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An algorithm for detection and isolation of faulty vibration modes based on a spectral decomposition of the plant time response

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Abstract

This work presents an algorithm for detection and isolation of faulty vibration modes based on the spectral decomposition of the plant time response. A Hankel matrix is built from plant output measurements and its singular values are used to detect and identify plant parameters related to faulty vibration modes. It is shown that the detection (alarm generation) and isolation (alarm interpretation) tasks are easily performed based on the proposed algorithm. Examples are finally presented to illustrate the performance and application of the proposed algorithm.

Resumen

Este trabajo presenta un algoritmo para la detección y aislamiento de modos de vibración defectuosos, el algoritmo es basado en la descomposición espectral de la respuesta de tiempo de la planta. Una matriz de Hankel se construye a partir de mediciones de la salida de la planta. Los valores singulares de esta matriz se usan para detectar e identificar los parámetros de la planta relacionados con los modos de vibración defectuosos. Se muestra como la detección (generación de la alarma) y el aislamiento (interpretación de la alarma) de la falla son tareas fácilmente realizadas con el algoritmo propuesto. Finalmente, se presentan ejemplos que ilustran el desempeño y aplicación del algoritmo propuesto.

1 Introduction

The development of safer and more reliable control systems has been an increasingly need in the last decades. To full-fill the modern standards, the control systems design must include fault detection and isolation issues at their very early design stage, Delmaire, Cassar and Staroswiecki (1995), Iserman (1984). The ultimate goal of these systems is to reach a fault-tolerant control (FTC) environment.

Fault detection and isolation (FDI) schemes are implemented as real-time algorithms whose inputs are plant output observations. They are used for a) fault detection: to decide whether the plant is in a normal operating condition or in a faulty one and b) fault isolation: to point out and identify the kind of the fault (if present) among a given fault set. Following the FDI diagnosis, on-line procedures are usually needed for FTC purpose, while off-line procedures could be used for maintenance purpose.

During the last decades, the international scientific community has presented several fine works, Iserman (1984), Yu and Shields (1995). Two main streams can be identified, control related techniques and artificial intelligence based methods. System theory, signal processing or artificial intelligence approaches have been extensively used according to the available data format. Most of the model-based and non-model-based techniques have been developed based on the comparison of the data produced by the real-time plant operation with some previously obtained knowledge of the system.

This paper presents a novel FDI algorithm based on the singular value decomposition of a Hankel matrix built from plant output measurements. The main feature of the proposed algorithm is that it does not rely on plant models. All it is required is a plant signature that can be experimentally obtained. The paper is organized as follows: Section 2 includes some comments on the FDI problem; Section 3 presents the basic formulation of the Eigensystem Realization Algorithm (ERA); Section 4 introduces the singular values based fault detection and isolation (SVFDI) algorithm; Section 5 explores the SVFDI algorithm features through experimental results; and finally, Section 6 presents final comments and conclusions.

2 Some comments on the FDI problem

FDI algorithms use the plant input-output measurements to implement a two-steps procedure: the fault detection and the isolation tasks. The first step is the fault detection step or alarm generation. The problem of the alarm generation is to decide whether the system is in a normal operating condition or not. The set of output measurements along with a previously obtained knowledge of the system constitute the algorithm inputs while a set of generated alarms are the algorithm outputs. The second step consists on the alarms interpretation. The main issue in this case is to correctly decide which faults are present (fault isolation) chosen from a pre-defined fault set. It is also of one's interest to establish their characteristics such as occurrence time, fault size, class, consequences, etc. The input is the set of alarms and the output is the faults isolation, characterization and diagnosis. In the case of FTC, further analysis is usually required to determine whether the system is still capable to perform properly after the failure. The algorithm performance is an important issue that must always be considered. The decisions taken at every step of the FDI problem solution might include and accumulate evaluation errors. The measured variables may include noise and load perturbations that might obscure system failures. Also the knowledge one has about the system normal operation might include uncertainties. Detection errors and false alarms can be confirmed by their probability of occurrence. Incomplete isolation and false isolation errors can be evaluated by comparison based on the faulty events probability of occurrence.

3 The basics of the Era Algorithm

This section presents the basic formulation of the Eigensystem Realization Algorithm (ERA), as originally proposed by Juang and Pappa in 1985. Since then, the scientific community has proposed several modifications and improvements. The ERA algorithm is a very reliable computational procedure originally proposed for modeling of dynamic systems. For the sake of simplicity and without lost in the argument, this work focuses on the original algorithm. In the following, all posterior algorithm improvements, less important derivation steps and several results have been omitted. Consider a state space realization for a linear time-invariant discrete-time dynamic system given by

$$\begin{aligned} x(k+1) &= A(\theta)x(k) + B(\theta)u(k) \\ y(k) &= C(\theta)x(k) + v(k) \end{aligned} \quad (1)$$

where, $[A, B, C]$ defines a discrete-time state space realization, x is a n -dimensional state vector, u an m -dimensional control input, y a p -dimensional measurement vector and v represents measurement noise. The system impulse response sequence is given by

$$h(k+1) = y(k+1) = CA^k B \quad (2)$$

A Hankel matrix can be constructed from the impulse response sequence as

$$H(k) = \begin{bmatrix} h(k+1) & h(k+2) & h(k+3) & \dots \\ h(k+2) & h(k+3) & h(k+4) & \dots \\ h(k+3) & h(k+4) & h(k+5) & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} = \begin{bmatrix} CA^k B & CA^{k+1} B & CA^{k+2} B & \dots \\ CA^{k+1} B & CA^{k+2} B & CA^{k+3} B & \dots \\ CA^{k+2} B & CA^{k+3} B & CA^{k+4} B & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (3)$$

also

$$H(0) = \begin{bmatrix} CB & CA^1 B & CA^2 B & \dots \\ CA^1 B & CA^2 B & CA^3 B & \dots \\ CA^2 B & CA^3 B & CA^4 B & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} = \begin{bmatrix} \bar{C} & \bar{B} \end{bmatrix} \quad (4)$$

with

$$\bar{C} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad \& \quad \bar{B} = \begin{bmatrix} B & AB & A^2 B & \dots & A^{n-1} B \end{bmatrix} \quad (5)$$

where \bar{C} and \bar{B} are the observability and controlability matrices, respectively. Also, it should be noted that, usually, $H(0)$ is not square and that $\dim(H(0)) = np \times nm$ and $\text{rank}(H(0)) \leq n$. From the singular value decomposition (SVD)

$$H(0) = M \Sigma N^T = M \begin{bmatrix} D & | & 0 \\ \hline 0 & | & 0 \end{bmatrix} N^T = M \begin{bmatrix} I_n \\ 0 \end{bmatrix} D [I_n \ | \ 0] N^T \quad (6)$$

then

$$H(0) = P D Q^T = \bar{C} \bar{B} \quad (7)$$

and

$$H^+ = N \Sigma^+ M^T = Q D^{-1} P^T \quad (8)$$

It is known that, there exist matrices E_p and E_m such that

$$h(k+1) = E_p^T H(k) E_m \quad ; \quad H(k) = \bar{C} A^k \bar{B} \quad (9)$$

and that

$$H(0) = \bar{C} \bar{B} = P D Q^T \quad (10)$$

then

$$\begin{aligned} h(k+1) &= E_p^T H(k) E_m = E_p^T \bar{C} A^k \bar{B} E_m \\ &= [E_p^T] [P D] [D^{-1} P^T] [H(k)] [Q D^{-1}] [D Q^T] [E_m] \\ &= [E_p^T P D^{1/2}] [D^{-1/2} P^T H(1) Q D^{-1/2}]^k [D^{1/2} Q^T E_m] \end{aligned} \quad (11)$$

finally, a minimal order realization can be found as

$$A = [D^{-1/2} P^T H(1) Q D^{-1/2}] \ ; \ B = [D^{1/2} Q^T E_m] \ \& \ C = [E_p^T P D^{1/2}] \quad (12)$$

Besides that, Juang and Pappa have also proposed two quantitative criteria to eliminate modal frequencies created by measurement noise, Juang and Pappa (1986).

4 The singular values based fault detection and isolation algorithm

The proposed SVFDI algorithm can be seen as a generalization of the ERA algorithm (originally applied for model identification). It will be shown later that in the case of the SVFDI problem there is no need for a plant model, all one needs is the singular values of the Hankel matrix built from the plant time response, as shown in the previous section.

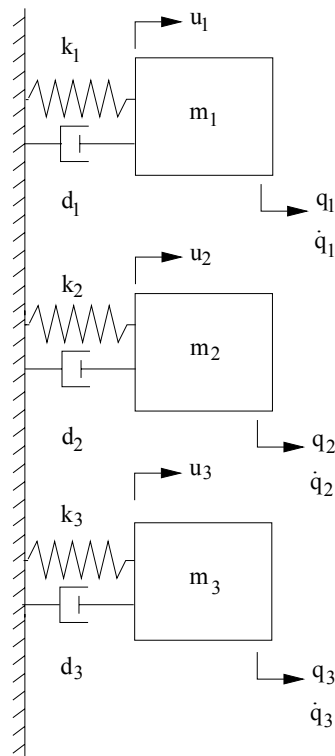
In the following and for the purposes of the SVFDI algorithm we shall call an observable plant parameter if any drift from its nominal value can be detected from output measurements. Also, it is assumed that in a close neighborhood of its nominal value a parameter drift will “friendly” affect the singular values of the Hankel matrix. Finally, it is also clear that observability and controllability properties of the plant (as their standard definitions) also play important roles in the performance of the SVFDI algorithm. In this context, the set of singular values can be considered a natural choice for detecting parametric drifts and failures. The singular values set can be interpreted as an image of the plant parameters. Assuming this fact, it can be established a relationship between the singular values and the plant parameters using standard correlation analysis and use these singular values as flags to indicate any parameter drift from its nominal value. The choice of singular

values as a measure to detect parametric drift is due to the fact that its nature (real positive numbers) does not change as natural frequencies and eigenvalues do when plant parameters change. Under “normal” operational conditions any change of plant parameters values would affect the system dynamics and in a final analysis the singular values of the Hankel matrix.

In a statistic framework, correlation does not imply causality meaning that correlation cannot be validly used to infer a causal relationship between variables. Consequently, correlation between variables is a necessary but not a sufficient condition to establish a causal relationship. However, having established causality, and in a close-enough neighborhood of the nominal plant, correlation can be taken as the natural choice for analysis. The correlation analysis will deliver a mapping of the plant parameters drifts into the singular values set of the Hankel matrix built from the plant time response. The proposed procedure for fault detection and isolation is depicted in the following section through examples. To illustrate the features of the proposed technique, two lumped parameter models have been chosen as shown next.

5 Experimental results

Example I - Let us consider the spring-mass-dashpot system shown in Figure 1.



$$T = \frac{1}{2} \{ m_1 \dot{q}_1^2 + m_2 \dot{q}_2^2 + m_3 \dot{q}_3^2 \}$$

$$V = \frac{1}{2} \{ k_1 q_1^2 + k_2 q_2^2 + k_3 q_3^2 \}$$

$$R = \frac{1}{2} \{ d_1 \dot{q}_1^2 + d_2 \dot{q}_2^2 + d_3 \dot{q}_3^2 \}$$

T is the kinetic energy,
V is the potential energy and
R is the Raleigh dissipation function.

Parameter	Value
$m_1 =$	1
$m_2 =$	1
$m_3 =$	1
$d_1 =$	0.0600
$d_2 =$	0.0200
$d_3 =$	0.0020
$k_1 =$	16
$k_2 =$	6.0
$k_3 =$	0.4

Figure 1. The Spring-Mass-Dashpot System for Example I.

Then, the differential equations of motions are given by

$$m_1 \ddot{q}_1 + d_1 \dot{q}_1 + k_1 q_1 = u_1$$

$$m_2 \ddot{q}_2 + d_2 \dot{q}_2 + k_3 q_2 = u_2$$

$$m_3 \ddot{q}_3 + d_3 \dot{q}_3 + k_3 q_3 = u_3$$

in matrix form, with $u_1 = u_2 = u_3 = u$, one has

$$M \ddot{q} + D \dot{q} + K q = F u \quad \text{with}$$

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}; \quad D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}; \quad K = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \quad \& \quad F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

then, a state space representations can be written as

$$\dot{x} = [A] x + [B] u; \quad y = [C] x; \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad \text{with}$$

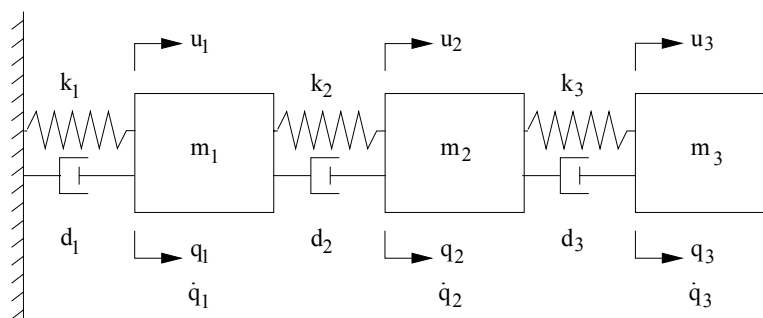
$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \quad \& \quad C = [I \quad 0]$$

then

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -16.0000 & 0 & 0 & -0.0600 & 0 & 0 \\ 0 & -6.0000 & 0 & 0 & -0.0200 & 0 \\ 0 & 0 & -0.4000 & 0 & 0 & -0.0020 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x \quad \text{with} \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

Example II – Let us consider the system presented in Figure 2.



$$T = \frac{1}{2} \{ m_1 \dot{q}_1^2 + m_2 \dot{q}_2^2 + m_3 \dot{q}_3^2 \}$$

$$V = \frac{1}{2} \{ k_1 q_1^2 + k_2 (q_2 - q_1)^2 + k_3 (q_3 - q_2)^2 \}$$

$$R = \frac{1}{2} \{ d_1 \dot{q}_1^2 + d_2 (\dot{q}_2 - \dot{q}_1)^2 + d_3 (\dot{q}_3 - \dot{q}_2)^2 \}$$

T is the kinetic energy,
 V is the potential energy and
 R is the Raleigh dissipation function.

Parameter	Value
$m_1 =$	1
$m_2 =$	1
$m_3 =$	1
$d_1 =$	0.0600
$d_2 =$	0.0200
$d_3 =$	0.0020
$k_1 =$	16
$k_2 =$	6.0
$k_3 =$	0.4

Figure 2. The Spring-Mass-Dashpot System for Example II.

In this case, the differential equations of motions are given by

$$\begin{aligned}
 m_1 \ddot{q}_1 + (d_1 + d_2) \dot{q}_1 - d_2 \dot{q}_2 + (k_1 + k_2) q_1 - k_2 q_2 &= u_1 \\
 m_2 \ddot{q}_2 - d_2 \dot{q}_1 + (d_2 + d_3) \dot{q}_2 - d_3 \dot{q}_3 - k_2 q_1 + (k_2 + k_3) q_2 - k_3 q_3 &= u_2 \\
 m_3 \ddot{q}_3 - d_3 \dot{q}_2 + d_3 \dot{q}_3 - k_3 q_2 + k_3 q_3 &= u_3
 \end{aligned}$$

in matrix form, with $u_1 = u_2 = u_3 = u$, one has

$$M \ddot{q} + D \dot{q} + K q = F u \quad \text{with}$$

$$D = \begin{bmatrix} d_1 + d_2 & -d_2 & 0 \\ -d_2 & d_2 + d_3 & -d_3 \\ 0 & -d_3 & d_3 \end{bmatrix} \quad \& \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

and with M, F, C and x defined as in the previous example, then

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -22.0000 & 6.0000 & 0 & -0.0800 & 0.0200 & 0 \\ 6.0000 & -6.4000 & 0.4000 & 0.0200 & -0.0220 & 0.0020 \\ 0 & 0.4000 & -0.8000 & 0 & 0.0020 & -0.0040 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = [C] x \quad \text{with} \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

It should be notice that in both examples the plant observability matrix is ill conditioned as shown in Table 1. Despite of that the proposed technique still delivers good results. Table 2 presents the systems eigenvalues and the nominal singular values of the plants.

Table 1. Conditioning Numbers for Examples I and II.

Uncoupled System Observability Matrix Conditioning Number	Coupled System Observability Matrix Conditioning Number
$\gamma = 249.2320$	$\gamma = 454.7896$

Table 2. System Eigenvalues and Singular Values for Examples I and II.

Uncoupled System Eigenvalues	Coupled System Eigenvalues	Uncoupled System Singular Values	Coupled System Singular Values
-0.0300 + j 3.9999	-0.0431 + j 4.9030	56.4659	46.4186
-0.0300 - j 3.9999	-0.0431 - j 4.9030	51.7111	39.5797
-0.0100 + j 2.4495	-0.0080 + j 2.0974	0.1352	24.5874
-0.0100 - j 2.4495	-0.0080 - j 2.0974	0.0000	24.1515
-0.0010 + j 0.6325	-0.0019 + j 0.8715	0.0000	10.6838
-0.0010 - j 0.6325	-0.0019 - j 0.8715	0.0000	6.9848

Figures 3 through 5 present several dynamic results of the plants used in the Examples I and II. They are placed side by side for comparison purposes.

The Hankel matrix in both cases was built from the impulse responses results shown in Figures 3a and 3b, respectively.

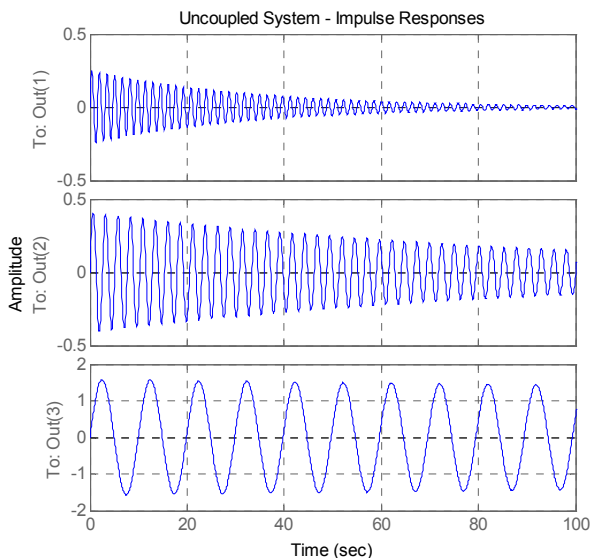


Figure 3a. Impulse Responses for Example I.

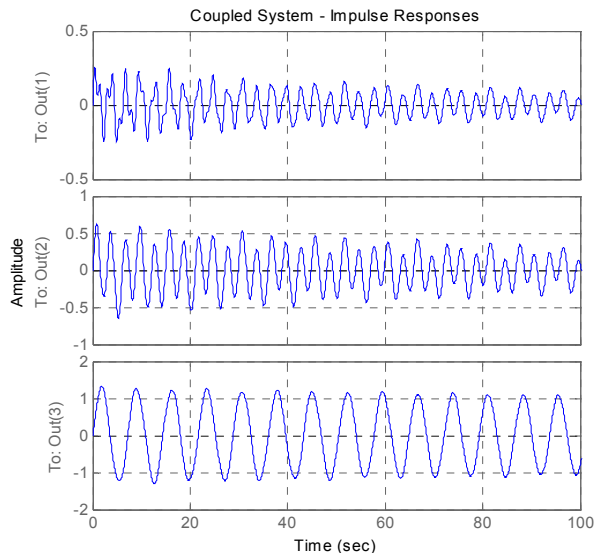


Figure 3b. Impulse Responses for Example II.

Figures 4a and 4b present the frequency responses for Examples I and II, respectively.

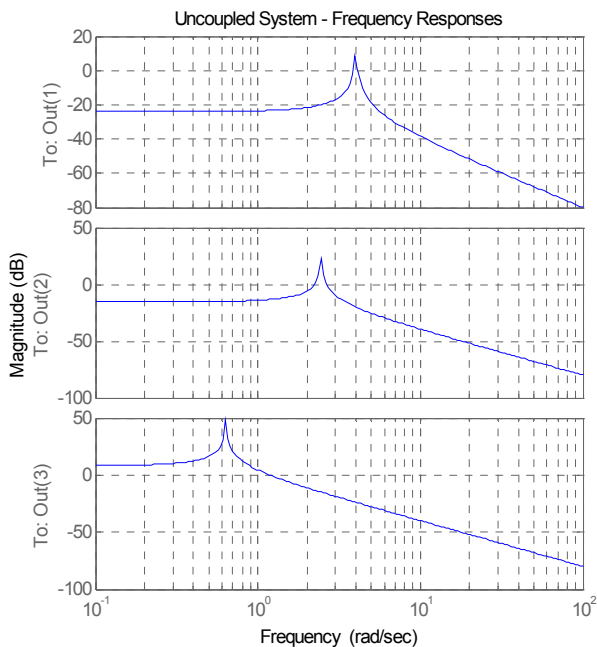


Figure 4a. Frequency Responses for Example I.

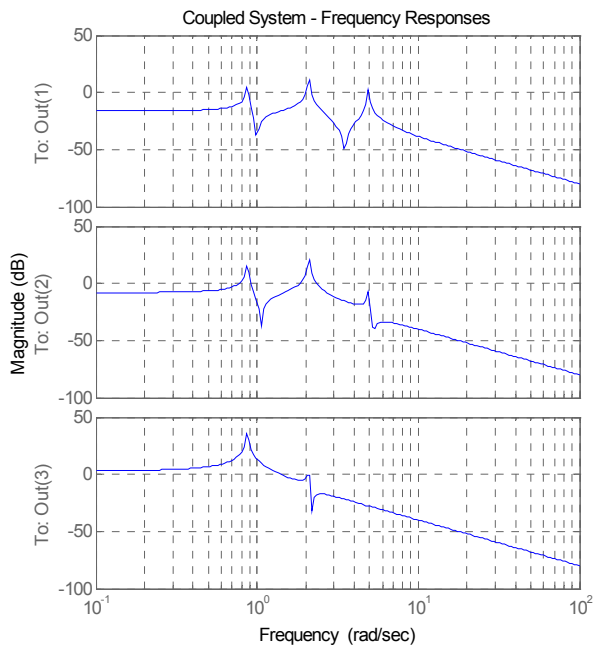


Figure 4b. Frequency Responses for Example II.

Figures 5a and 5b present the power spectrum densities for Examples I and II, respectively.

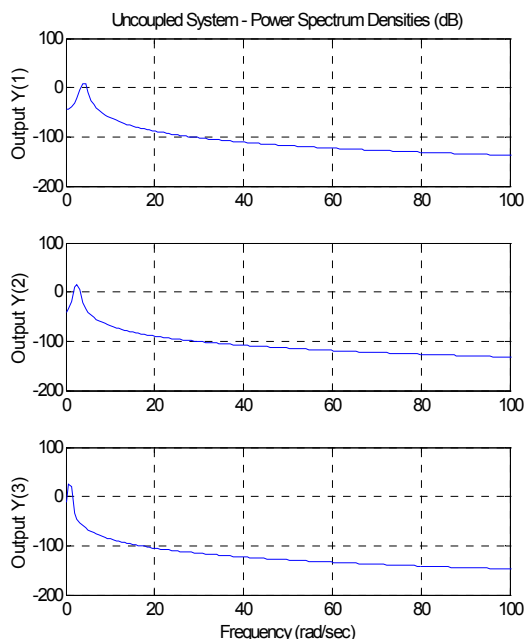


Figure 5a. Spectrum Densities for Example I.

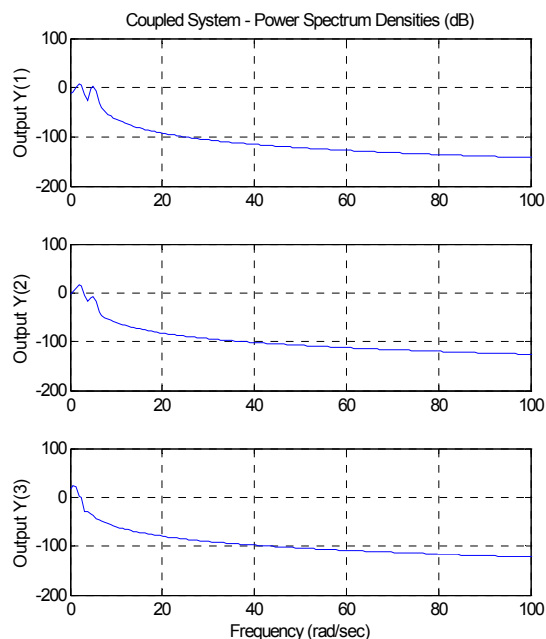


Figure 5b. Spectrum Densities for Example II.

Using standard regression analysis techniques, the correlation coefficients between plant parameters and the singular values of the Hankel matrix were calculated and normalized such that it has been assigned the value of “1” to the greatest coefficient and “0” to the smallest. The results are depicted on Table 3. A “one” would mean strong correlation and a “zeros” a weak or inexistent correlation.

Table 3 can be used to select the best singular values to be used as “flags” based on their correlation with plant parameters. From Table 3 one can built Table 4 that presents the structural sensitivity of the singular values with respect to parameter drifts.

Table 3. Correlation Coefficients for Examples I and II.

Correlation Coefficients for the Uncoupled System							Correlation Coefficients for the Coupled System						
	SV1	SV2	SV3	SV4	SV5	SV6		SV1	SV2	SV3	SV4	SV5	SV6
m_1	0.901	1	0.353	0.082	0	0.077	m_1	0.157	0	0.493	0.424	1	0.905
d_1	1	1	1	0.714	0	0.378	d_1	0.723	1	0.730	0.624	0	0.298
k_1	0.964	1	0	0.341	0.506	0.592	k_1	1	0.838	0	0.792	0.931	0.610
m_2	0.657	0.657	0.657	0.738	0	1	m_2	1	0.994	0.414	0.361	0	0.391
d_2	1	1	1	0.655	0	0.204	d_2	0.841	0.975	0.604	0.485	0	1
k_2	0.029	0.029	0.029	1	0	0.782	k_2	0	0.594	0.993	1	0.644	0.199
m_3	0.200	0.200	0.200	0	1	0.478	m_3	0.691	0.710	1	0.405	0	0.864
d_3	1	1	1	0	0.516	0.484	d_3	0.479	0.647	0.999	1	0.409	0
k_3	0.102	0.102	0.102	0.960	1	0	k_3	0.998	1	0	0.998	0.976	0.954

Table 4. Structural Sensitivity Coefficients for Examples I and II.

Structural Sensitivity Coefficients for the Uncoupled System							Structural Sensitivity Coefficients for the Coupled System						
	SV1	SV2	SV3	SV4	SV5	SV6		SV1	SV2	SV3	SV4	SV5	SV6
m_1		1					m_1					1	
d_1	1	1	1				d_1		1				
k_1	1	1					k_1	1					
m_2						1	m_2	1	1				
d_2	1	1	1				d_2		1				1
k_2				1			k_2			1	1		
m_3					1		m_3			1			
d_3	1	1	1				d_3			1	1		
k_3				1	1		k_3	1	1		1	1	1

6 Final comments and conclusions

This paper presented a fault detection and isolation algorithm based on plant output measurements. In a close neighborhood of the nominal plant values regression analysis has shown to be the proper choice to link the Hankel matrix singular values with plant parameters. An important feature of the SVFDI algorithm is that its formulation does not require a plant model. Having obtained a nominal plant image through the singular values of the Hankel matrix; this image can be used to determine, by comparison, any value drift of the plant parameters. Two functional levels of SVFDI procedure can be distinguished, namely alarm generation and alarm interpretation. At the alarm generation level (detection) the SVFDI algorithm naturally displays plant failure through the change of the singular values structure and values. At the alarm interpretation level (isolation) the SVFDI algorithm delivers an image of the plant parameters through the singular values allowing the identification of the faulty parameter. Finally, the proposed SVFDI algorithm was applied to ill conditioning plants showing outstanding performance in solving both, detection and isolation problems.

7 References

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