Inertial shaker as hybrid active/passive dynamic vibration absorber

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ABSTRACT

The reduction of structural vibrations to minimize the emission of airborne noise, is a commonly known engineering issue. Beside passive methods, active methods known as Active Vibration Control (AVC) or Active Structural Acoustic Control (ASAC) become more important. Their advantage is the use of lower mass in comparison to passive methods. The active control is based on the assumption of linear system dynamics, which is not always sufficient for real systems. In this paper an inertial shaker reduces vibrations of a single degree of freedom system. The parameters for the passive part are determined analytically and numerically. A PD controller is designed for the active part. The combined system is simulated. The total absorption is compared to the passive absorber. The simulation shows good performance of the combined system for the vibration as well as for the relative movement between the absorber and the main mass.

Keywords: Noise Vibration Harshness, vibration absorber, Annoyance
I-INCE Classification of Subject Number: 30

1. INTRODUCTION

The suppression of vibrations has different reasons, for instance the improvement of stability of materials and products, reduction of noise or improvement of working conditions. Numerous methods to reduce vibrations are developed over the years. The first method were passive dynamic vibration absorber, developed by H. Frahm 1909 [1]. Furthermore, active methods get popular, for example the usage of piezo elements on different main structures, like cantilever beams [2] or cantilever plates [3]. The research for active vibration control of plates with piezo elements is reviewed in [4]. Beside piezo elements, voice coil devices are used for vibration reduction in different forms, with force application to the main structure [5,6] or to a special form of the absorber [7].

This paper deals with a simplified situation, see Figure 1.1. The main structure is regarded as a single mass m_1 which is supported by four springs with overall stiffness k_1 to two stiff crossbeams. A modal exciter generates an excitation force F_e. An inertial shaker with mass m_2 acts as active dynamic vibration absorber (DVA). It consists of a permanent magnet which is connected by two plate springs to the stiff housing, see Figure 1.2. A current-carrying coil generates an electromagnetic force between housing and permanent magnet.

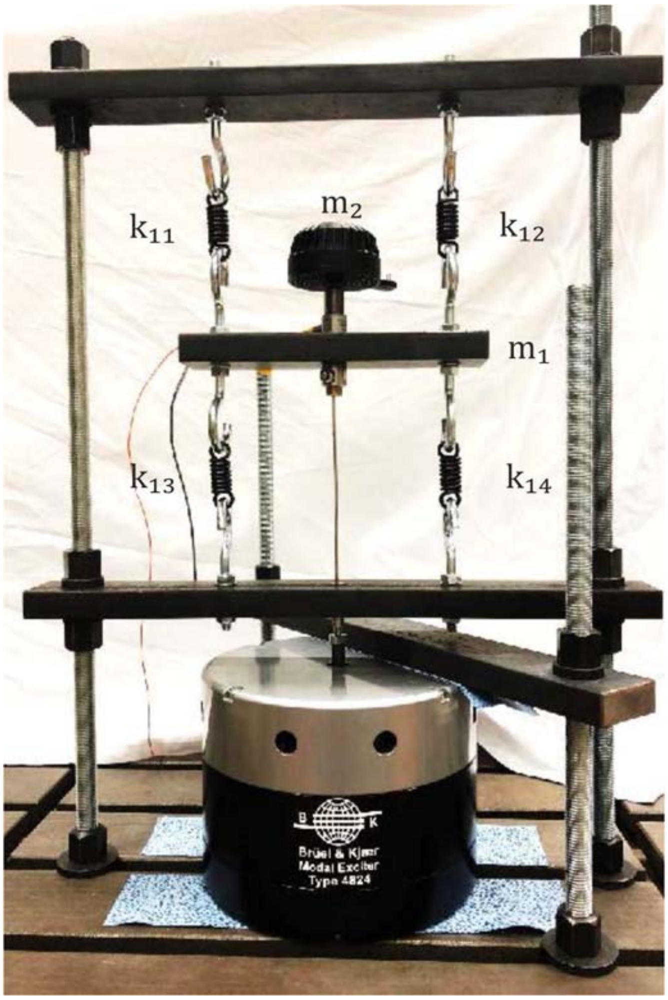

Figure 1.1: Experimental setup with inertial shaker

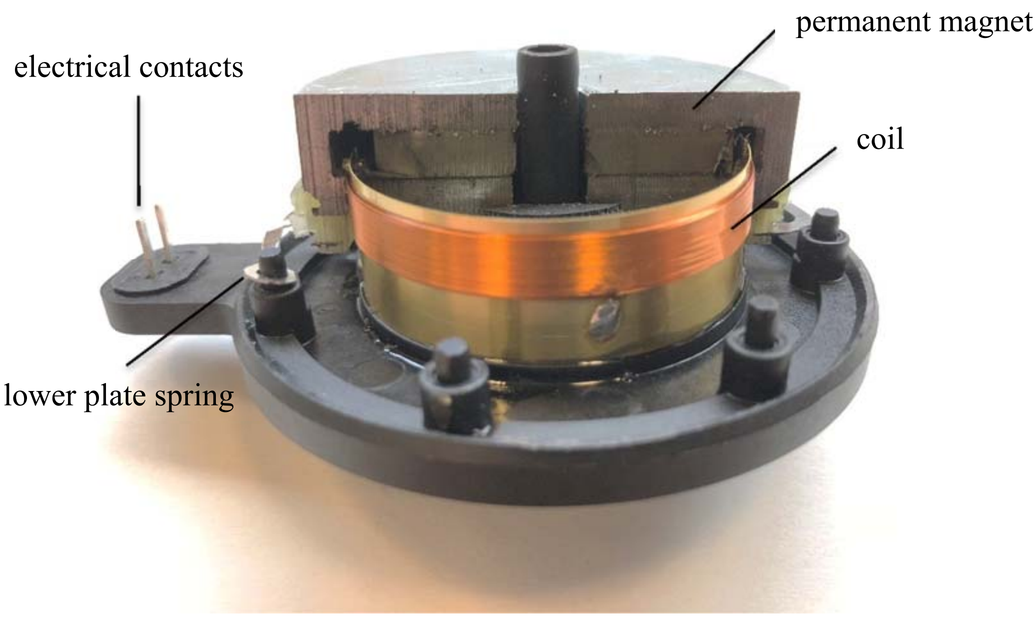

Figure 1.2: DVA without housing and upper spring in sectional view

2. SYSTEM MODEL OF ACTIVE DUAL MASS SYSTEM

Figure 2.1 shows the electro-mechanical model. The system consists of the mass of the main structure m_1 and the mass m_2 of the dynamic vibration absorber (DVA). Both are connected by the spring with stiffness k_2 and the damper with coefficient b_2. The external force F_e acts on the main mass. The main mass is supported by the spring with stiffness k_1 and the damper with coefficient b_1. The coil is modelled by resistance R and inductance L. The converter constant θ couples the electric circuit with the mechanic dual mass system. The voltage U is applied between the electrical contacts at the coil.

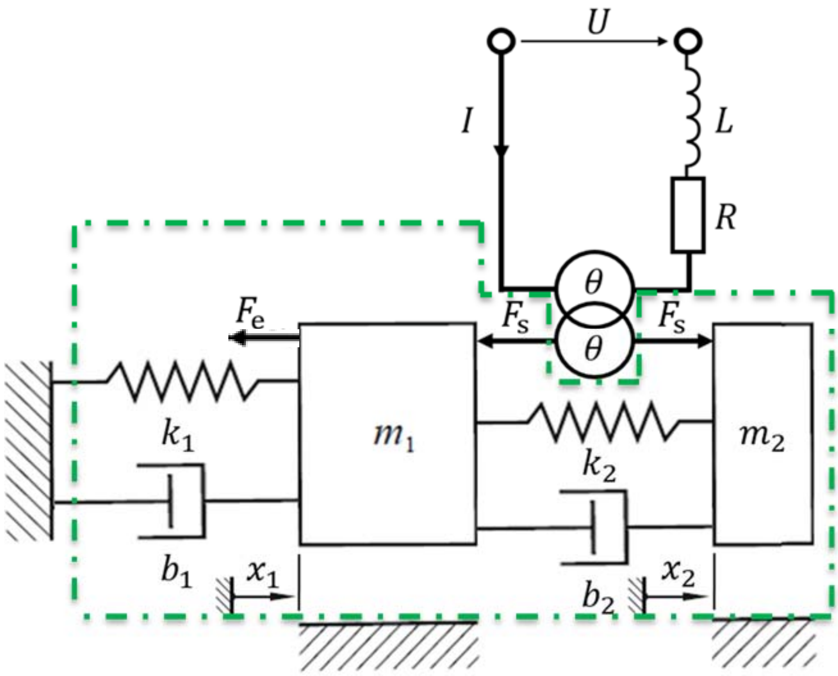

Figure 2.1: System model

Table 1.1: Physical system parameters

Name	Parameter	Value	Unit
main mass	m_1		kg
main stiffness	k_1	200	kN/m
main damping	b_1		Ns/m
DVA mass	m_2		kg
DVA stiffness	k_2		kN/m
DVA damping	b_2		Ns/m
converter constant	θ	179	N/A
coil resistance	R	10	Ω
coil inductance	L	0.4482	mAs
excitation force amplitude	$\hat{F}_e$		N

2.2 Dimensionless quantities

We introduce the damping ratios

$$D_i = \frac{b_i}{2\sqrt{k_i m_i}}, \qquad i = 1{,}2 \tag{1}$$

and the angular frequencies

$$\omega_i = \sqrt{k_i/m_i}, \qquad i = 1{,}2\ . \tag{2}$$

Dimensionless quantities lead to simplified equations of motion. Therefore we choose the reference frequency $\omega_r = \omega_1 = \sqrt{k_1/m_1}$ and the reference mass $m_r = m_1$. We also introduce the reference displacement $x_r = \hat{F}_e/k_1$, given by the static displacement of the main mass due to the excitation force amplitude $\hat{F}_e$.

Related (dimensionless) quantities are marked by a tilde ~ on top. We define the dimensionless time

$$\tilde{t} = \omega_1 t \tag{3}$$

and the related frequency and excitation frequency

$$\widetilde{\omega}_2 = \omega_2/\omega_1,\ \widetilde{\Omega} = \Omega/\omega_1. \tag{4}$$

The motion related to the reference motion and their derivatives with respect to the dimensionless time are

$$\tilde{x}_i = \frac{x_i}{x_r}, \qquad \dot{\tilde{x}}_i = \frac{d}{d\tilde{t}}\frac{x_i}{x_r}, \quad \ddot{\tilde{x}}_i = \frac{d}{d\tilde{t}^2}\frac{x_i}{x_r}, \quad i = 1{,}2 \tag{5}$$

The excitation force is related to its amplitude

$$\tilde{F}_e = \frac{F_e}{k_1 x_r} \tag{6}$$

We also introduce the following electrical quantities, the dimensionless coil current

$$\tilde{I} = \frac{\theta I}{k_1 x_r} \tag{7}$$

the dimensionless coil voltage

$$\widetilde{U} = \frac{\theta U}{R_c k_1 x_r} \tag{8}$$

the dimensionless converter constant

$$\tilde{\theta} = \frac{\theta^2 \omega_1}{R\, k_1} \tag{9}$$

and the dimensionless time constant of the coil

$$\tilde{\tau} = \tau\omega_1 = \frac{L}{R}\omega_1 \tag{10}$$

2.2 State-space model

The equations of motion of the electro-mechanical system can be written in state-space form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \quad (11)$$

with the state vector

$$\boldsymbol{x} = [\tilde{x}_1 \quad \tilde{x}_2 \quad \dot{\tilde{x}}_1 \quad \dot{\tilde{x}}_2 \quad \tilde{I}]^T \quad (12)$$

the system matrix

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -(1+\widetilde{m}_2\widetilde{\omega}_2^2) & \widetilde{m}_2\widetilde{\omega}_2^2 & -(2D_1 + 2D_2\widetilde{m}_2\widetilde{\omega}_2) & 2D_2\widetilde{m}_2\widetilde{\omega}_2 & -1 \\ \widetilde{\omega}_2^2 & -\widetilde{\omega}_2^2 & 2D_2\widetilde{\omega}_2 & -2D_2\widetilde{\omega}_2 & \frac{1}{\widetilde{m}_2} \\ 0 & 0 & \tilde{\theta}/\tilde{\tau} & -\tilde{\theta}/\tilde{\tau} & 1/\tilde{\tau} \end{bmatrix} \quad (13)$$

the input matrix

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 1/\tilde{\tau} \end{bmatrix} \quad (14)$$

and the input vector

$$\boldsymbol{u} = \begin{bmatrix} \tilde{F}_e \\ \tilde{U} \end{bmatrix}. \quad (15)$$

As elements of the output vector $\boldsymbol{y}$ the displacement $\tilde{x}_1$ and the elongation of the DVA spring $\tilde{x}_2 - \tilde{x}_1$ are chosen. Therefore we get the observer matrix

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (16)$$

and the direct input-output matrix

$$\boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (17)$$

Table 2.1 contains all parameters of the state space model. The dimensionless coil time constant and converter constant are calculated from Equation (9), (10) with the measured parameters from Table 1.1 and the measured natural frequency ω_1 = 251.3 rad/s. The DVA parameters D_2 and $\widetilde{\omega}_2$ are optimized in Chapter 3.

Table 2.1: Dimensionless system parameters

Parameter	Value	
D_1	0.005	measured
D_2		
$\widetilde{m}_2$	0.1	estimated
$\widetilde{\omega}_2$		
$\tilde{\theta}$	4.0222	measured
$\tilde{\tau}$	0.0118	measured

3. PASSIVE DYNAMIC VIBRATION ABSORBER DESIGN

We design the passive DVA. This means that the electrical state vanishes $\tilde{I} \equiv 0$. In a first design we assume that $\tilde{m}_2 \ll 1$ and $D_1 \cong 0$. Following the analytic approach of [8] then leads to the optimal natural angular frequency $\tilde{\omega}_2 = 0.8222$ and the optimal damping ratio $D_2 = 0.0911$, compare [9].
For the second design we use realistic values of $\tilde{m}_2 = 0.1$ and $D_1 = 0.005$. A numeric optimization following the method of [10] results in the optimal values of the natural angular frequency $\tilde{\omega}_2 = 0.907$ and the damping ratio $D_2 = 0.0928$. Details are given in [9].

Table 3.1: Optimal passive DVA system parameters [9]

Name	**Parameter**	**Analytical opt.**	**Numerical opt.**
damping ratio	D_1	$\cong 0$	0.005
mass	$\tilde{m}_2$	$\ll 1$	0.1
damping ratio	D_2	0.0911	0.0928
natural angular frequency	$\tilde{\omega}_2$	0.8222	0.907

4 CONTROL DESIGN

4.1 Model of active dynamic vibration absorber

The active dynamic vibration absorber is modelled in Simulink. Therefore the state space model (11) is introduced by the block “coil-two-mass-system”. Figure 4.1 shows the control loop with the variable x_1 to be controlled by the transfer function G_R.

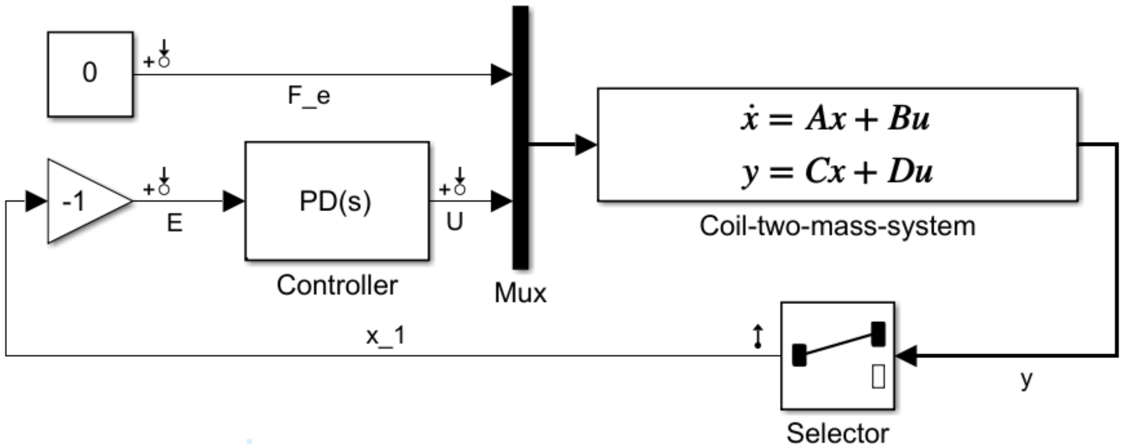

Figure 4.1: Closed loop system – SIMULINK model

An amplifier connected to the coil contacts supplies the voltage U. This leads to current I and due to the coil resistance R to high energy dissipation in the electrical part of the system. Figure 4.2 shows the open loop disturbance and control frequency response. Due to the high electrical damping only one resonance peak appears.

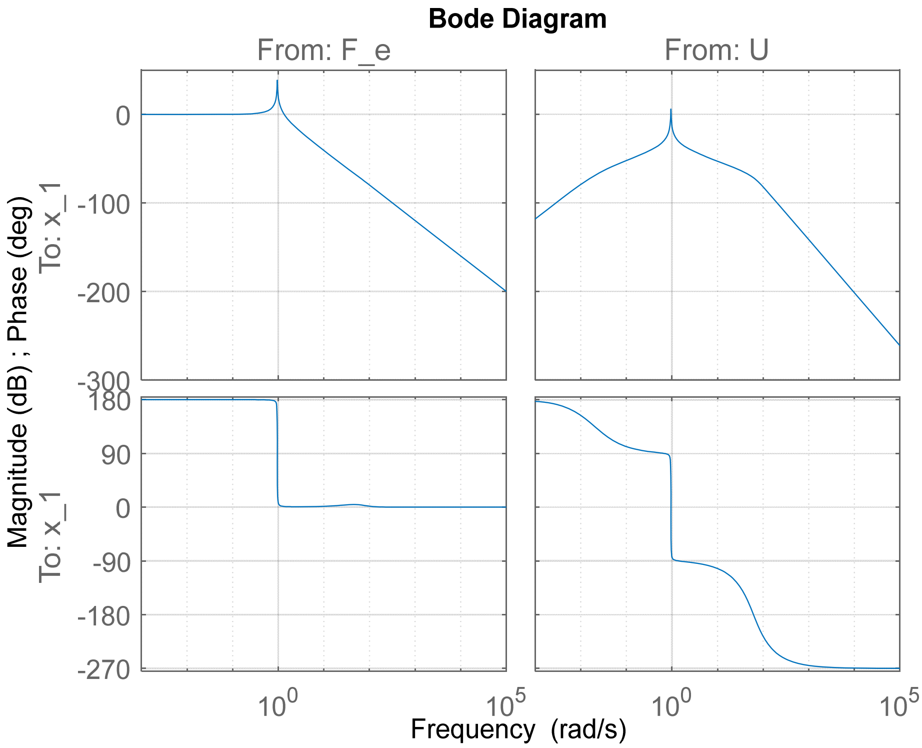

Figure 4.2: Open loop disturbance $\tilde{X}_1(s)/\tilde{F}_e$ (left) and control $\tilde{X}_1(s)/\tilde{U}(s)$ (right) frequency response

4.2 PD Controller

We introduce a PD controller with the transfer function in the form

$$G_R(s) = \frac{\tilde{U}(s)}{\tilde{X}_1(s)} = \frac{K_P(1 + T_V s + T_D s)}{1 + T_D s} \tag{18}$$

The controller parameters

$$K_P = 1000, T_D = 0.001 \text{ and } T_V = 0.01 \tag{19}$$

are determined by a heuristic frequency-curve design method [11]. This PD controller fulfils the criterions for stability, controllability and observability for the state space system, see [9]. Figure 4.3 shows the disturbance transfer function $\tilde{X}_1(s)/\tilde{F}_e$. The resonance is greatly reduced.

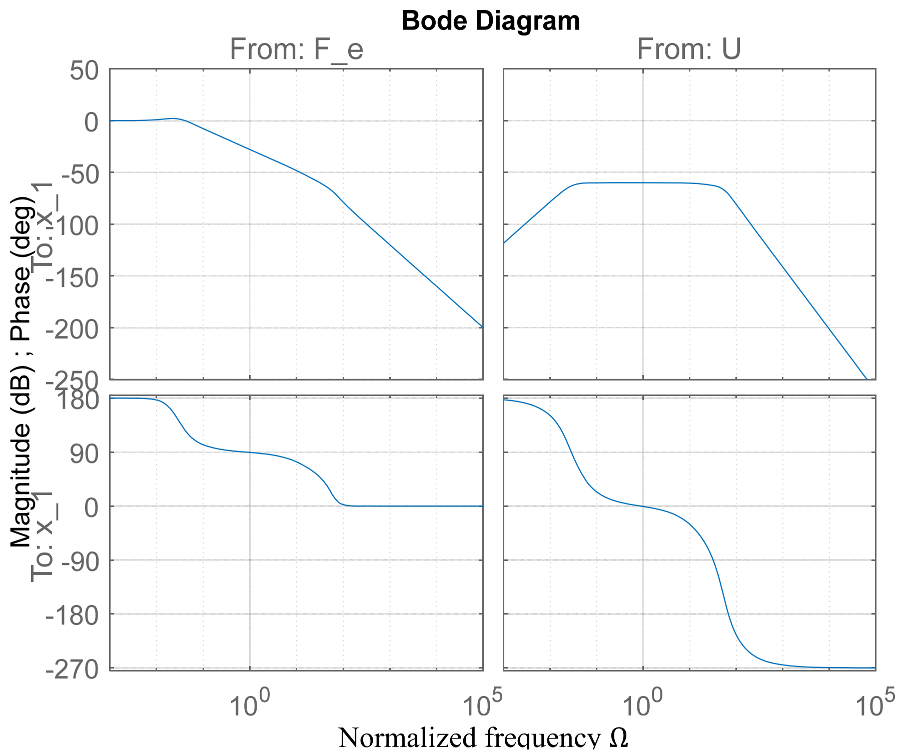

Figure 4.3: Closed loop disturbance $\tilde{X}_1(s)/\tilde{F}_e$ (left) and control $\tilde{X}_1(s)/\tilde{U}(s)$ (right) frequency response with PD-control

5. RESULTS

Figure 5.1 shows the normalized displacement $\tilde{x}_1$ of the main mass m_1 excited by the external force F_e. The single degree of freedom (DOF) main system with no DVA (blue line) shows a weakly damped resonance region around $\tilde{\Omega} = \Omega/\omega_1 = 1$. Adding a DVA results in a two DOF system with two resonances. The amplitude peak of the main system is greatly reduced, but we observe a slight amplification of the amplitude for frequencies below and above the resonance region. A numeric optimisation delivers DVA parameter which result in a further reduction of the maximum amplitude by approx. 1 dB, see Figure 5.1b. Adding a coil to the two mass system brings an additional (electric) state variable. Controlling the current by the PD-controller results in a drastically better system response in the whole frequency above $\tilde{\Omega}$ >10^-1.6=0.025, see purple curve in Figure 5.1a.

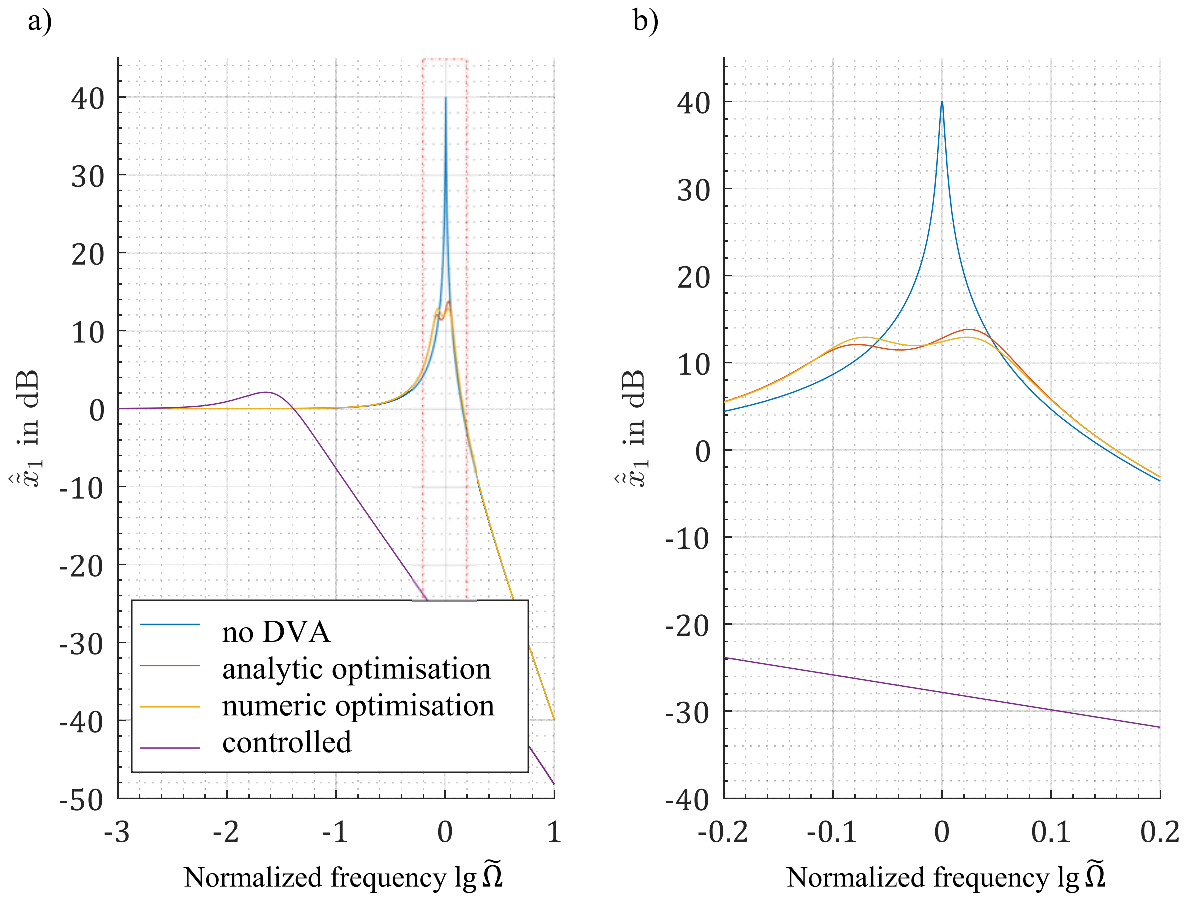

Figure 5.1: a) Frequency response $\tilde{x}_1/\tilde{F}_e$ b) enlargement, adapted from [9]

6. CONCLUSION

This study shows good vibration reduction of a single mass system by adding a DVA. Optimisation of two parameters (damping ratio and eigenfrequency) of the DVA results in a good reduction of the maximum of the response curve. But this deteriorates slightly the vibration outside the resonance region. The controlled active DVA leads to a much better vibration reduction. These results have to be considered carefully, because they are calculated with a linear system model. Nonlinear system behaviour (springs, damper, mechanical and electrical limit stops) and sensor noise are not considered.

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