PREDICTION OF ACOUSTIC WAVE PROPAGATION IN UNDERWATER STEP PROBLEMS VIA THE METHOD OF FUNDAMENTAL SOLUTIONS

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Abstract
In this paper, the Method of Fundamental Solutions (MFS) is used to simulate in the frequency domain the two-dimensional acoustic wave propagation in a waveguide of infinite extent, taking into account constant depth in each section of the sea. The time domain responses are obtained through an inverse Fast Fourier Transform (FFT) of results computed in the frequency domain. In the numerical model only a vertical interface between the sub-regions of different depth is discretized and a Green's function that takes into account the presence of free and rigid surfaces is used. This Green's function is obtained by eigenfunction expansion. The proposed model is compared with a classic one based on the Boundary Element Method (BEM). Numerical examples are performed in order to demonstrate the efficiency of the proposed model in the analysis of acoustic wave propagation in shallow water problems containing a step on the seabed. The results will show that the MFS is a very efficient tool for such problems.

Keywords: Method of Fundamental Solutions, Green's function, underwater problem.

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1 Introduction

Several analytical and numerical methods have been applied to simulate and analyse underwater acoustic wave propagation. Jensen et al. [1] discusses in detail the different methodologies applied to solve the problem of wave propagation in acoustic environments that have interested many researchers over past decades. Some of the well-known methods are based on the acoustic ray theory [2], the normal modes method [3] and the parabolic equation [4].

In recent years, meshless methods have attracted great interest of scientists and researchers. The Method of Fundamental Solutions (MFS) is one of these methods and it has been applied with success for scattering or radiation problems. Mathematically, the MFS is a very simple technique based on the prior knowledge of fundamental solutions, but not requiring the numerical and analytical integrations that need to be performed in the Boundary Element Method (BEM). One disadvantage of the MFS is the determination of the position of the pseudo-boundary on which the singularities are placed. Karageorghis [5] has proposed a simple algorithm for estimating an optimal pseudo-boundary for

certain boundary value problems. Costa et al. [6, 7] have shown that, despite its simplicity, the MFS is a very interesting tool to efficiently predict wave acoustic propagation in shallow water.
In this paper, the Method of Fundamental Solutions (MFS) is used to analyse, in the frequency domain, the two-dimensional acoustic wave propagation in a shallow water configuration, considering a step up on the bottom of the sea. Time domain signals are computed by means of an inverse fast Fourier transform of the numerical results obtained in the frequency domain. Green's function is used to decrease the number of discretized surfaces and consequently reducing the computational cost of the proposed models. The numerical model is developed using a sub-region technique, where only the vertical interface between the sub-regions of different depth has to be discretized. This Green's function is obtained by eigenfunction expansion. A set of numerical examples is performed in order to demonstrate the efficiency of the proposed model in the analysis of acoustic wave propagation in shallow water problems containing a step on the seabed. In addition, a detailed discussion on the performance of this formulation is carried out with the aim of showing that the MFS is a very efficient tool for solving the acoustic step problem in shallow water.

2 Governing equation

The problem of two-dimensional acoustic wave propagation in a region Ω of infinite extent in the longitudinal z-direction is analysed, taking into account the presence of a step on the bottom of the sea, as shown in Fig. 1. If the sound velocity is constant, the source of the acoustic disturbance is time-harmonic and the medium in the absence of perturbations is quiescent, the problem is governed by the Helmholtz equation which can be written as:

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = -Q\delta(\mathbf{x} - \boldsymbol{\xi}^s) \quad \text{in } \Omega\,, \tag{1}$$

where $\nabla^2 = \partial/\partial x^2 + \partial/\partial y^2$; $p(\mathbf{x})$ is the acoustic pressure; Q is the magnitude of the acoustic source $\boldsymbol{\xi}^s$ located at (x_{ξ^s}, y_{ξ^s}); $\mathbf{x}$ is the observation point located at (x, y); $\delta(\mathbf{x} - \boldsymbol{\xi}^s)$ is the Dirac delta function, and $k = 2\pi f/c$ is the wave number, with f being the excitation frequency and c the propagation velocity of the acoustic domain.

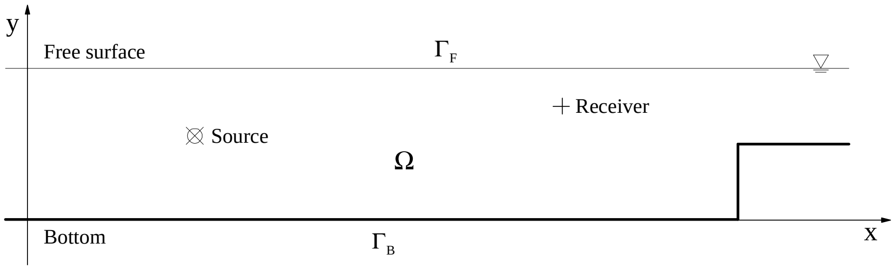

Figure 1 – Geometry of the problem.

The boundary conditions for the above described problem are given by Dirichlet condition ($p(\mathbf{x}) = 0$ in Γ_F) and Neumann condition ($v_n(\mathbf{x}) = (\mathrm{i}/\omega\rho)\partial p(\mathbf{x})/\partial\mathbf{n} = 0$ in Γ_B), being $\omega = 2\pi f$ the angular frequency, ρ the density of the acoustic medium, $\mathbf{n}$ the unit outward normal vector and Γ_F and Γ_B the free and bottom surfaces, respectively.

3 Green's function

The Green's function denominated $G_{\mathrm{M}}(\boldsymbol{\xi},\mathbf{x})$ directly satisfies the boundary conditions on the flat rigid bottom and the free surface. The Green's function can be written in terms of normal modes [8] as:

$$G_{\mathrm{M}}(\boldsymbol{\xi},\mathbf{x})=\frac{\mathrm{i}}{H}\sum_{m=1}^{\infty}\sin\left[k_{ym}(Y_{\mathrm{F}}-y_{\xi})\right]\sin\left[k_{ym}(Y_{\mathrm{F}}-y)\right]\frac{e^{\mathrm{i}k_{xm}|x-x_{\xi}|}}{k_{xm}}. \quad (2)$$

where the source point $\boldsymbol{\xi}$ is located at (x_{ξ},y_{ξ}) and H is the depth of the waveguide $(Y_{\mathrm{F}}-Y_{\mathrm{B}})$, as defined in Fig. 2. The parameters $k_{xm}=\sqrt{k^2-k_{ym}^2}$ and $k_{ym}=(m-1/2)\pi/H$ are horizontal and vertical wavenumbers, respectively.

Figure 2 – Geometry of the waveguide.

It is important to note that the exponential term in Eq. (2), makes the convergence of the Green's function decrease quickly when k_{xm} becomes an imaginary number (evanescent modes), but when the exponential term is equal to 1.00, the convergence of the series is much slower.

4 Method of Fundamental Solutions

In this sub-section, a single proposed model was defined using the MFS, as schematically illustrated in Fig. 3. This model makes use of the sub-region technique, with the Green's function denominated $G_{\mathrm{M}}(\boldsymbol{\xi},\mathbf{x})$ applied for each sub-region, requiring only the discretization of the vertical interface $\Gamma_{\mathrm{c}}\cup\Gamma_{\mathrm{step}}$. In both regions, the Green's function that satisfies the rigid bottom and free flat surface is assumed. Thus, imposing continuity of the acoustic pressure and normal component of the velocity at each interface point $\mathbf{x}$, the following equations can be obtained:

$$\sum_{n=1}^{NVS_c+NVS_s}\mathrm{A}_n G_{\mathrm{M}}^{\mathrm{H}_1}(\boldsymbol{\xi}_n,\mathbf{x})+(1-Q)G_{\mathrm{M}}^{\mathrm{H}_1}(\boldsymbol{\xi}^s,\mathbf{x})=\sum_{n=1}^{NVS_c}\mathrm{B}_n G_{\mathrm{M}}^{\mathrm{H}_2}(\boldsymbol{\xi}_n,\mathbf{x})+QG_{\mathrm{M}}^{\mathrm{H}_2}(\boldsymbol{\xi}^s,\mathbf{x}), \quad (3)$$

$$\sum_{n=1}^{NVS_c+NVS_s}\mathrm{A}_n\frac{\partial G_{\mathrm{M}}^{\mathrm{H}_1}(\boldsymbol{\xi}_n,\mathbf{x})}{\partial\mathbf{n}_1}+(1-Q)\frac{\partial G_{\mathrm{M}}^{\mathrm{H}_1}(\boldsymbol{\xi}^s,\mathbf{x})}{\partial\mathbf{n}}=\sum_{n=1}^{NFV_c}\mathrm{B}_n\frac{\partial G_{\mathrm{M}}^{\mathrm{H}_2}(\boldsymbol{\xi}_n,\mathbf{x})}{\partial\mathbf{n}}+Q\frac{\partial G_{\mathrm{M}}^{\mathrm{H}_2}(\boldsymbol{\xi}^s,\mathbf{x})}{\partial\mathbf{n}}, \quad (4)$$

where A_n and B_n are the amplitudes to be determined for each virtual source; $Q=1$ if the real source is placed in region Ω_2 and $Q=0$ if the real source is placed in region Ω_1; $G_{\mathrm{M}}^{\mathrm{H}_1}(\xi^s,\mathbf{x})$ and $G_{\mathrm{M}}^{\mathrm{H}_2}(\xi^s,\mathbf{x})$ are the incident fields regarding the acoustic pressure generated by the real source; NVS_{c} is the number of virtual sources placed at each sub-region and NVS_s is the number of virtual sources positioned in region Ω_1; $G_{\mathrm{M}}^{\mathrm{H}_1}(\xi_n,\mathbf{x})$ and $G_{\mathrm{M}}^{\mathrm{H}_2}(\xi_n,\mathbf{x})$ refer to the Green's functions for a flat rigid bottom and flat free surface, whose details were given in the previous section.

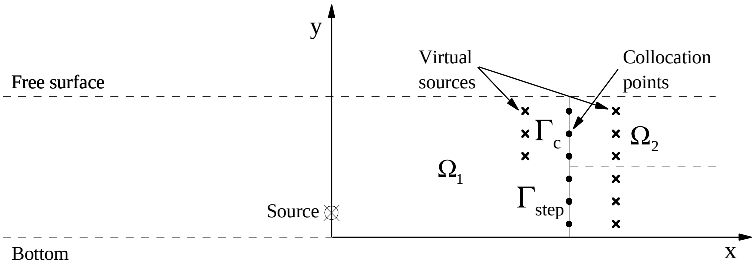

Figure 3 – Geometry of the numerical model.

Imposing the Neumann condition $(v_n(\mathbf{x})=0)$ at each collocation point $\mathbf{x}$ of the vertical boundary Γ_{step}, the following equation can be written:

$$\sum_{n=1}^{NVS_s+NVS_c} \mathrm{A}_n \frac{\partial G_{\mathrm{M}}^{\mathrm{H}_1}(\xi_n,\mathbf{x})}{\partial \mathbf{n}} = -(1-Q)\frac{\partial G_{\mathrm{M}}^{\mathrm{H}_1}(\xi^s,\mathbf{x})}{\partial \mathbf{n}}, \tag{5}$$

Therefore, a linear system of NVS_s+2NVS_c equations on NVS_s+2NVS_c unknowns may be written. Once this system of equations is solved for the relevant unknown amplitudes, the response at any point of the domain may be obtained by using the following equations:

$$\sum_{n=1}^{NVS_c+NVS_s} \mathrm{A}_n G_{\mathrm{M}}^{\mathrm{H}_1}(\xi_n,\mathbf{x}) + (1-Q)G_{\mathrm{M}}^{\mathrm{H}_1}(\xi^s,\mathbf{x}) \quad \text{region } \Omega_1, \tag{6}$$

$$\sum_{n=1}^{NVS_c} \mathrm{B}_n G_{\mathrm{M}}^{\mathrm{H}_2}(\xi_n,\mathbf{x}) + QG_{\mathrm{M}}^{\mathrm{H}_2}(\xi^s,\mathbf{x}) \quad \text{region } \Omega_2. \tag{7}$$

5 Verification of the numerical model

In order to verify the behaviour of the proposed MFS model used in this work, the described formulation was implemented and a test case was analysed. We consider the geometry depicted in Fig. 4(a), consisting of a flat waveguide containing a step on the bottom of the sea. For this configuration, the response is generated by a point source positioned at $5.0\ \mathrm{m}$ depth and computed at one horizontal line of receivers, placed at a depth of $2.5\,\mathrm{m}$. The acoustic medium is assumed to be water, with density of $1000\,\mathrm{kg/m^3}$ and a sound propagation velocity of $1500\ \mathrm{m/s}$.

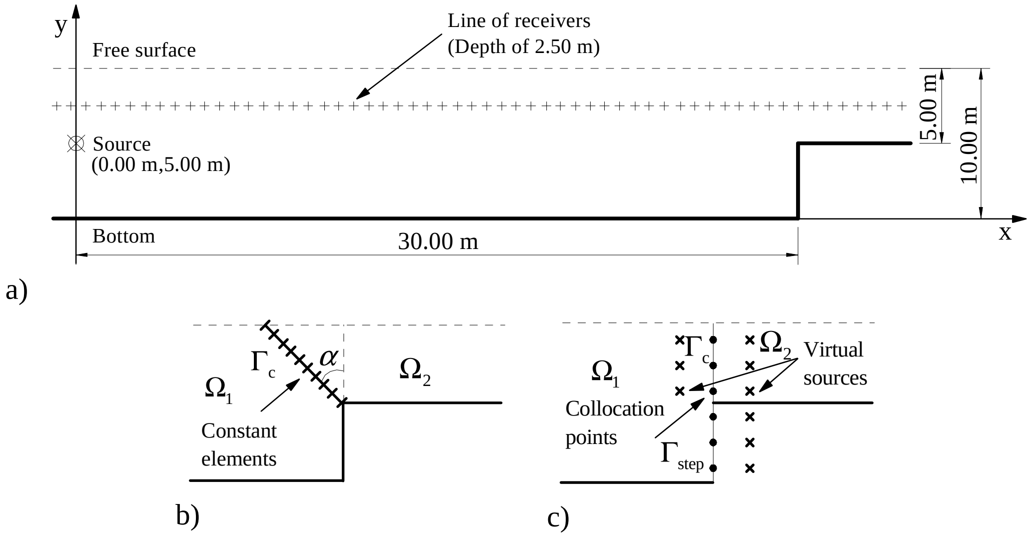

Figure 4 – a) Geometry of the problem, b) BEM Model: geometry with discretized inclined interface and c) MFS Model: geometry and position of collocation points and fictitious sources.

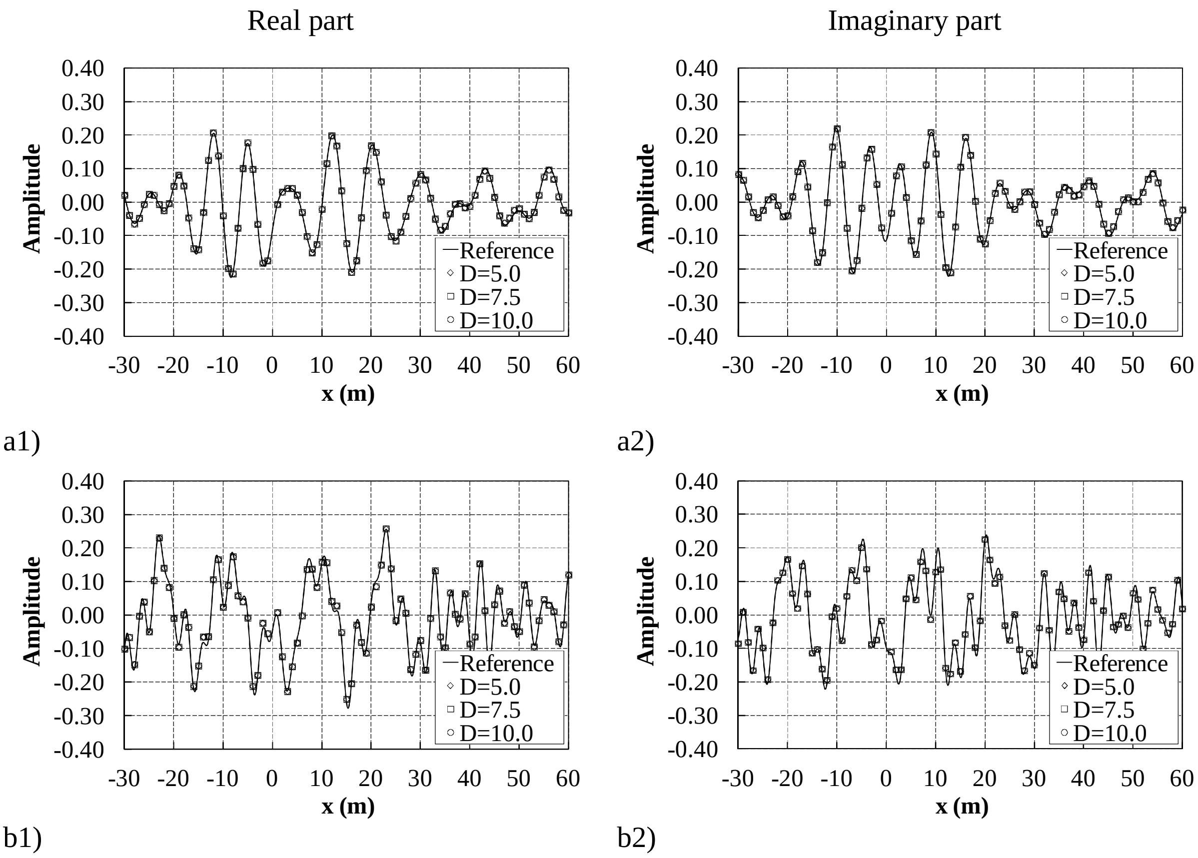

Figure 5 – Response calculated at one horizontal line of receivers placed at a depth of 2.5 m for the reference and proposed models: a) 250 Hz and b) 500 Hz.

The reference model requires the discretization of an inclined interface with an angle of $\alpha = 2.3°$ to the left side, as illustrated in Fig. 4(b). To build the MFS model, boundary points are located over a vertical interface and two sets of virtual source points may be defined (see Fig. 4(c)).
In MFS model, the number of collocation points is defined as a function of the frequency, by using a relation between the incident wavelength and the distance between collocation points, equal to a minimum of 5. For the BEM model, a very large number of elements was used to ensure the accuracy of the reference solution. Figure 5 displays the response computed for frequencies of $250\ \text{Hz}$ and $500\ \text{Hz}$ using the reference and proposed models, for a set of receivers located at a depth of $2.50\ \text{m}$.
To evaluate the sensitivity of the MFS model with respect to the distance between virtual sources and vertical interface, the responses were computed assuming different distances between the interface and virtual sources. Here, the results of the MFS model were obtained for the distances of 5.0, 7.5 and 10.0 times the distance between the collocation points and a relation of 5 was adopted. These results agree very well with the reference solution, allowing concluding that the proposed MFS model has a stable behaviour for a large range of distances between the interface and source points.

6 Numerical Examples

To illustrate the applicability of the proposed formulation, the problem shown in Fig. 6 is analysed using only the MFS model. In underwater acoustics, it is common to use the Transmission Loss ($\text{TL} = -10\log(p^2 / p_0^2)$ with p_0 being the pressure of the incident field generated by a point source at a distance of $1.00\ \text{m}$) to calculate the response of a given underwater system. It is thus important to assess the influence of the step on the bottom of the sea using the TL. Once again, the acoustic medium is assumed to be water.

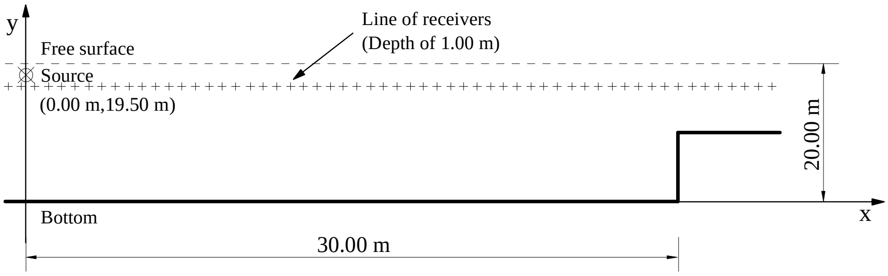

Figure 6 – Geometry of the numerical example.

Figure 7 displays the TL plots computed along the line of receivers placed 1.00 m below the free surface of the sea when an excitation source is at $(0.00\ \text{m}, 19.50\ \text{m})$. In these plots, the dashed curve refers to the reference case of a waveguide with free surface and flat rigid bottom, while the continuous curve refers to the configurations of the proposed MFS model. A vertical dashed line is also included to mark the transition between the two sub-regions of different depth of the waveguide. The transmission loss is computed for four different excitation frequencies, so that the response includes the contribution of the 1st ($50\ \text{Hz}$), 3rd ($100\ \text{Hz}$), 5th ($250\ \text{Hz}$) and 13th ($500\ \text{Hz}$) propagating modes of the waveguide with constant depth of $20.0\ \text{m}$.
For all frequencies, the TL responses of the waveguide containing a step up present significant differences in relation to the TL responses considering only a horizontal waveguide. It can be seen that the TL curves in this waveguide exhibit only few oscillations along the computed receivers for frequencies of $50\ \text{Hz}$ and $100\ \text{Hz}$, whereas for the frequencies of $250\ \text{Hz}$ and $500\ \text{Hz}$ the TL curves show a sequence of peaks corresponding to a large number of propagating modes within the system, particularly at the frequency of $500\ \text{Hz}$.

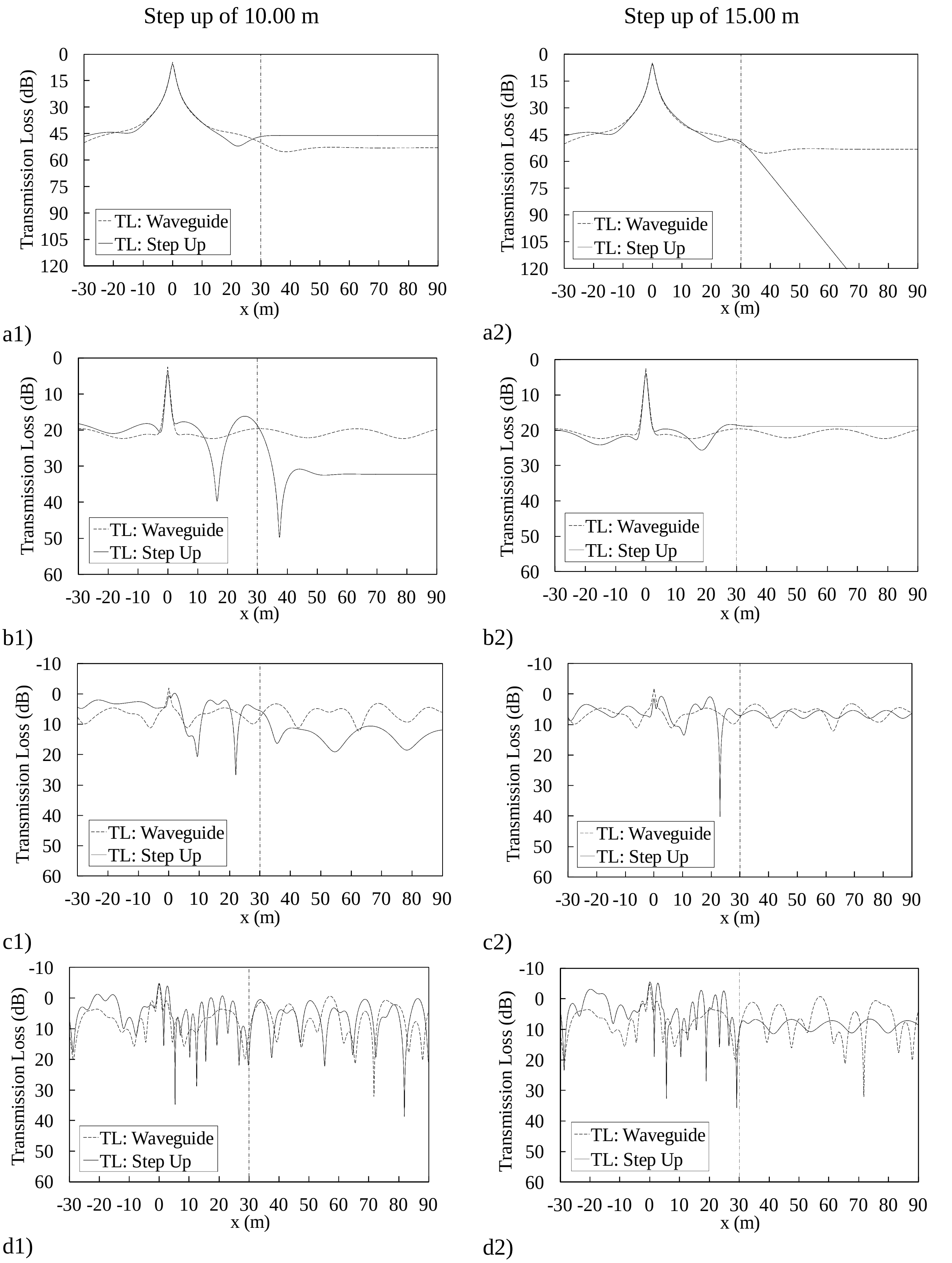

Figure 7 – TL in a waveguide containing a step up with different heights (10 meters and 15 meters) on the bottom of the sea for frequencies of: a) 50 Hz, b) 100 Hz, c) 250 Hz and d) 500 Hz.

However, when the waveguide containing a step up is considered, a pronounced interference occurs between the source and the step up related to the waves that are reflected by the step, particularly at the higher frequencies ($250\ \mathrm{Hz}$ and $500\ \mathrm{Hz}$). Therefore, the values computed for the TL tend to be closer to those responses calculated for the waveguide (see Figs. 7(c) and (d)). For this reason, the TL plots are more difficult to be analysed at the higher frequencies due to the pronounced interference between the reflections that occur within this system due to the presence of the step up. It is important to note that the lower frequencies produce less propagating modes with higher amplitudes, while the higher frequencies allow a large number of propagating modes.

For the lower frequencies ($50\ \mathrm{Hz}$ and $100\ \mathrm{Hz}$), the receivers receptors further away from the source have a constant TL, indicating that there are few propagating modes (see Figs. 7(a1), (b1) and (b2)), while Fig. 7(a2) displays a different characteristic, with an exponential increase of the TL when the wave travels farther away from the source. This behaviour indicates that there are no propagating modes and only evanescent waves occur, characterized by a pronounced and progressive decay of the energy in this part of the channel due to its lower depth.

Time domain responses are computed in shallow water acoustic problems for the identification of important features related to the presence of a step on the seabed, which are not clearly visible in frequency domain analyses. In this work, time domain signals are computed through the inverse fast Fourier transform. The pressure field generated by a point source in the spatial-temporal domain is assumed to be defined by a Ricker wavelet.

The analysis uses complex angular frequencies with $\zeta = 0.7\Delta\omega$ to avoid the aliasing phenomena. In the time domain this shift is later taken into account by applying an exponential window $e^{\zeta t}$ to the response [9].

A relation between the wavelength of the incident waves and the distance between collocation points of 5 was adopted. The distance between the virtual sources and the collocation points was 5 times the distance between the collocation points.

The calculations were performed over a frequency range from 2.0 to 1024.0 Hz, assuming a frequency step of 2.0 Hz, which gives a total time of $\mathrm{T} = 500\ \mathrm{ms}$. The pressure field was computed over a grid of receivers, equally spaced ($\Delta x = \Delta y = 0.25\ \mathrm{m}$). A sequence of snapshots displaying the pressure field computed within the channel at different time instants is presented to illustrate the acoustic wave propagation pattern. In the time domain analyses, the responses provided by a flat seabed of constant depth were displayed and used as a reference solution.

The channel is excited by a point source placed at $(0.00\ \mathrm{m}, 19.50\ \mathrm{m})$ and the acoustic wave propagation of a Ricker pulse generated by this point source with a characteristic frequency of 400 Hz is modelled. The first column illustrates pressure distribution snapshots in a waveguide with flat bottom and free surface while the second and third columns depict the plots in the presence of step of 10.00 m and 15.00 m, respectively. Figure 8 (a1-a3) displays a snapshot for the time instant $\mathrm{t} = 21.3623\ \mathrm{ms}$, where an incident pulse (P1) can be seen, although it is already combined with a first reflection from the rigid bottom of the sea. In the presence of the step, it is also possible to observe the first reflection from the bottom discontinuity (P2). Later, at $\mathrm{t} = 29.1748\ \mathrm{ms}$, the second reflection is visible (P3). In addition, at this time, the first reflection generated on the free surface can also be easily identified (P4), with inverted polarity (Fig. 8 (b1-b3)). At $\mathrm{t} = 34.0576\ \mathrm{ms}$, a diffracted pulse (P5) with a very low amplitude is generated at the bottom discontinuity (see Figs. 8(c2) and (c3)). In these figures, another diffracted pulse on the top of the step up is also visible (P6). In the plots of Figs. 8(d2) and (d3), at $\mathrm{t} = 46.7529\ \mathrm{ms}$, a reflected pulse (P7) in the rigid bottom can also be easily observed. As time passes (Figs. 8(e2) and (e3)), the pulses reflected and diffracted on the surfaces of the channel have generated wave fronts, some propagating towards the higher region of the channel and others propagating towards the lower region of the channel.

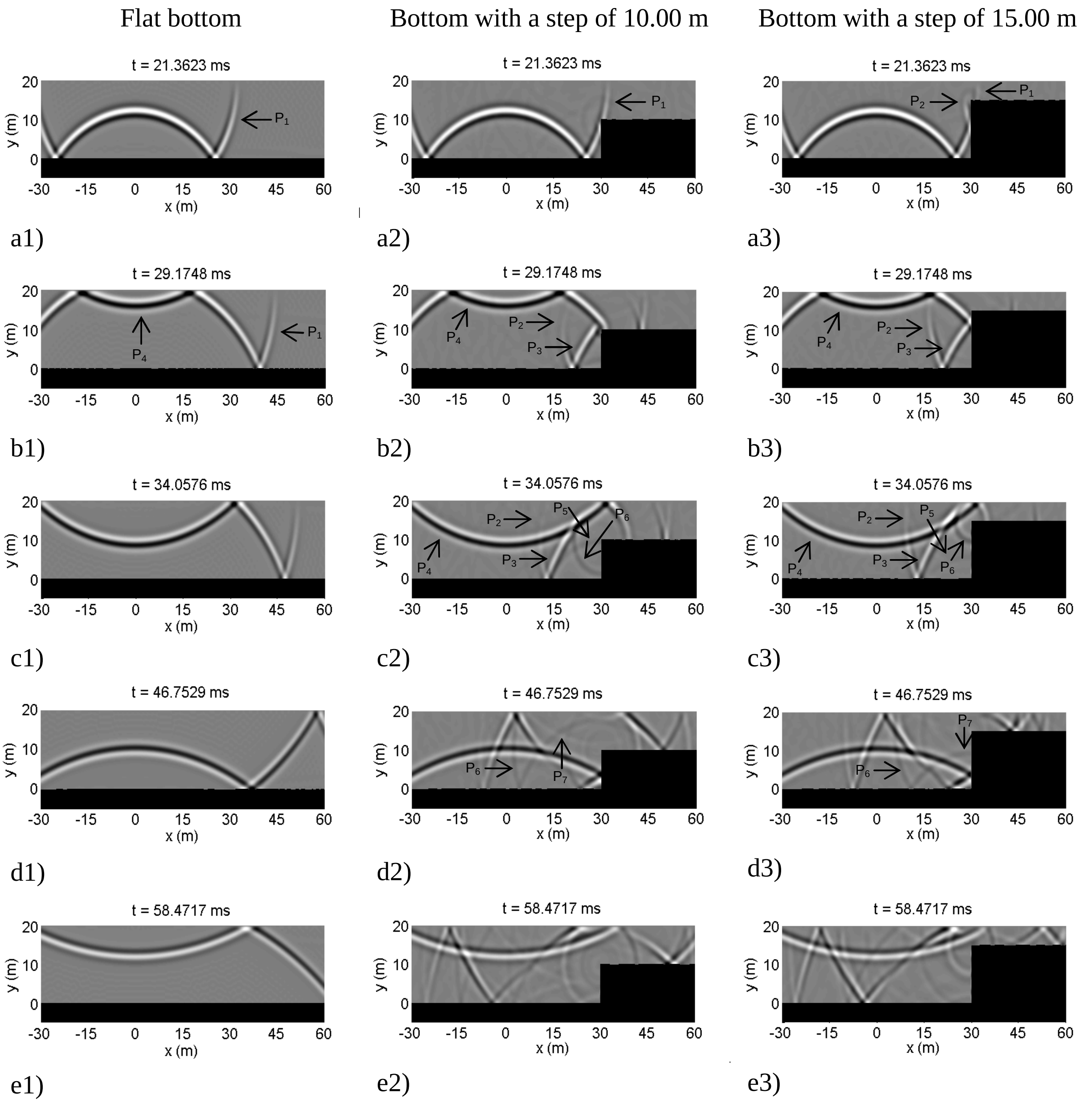

Figure 8 – Time domain responses when a point source excites the channel without and with a step up on the bottom of the sea for different instants: a) 21.3623 ms; b) 29.1748 ms; c) 34.0576 ms; d) 46.7529 ms and e) 58.4717 ms.

It is important to note that several wave fronts are generated by reflections on the waveguide, and by reflections and diffractions on the bottom discontinuity between the two regions of the channel, and so, as time elapses, a growing number of reflections and diffractions are registered in the higher region of the channel. This behaviour is more pronounced when a step up of 15.00 m is considered.

7 Conclusions

In this paper, the Method of Fundamental Solutions was used to simulate two-dimensional acoustic wave propagation in shallow water considering a step up on the bottom of the sea, in the frequency domain. The time domain responses were obtained through an inverse fast Fourier transform of numerical results computed in the frequency domain. Appropriate Green's function was employed, decreasing the required discretization and reducing the computational cost of the proposed model. This Green's function was obtained by a classic eigenfunction expansion. An excellent agreement occurred between the reference and proposed models. The numerical examples demonstrated the use of the proposed model in the analysis of wave propagation in shallow water. In addition, the presented cases of a waveguide containing a step up with different heights allowed identifying relevant differences in the sound propagation patterns in the underwater acoustic problem both in the frequency and in the time domain.

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