COMPARATIVE STUDY OF SIMULATION METHODS FOR THE QUANTIFICATION OF THE ACOUSTIC INSULATION PROVIDED BY PERIODIC STRUCTURES

PACS: 43.50 Gf, 43.28. Fp

Parrilla Navarro, M. J[1]; Redondo, J.[1]; Sánchez-Pérez, J.V.[2]; Peiró-Torres, M.P.[2]

[1]Instituto para la Gestión integral de Zonas Costeras. -Universitat Politécnica de València. Campus de Gandía. C/ Paranimf, 1. Grao Gandía (Spain). maparna@epsg.upv.es;
[2]Centro de Tecnologías Físicas: Acústica, Materiales y Astrofísica. -Universitat Politécnica de València. Camino de Vera s/n. Valencia (Spain)

Keywords: Multiple Scattering; Finite Elements Method; Finite Difference Time Domain; Sonic Crystals; Computational Cost; Simulation; Uncertainty.

RESUMEN

Actualmente existen diferentes métodos para realizar simulaciones acústicas. La conveniencia de utilizar un método u otro depende del sistema que se estudia en cada caso. En este trabajo se realiza una comparativa entre diferentes métodos (Múltiple Scattering, Método de Elementos Finitos y Método de Diferencias Finitas en el Dominio del Tiempo) aplicados a la cuantificación del aislamiento proporcionado por estructuras periódicas (cristales de sonido). Se considera un estudio sistemático sobre tiempo computacional, la precisión y el coste computacional.

ABSTRACT

Nowadays there are different available methods to perform simulations in acoustics. The suitability of them strongly depends on the system studied in each case. In the present work, a comparison between different methods (Multiple Scattering, Finite Elements Method and Finite Difference Time Domain) is carried out particularized to the quantification of the isolation provided by periodic structures (Sonic crystals). It has been considered a systematic study about computational time, precision and computational cost.

INTRODUCTION

A Sonic Crystal (SC) is a periodic array of cylindrical acoustic scatters with radius *r* separated by a predetermined lattice constant, and embedded in a fluid [1]. The first works simulating these structures started using the Plane Waves Expansion theory (PWE) by Yablonovitch [2] and John [3] in 1987. At the end of 20th century, Multiple Scattering Theory (MST) started to develop in acoustics, with the work of Sánchez-Pérez *et al* [4].

In addition, several numerical methods have been well studied by the scientific community. Different researchers have been using these methods to study the behaviour of phononic crystals. As an example, Cao *et al.* [5] used the Finite Difference Time Domain Method (FDTD), which, according to the authors, was an effective technique for the band-structure calculations of 2D phononic crystals. Some published works also document the use of the Finite Elements Method (FEM) for the analysis of periodic structures. Wang *et al.* [6] used FEM to study the generation of large band-gaps by periodic structures. Some of recent works like Sánchez-Pérez *et al.* [7] used FEM to define a two-step 2D model for designing of sonic crystal barriers. In Liu, *et al.* [8], a wavelet-based FEM was used to investigate the band structure of 1D phononic crystals [9]. In this work, the study is focussing on the comparison of the methods of Multiple Scattering, Finite Elements Method and Finite Difference Time Domain Method.

STATE OF ART OF SIMULATION METHODS

MULTIPLE SCATTERING HISTORY

The first author who studied this method was Záviska in 1913 [10]. He described the method in 2D acoustic field for the scattering of finite arrays. This method was applied in 1914 by Ignatowsky [11] for the case of normal incidence on an infinite row of cylinders. Multiple scattering can be understood as an interaction of wave fields with two or more obstacles. Multiple Scattering Theory solves the problem considering that the field scattered by one obstacle induces further another scattered field to the other obstacles, these obstacles induce, in the same way, further scattered fields to all the other obstacles, and so on. This characterizes Multiple Scattering Theory as a self-consistent method, being applicable to randomly or periodically-spaced cylinders. The first work in acoustics with MST was in 2001 by Chen *et al* [12].

FINITE ELEMENTS METHOD HISTORY

This method was originated from the need to solve complex problems of elasticity and structural analysis in civil and aeronautical engineering. Its development dates to the work by A. Hrennikoff [13] and R. Courant [14] in the early 1940s. Typical areas of interest include structural analysis, heat transfer, fluid flow, mass transport, electromagnetic potential and acoustics. The finite elements method formulation of the problem results in a system of algebraic equations. The method provides approximate values of the unknowns to a discrete number of points over the domain [15]. To solve the problem, the methodology subdivides the large problem into smaller ones or simpler parts that are called finite elements. The simple equations which model these finite elements are then assembled into a larger system of equations which models the entire problem.

FINITE DIFFERENCE TIME DOMAIN METHOD HISTORY

The finite-difference time-domain method (FDTD) is possibly the simplest one of the full-wave techniques used to solve problems in electromagnetics, both conceptually and in terms of implementation. The FDTD method employs finite differences as approximations to both the spatial and temporal derivatives which appear in Maxwell's equations. The technique was firstly proposed by K. Yee [16]. The originality of the idea of Yee resides in the allocation in space of the electric and magnetic field components, during recent times the procedure it has developed and became better.

SONIC CRYSTAL UNDER ANALYSIS

In order to perform a comparison between the different methods exposed above, we have established a simple sonic crystal structure to be analyzed. Every principal parameters of each method will be varied to study its effect. The structure is composed of 7 rows and 4 columns of circular scatters, with a lattice constant such that the first band gap is localized at 1000 Hz (Figure 1). We have chosen 4 columns given that it is the minimal value required to observe periodicity effects [17], and 7 rows to obtain a width of 1.2 m in the experimental section which is a standard measure in the building sector. The position of band gap is chosen on that frequency in which the target frequency range for traffic noise spectrum is centred, at 1000 Hz [17]. The filling fraction (*ff*) of the structure will be varied from 0.6 to 0.9 and the uncertainty will be averaged to obtain a single value.

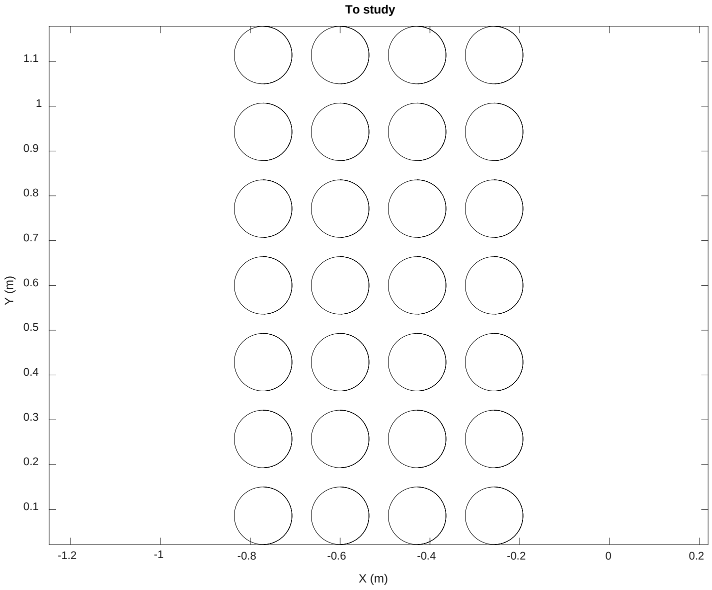

Figure 1. Structure to study with 0.75 of ff

In order to quantify the performance of the sonic crystal under analysis we have used the parameter known as Insertion Loss, defined as (1):

$$IL = 20 * \log\left(\frac{P_{direct}}{P_{interfered}}\right) \qquad (1)$$

Where P_{direct} is the pressure level without the barrier and $P_{interfered}$ the pressure level with barrier. The range of simulated frequencies was from 100 Hz to 5000 Hz, frequencies provided by the normalized traffic noise spectrum [18].

SIMULATIONS RESULTS

The uncertainty of calculations has been considered by comparison with the best case in each simulation technique. All the simulations of this work were performed in a PC with 8 cores of i7-7700HQ at 2.8 GHz and 16 GB DDR4-2400 RAM

MULTIPLE SCATTERING SIMULATION

The first parameter to characterize was the order of the calculations, in other words, how many scattered fields are taken into account. Figure 2 illustrates the effect of the order in the accuracy of the method. It can be seen that order 5 is enough to obtain a very low uncertainty. Actually, increasing the order has nearly no effect.

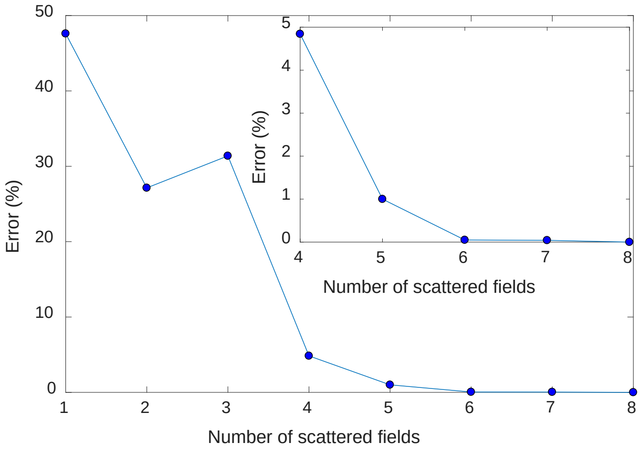

Figure 2. Number of scattered fields

With the order set to 5, the next parameter to study is how many induvial frequencies per band have to be considered.

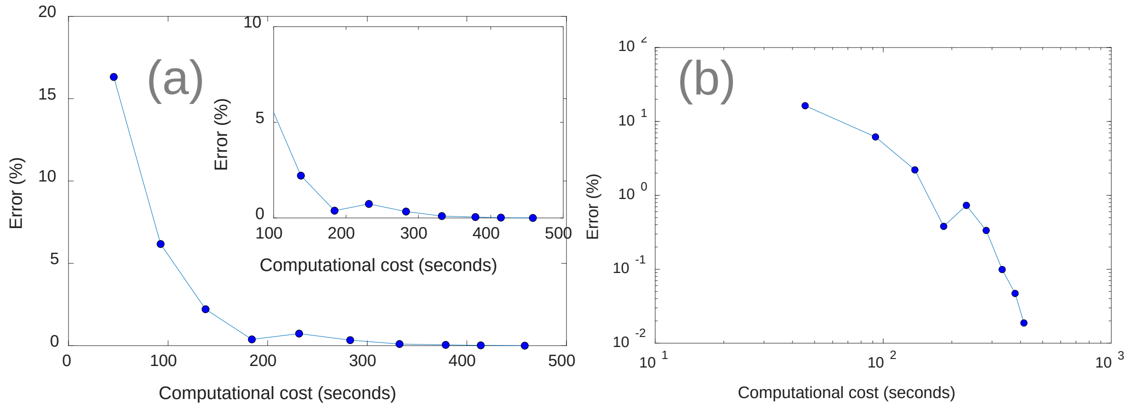

Figure 3. Uncertainty versus computational cost for MS. (a): Linear axis. (b): Logarithmic scale

Considering 4 frequencies per one third octave band (fourth point in Figure 3) the uncertainty is less than 3%. In this case the computational cost is about 180 seconds. Increasing the number of frequencies does not cause a significative reduction of the uncertainty but increases unnecessarily the computational cost. Then, we understand that this is the best compromise point between uncertainty and computational cost for this method.

FINITE ELEMENTS METHOD

This simulation method was performed with the commercial software COMSOL Multiphysics. The first parameter to study was the size of each simpler part. Considering the maximum frequency of work, it can be calculated the minimum size of each element. First, it was studied the size of each element. In this paper was considered 8 elements for wave length and the maximum work frequency was 5000 Hz, so, it was performed simulations with 3 frequencies per one third octave band and 3 sizes of elements, (for 3000 Hz, 4000 Hz and 5000 Hz, or 0.0143 m, 0.0107 m and 0.0086 m).

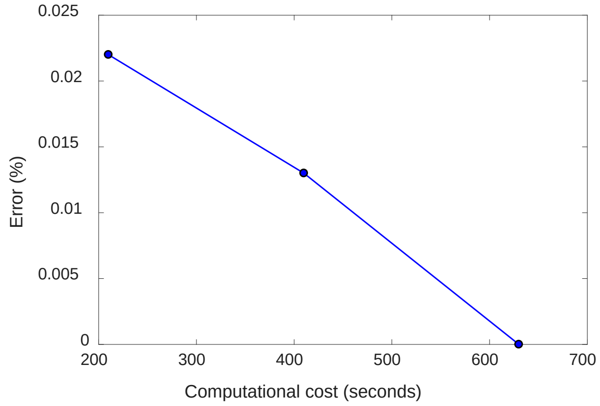

Figure 4. Size of the elements FEM

As it can be seen in Figure 4, the accuracy of the simulation is less than 3% for all cases, so we will use the maximum size of element, (the first point in figure 4) that corresponds to a size of 0.0143 m, because with higher sizes, the computational cost increases unnecessary.

Other variable parameter in FEM is the number of frequencies to simulate (like in multiple scattering). The same distribution of simulation frequencies used in multiple scattering was used for FEM. In the next figure we can see the uncertainty vs computational cost. Each point represents how many frequencies per one third octave band were used, (from 1 to 10).

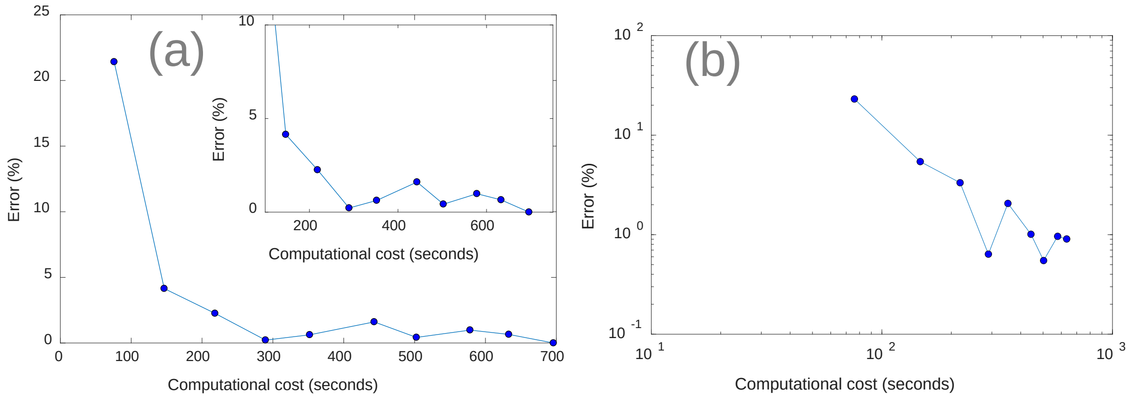

Figure 5. Uncertainty versus computational cost for FEM. Left (a): Linear axis. Right (b): Logarithmic scale

Figure 5 shows that considering three frequencies per one third octave band is enough to obtain an uncertainty lower than 3%. Increasing the number of simulation frequencies causes an unnecessary increase of the computational cost.

FINITE DIFFERENCE TIME DOMAIN

In an analogous way to the case of FEM, the most important parameter is the size of the elements. A smaller size of elements provides greater precision in the simulation but also requires more computational cost. The size of the elements was chosen in order to obtain 10 to 35 elements per wavelength. It is worth noting that the smallest wavelength to be considered is about 6 cm (that corresponds to the higher frequency, $4000*\sqrt{2}$ Hz).

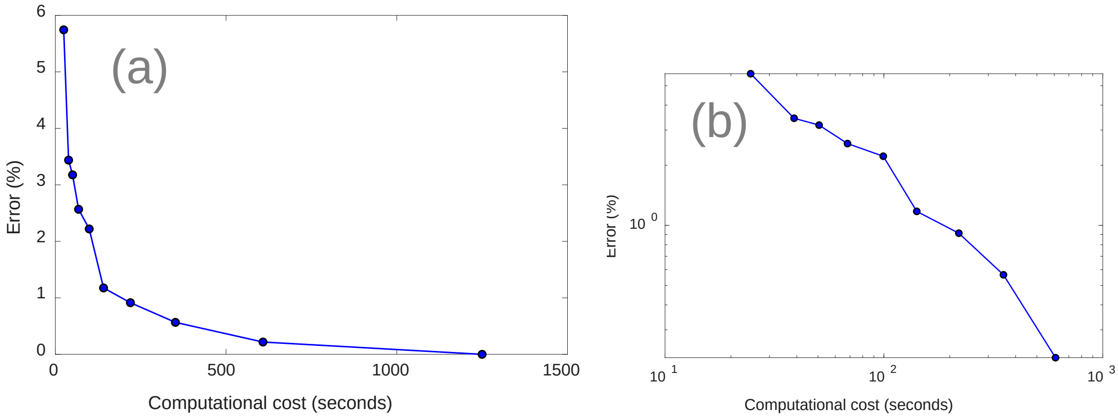

Figure 6. Uncertainty versus computational cost for FDTD. Left (a): Linear axis. Right (b): Logarithmic scale

As can be seen in Figure 6, considering around 12 elements per wavelength, (the second point in Figure 6), we can find equilibrium between computational cost and uncertainty. So, increasing the number of elements per wavelength increases the computational cost unnecessary.

COMPARATIVE ANALYSIS

In order to clarify which method can calculate the parameter of Insertion Loss (IL) with less computational cost and better accuracy, a comparison between the values of "IL" for the case of ff=60% and 75% it was carried out, the results are shown in the next Figure:

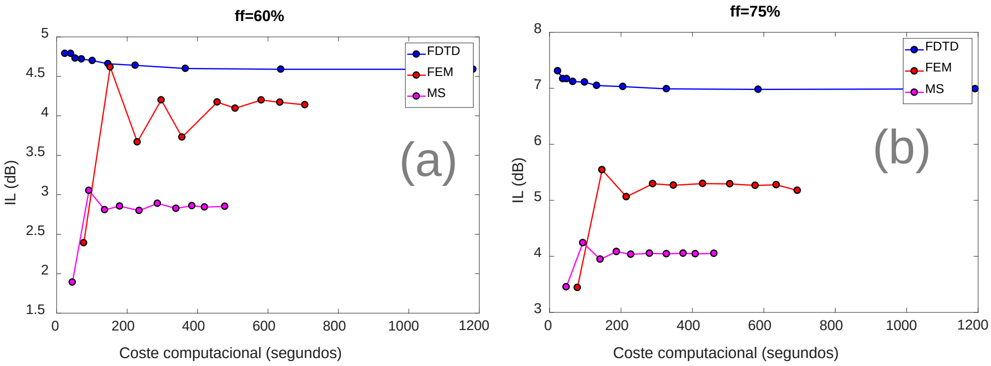

Figure 7. IL calculations. Left (a): For ff=60%. Right (b): For ff=75%

As we can be seen, on one hand the values of IL are different in all cases, and for values of ff higher, the difference is greater. On the other hand, we also can see that the value of IL converges faster in FDTD than in FEM. This can be seen better in the next figure (Figure 8).

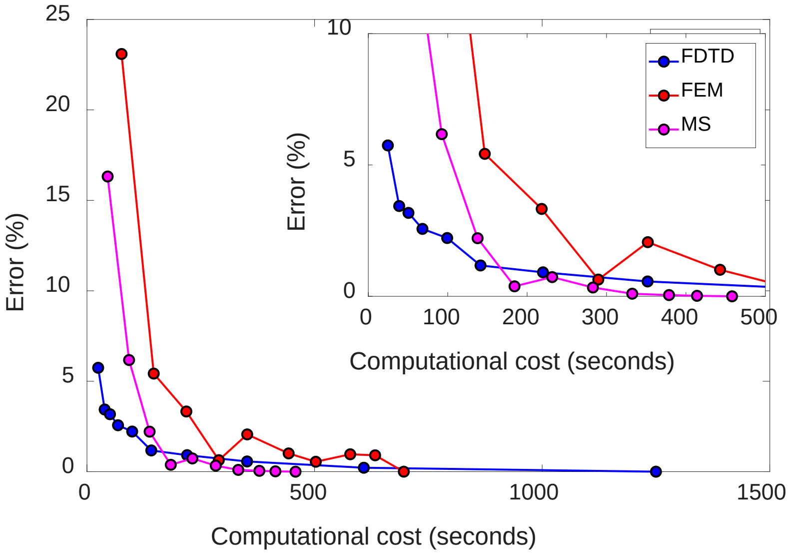

Figure 8. Computational cost of FDTD, FEM and MS

It can be seen in Figure 8 that FDTD need less computational cost to obtain less uncertainty in its results than the other two methods. This is the method that we are going to choose like the best method to simulate acoustic barriers based on periodic structures.

CONCLUSIONS

In this work, different simulation methods have been compared in order to clarify which would be used with less computational cost and would provide better accuracy in simulation of acoustic barriers based on periodic structures.

Attend to uncertainty results; any of the three methods studied could be used in this type of simulations. But, thanks to a comparative analysis study (Figure 8), we can conclude that Multiple Scattering has a low computational cost, but the values of IL are very different from the other simulation methods, this will be because MST is a semi-analytic method. Also, for first time in years, we have seen that with FEM and FDTD, we obtain different values very similar, and FEM needs more computational cost than FDTD. This was unexpected for us, and to conclude which method is more advised to make that simulations, we need experimental measures. In the future, we are going to perform these experimental measures, that couldn’t be performed because budget and logistic problems. Thus, FDTD converge the values of IL faster than FEM and MST.

We conclude that, in future works, is advised to perform simulations of acoustic barriers based on periodic structures using FDTD method.

ACKNOWLEDGEMENT

This work was partially supported by the Spanish “MINISTERIO DE ECONOMIA, INDUSTRIA Y COMPETITIVIDAD” under the project TEC2015-68076-R.

REFERENCES

1. Martínez Sala, R., Sancho, J., Sánchez, J. V., Gómez, V., Llinares, J., Meseguer, F., 1995. Sound-attenuation by sculpture. nature, 378(6554), 241- 241.
2. Yablonovitch E., Phys. Rev. Lett. 58, 2059 (1987)
3. John S., Phys. Rev. Lett. 58, 2486 (1987)
4. Sánchez-Pérez J. V., Rubio C., Martínez-Sala R., Sánchez-Grandía R. and Gómez V., Appl. Phys. Lett 81(27), 5240-5242 (2002)
5. Y. Cao, Z. Hou and Y. Liu, Convergence problem of plane-wave expansion method for phononic crystals, *Physics Letters A* 327, 2 (2004) 247-253.
6. Y.F. Wang, Y.S. Wang and X.X. Su, Large bandgaps of two-dimensional phononic crystals with cross-like holes, J. App. Phys. 110, 11 (2011)
7. J.V. Sánchez-Pérez, C. Rubio-Michavila and S. Castiñeira-Ibañez, Towards the development of a software to design acoustic barriers based on Sonic Crystals: an overlapping model, Proceedings of Euronoise 2015 (2015)
8. M. Liu, J. Xiang, H. Gao, Y. Jiang, Y. Zhou and F. Li, Research on band structure of one-dimensional phononic crystals based on wavelet finite element method, CMES - Comput. Model Eng. Sci. 97, 5 (2014).
9. L Godinho, P. Amado-Mendes, A. Pereira. And D. Soares Jr. An efficient mfs formulation for the analysis of acoustic scattering by periodic structures. (2017)
10. F. Zaviska. Über die beugung elektromagnetischer wellen an parallelen, unendlich langen kreisylindern. Ann. Phys., 40:1023, 1913.
11. W. von Ignatowsky. Zur theorie der gitter Ann. Phys. (1914).
12. Chen, Y. Y., Ye, Z., 2001. Theoretical analysis of acoustic stop bands in two-dimensional periodic scattering arrays. Physical Review E, 64(3), 036616.
13. Hrennikoff, Alexander "Solution of problems of elasticity by the framework method". Journal of applied mechanics (1941).
14. Courant, R. "Variational methods for the solution of problems of equilibrium and vibrations". Bulletin of the American Mathematical Societ (1943)
15. Daryl L. Logan. A first course in the finite element method. Cengage Learning. (2011).
16. Kane Yee "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media". IEEE Transactions on Antennas and Propagation. (1966)
17. J. Redondo, R. Picó, V. J. Sánchez-Morcillo, W. Woszczyk. Sound diffusers based on sonic crystals. The Journal of the Acoustical Society of America 2013 134:6, 4412-4417
18. EN 1793-3 “Road Traffic noise reducing devices – Test methods for determining the acoustic performance – Part 3: Normalized traffic noise spectrum” (1997).