the left plots. Backward scattering is shown on the right plots. Symbols and mark attenuation factors connected with scattering into and modes, respectively. Indices 0 and 1 correspond to the attenuation connected to scattering to the higher-order modes.

Figure 3 shows the frequency dependence of the partial attenuation factors caused by the forward and backward scattering of mode into itself and other higher-order modes for different values of the ratio $a/R1$. As a comparison, figure 3 also presents the partial attenuation coefficients for the Stoneley wave in the borehole. The following Gaussian roughness correlation function was used to obtain this set of results

$$W(x) = e^{-x^2/a^2}, \quad W(k) = a\sqrt{\pi}e^{-(ka)^2/4}. \tag{3}$$

The data for the partial attenuation coefficients are normalized by the factor $\sigma^2 a / R_1^4$ and presented for a range of ratios of the roughness correlation length to the inner pipe radius a/R_1. The modal attenuation due to the forward- and back-scattering is controlled by the roughness correlation length and by the inner pipe radius. Therefore, the frequency scales in these graphs are different and normalized by a/c_s and R_1/c_s, respectively. The results presented in figure 3 show that the modal attenuation due to the wall roughness is a combination of the forward- and back-scattering of the α-mode into itself and in other higher-order modes. In the case of the wall roughness with a small correlation length, i.e. $a/R_1 << 1$, the back-scattering is the dominant mechanism of attenuation. As the ratio a/R_1 increases the modal attenuation due to forward-scattering becomes more pronounced and dominates for $a/R_1 \cong 1$. The results also suggest that for $a/R_1 > 0.1$ the attenuation due to back-scattering is more pronounced in the low frequency range whereas the attenuation due to forward-scattering is more pronounced at the higher frequencies of sound.

Conclusions

In general, the effect of the wall roughness on the acoustic attenuation due to cross-modal scattering in the pipe is greater than that in the borehole. It is interesting to note that there is a strong resemblance in the spectra for the modal back-scattering coefficients in the borehole and in the pipe. However, the spectra for the forward-scattering coefficients for these two cases are markedly different, particularly in the low frequency regime. The results presented figure 3 suggest that in the low frequency regime in the pipe the attenuation of the α-mode in itself and in the α-mode due to back-scattering by the wall roughness can be described by a quadratic function of the frequency. A similar behavior is observed in the case of the Stoneley wave in the borehole. The behavior of the attenuation due to the forward-scattering of these modes shown in figure 3 is considerable different in the case of the pipe and borehole. The behavior of the forward-scattering attenuation of the α-mode is also quadratic and by several orders of magnitude exceeds the analogous attenuation of the Stoneley wave in the borehole.

The results obtained in this work can be used to develop a method for the invasive inspection of the wall quality in pipes and boreholes from attenuation data. The predicted modal frequency-dependent attenuation behavior is linked strongly to the statistical parameters of the wall roughness can be used to determine the roughness mean height and dominant correlation length.

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