PERFORMANCE OF SONIC CRYSTAL ACOUSTIC BARRIER WITH RESONANT SCATTERERS

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ABSTRACT
Sonic crystals have been regarded with interest for the attenuation of sound waves in a specific frequency range, since it is possible to define their geometry to match a given range of dominant noise frequencies. Besides the attenuation due to the geometric configuration (multiple internal scattering), the sound attenuation can be extended to a broader range of frequencies, making use of complementary resonance phenomena.
In the present article, a numerical formulation based on the Method of Fundamental Solutions is presented, with the objective of describing both phenomena and allowing the evaluation of different configurations for sonic crystal barriers.

RESUMEN
Los cristales de sonido son considerados con interés para la atenuación de ondas sonoras en rangos de frecuencias específicos, pudiéndose definir su geometría para que su efecto coincida con el rango de frecuencias dominantes de dado ruido. Además tal atenuación, debida a la configuración geométrica (dispersión interna múltiple), puede extenderse la atenuación acústica a una gama más amplia de frecuencias, debido a fenómenos de resonancia adicionales.
Este artículo presenta una formulación numérica basada en el Método de Soluciones Fundamentales, con el objetivo de describir ambos fenómenos y permitir la evaluación de diferentes configuraciones como barreras de cristales de sonido.

INTRODUCTION
Alongside questions regarding annoyance and loss of productivity, recent evidences seem to confirm previous suggestions that road traffic noise can also lead to cardiovascular problems [1]. In the European Union (EU) alone, it’s costs, concerning the effects in the health of the disturbed populations, may reach a total between one and six million DALYs (Disability-adjusted life years) [2]. In monetary terms, an evaluation of 40 billion €/year, again for the EU, has been suggested [3].
Regarding its mitigation, this noise can be disaggregated into different aspects relating to the generation, propagation and reception, enabling, in each of these steps, to advocate different actions that can contribute to such end. One such approach consists on using noise barriers, creating an obstacle in the propagation of the sound waves between the noisy roads and the buildings affected by this noise.

In the present paper the use noise barriers using cylindrical vertical elements, arranged to form a certain geometrical regular pattern (referred to as 'lattice' or 'array'), resulting in what has come to be known as Sonic Crystal (SC), will be addressed by means of numerical modeling of the sound attenuation provided by these structures.
One of the main features of SC barriers is its ability to restrain sound's propagation in certain specific frequency ranges, usually called 'band gaps', based on multiple interference phenomena of acoustic waves, which are scattered by the structure elements, (for this reason named scatterers), a process also known as 'Bragg interference'.
Additionally other acoustic mechanisms may also take place, filtering the incident sound to a greater or lesser extent, such as the use of non-rigid scatterers or elements covered with porous materials, providing improved capability, by adding sound absorption mechanisms [4; 5]. Another approach is to consider SCs with open scatterers capable of attributes similar to the, well-known, Helmholtz resonators, discussed in multiple related research work [6; 7].
After a brief description of the proposed numerical model, based on the Method of Fundamental Solutions, and its validation, this paper includes a general parametric study which is subsequently discussed in order to appreciate the applicability and efficiency of different type sonic-crystal barriers as a noise mitigation solution, namely roadside noise barriers.

ACOUSTIC WAVE PROPAGATION AND MODELING OF NOISE BARRIER PERFORMANCE

In a bi-dimensional space the propagation of sound can be analyzed, in the frequency domain, assuming its mathematical description by the Helmholtz equation, in its usual form:

$$\nabla^2 p(\mathbf{x},k) + k^2 \cdot p(\mathbf{x},k) = 0 \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{dy^2}$, $p(\mathbf{x},k)$ is the acoustic pressure at a location $\mathbf{x}=(x,y)$, $k=\omega/c$, $\omega=2\pi f$, f is the frequency and c is the propagation velocity within the acoustic domain.
Adopting the problem's governing formulation, specified in equation (1), it is then possible to define analytical solutions satisfying the equation for certain conditions.
One such instance relates to free-field conditions in which an infinite medium is considered and for which a two-dimensional pressure field is generated by a sound source located at point $\mathbf{x_0}$ of coordinates (x_0, y_0). This solution, known as the fundamental solution, allows the definition of the acoustic field in terms of pressure, $G(\mathbf{x},\mathbf{x}_0,k)$, and particle velocities, $H(\mathbf{x},\mathbf{x}_0,k,\vec{n})$, generated by the source at any receiver located at point $\mathbf{x}$ of coordinates (x,y) as:

$$G(\mathbf{x},\mathbf{x}_0,k) = -\frac{\mathrm{i}}{4} H_0^{(2)}(k\mathbf{r}) \tag{2}$$

and

$$H(\mathbf{x},\mathbf{x}_0,k,\vec{n}) = -\frac{k}{4\rho\omega} H_1^{(2)}(k\mathbf{r}) \frac{\partial \mathbf{r}}{\partial \vec{n}} \tag{3}$$

where $H_0^{(2)}$ and $H_1^{(2)}$ are Hankel functions, i is the imaginary unit, $\mathbf{r} = \|\mathbf{x} - \mathbf{x}_0\|$ is the distance between points $\mathbf{x}$ and $\mathbf{x}_0$, and $\vec{n}$ is the direction along which particle velocity is to be calculated.

The MFS formulation

In the present work, the Method of Fundamental Solutions (MFS) was selected to perform the numerical simulations related to the characterization of the pressure and velocity fields in a bi-dimensional problem of acoustic wave propagation. It is notably well suited to the analysis of a situation where the scatterers have a circular shaped cross section, allowing the physical boundary of the setting under study to be adequately defined by a set of so-called "collocation points", based on which a linear combination of fundamental solutions of the differential equation governing the problem will establish an approach to its solution. Several published works indicate that the MFS can provide a very accurate calculation of solutions for different physical problems, including some related to the field of acoustics [8; 9; 10], and wave propagation [11].
To solve the Helmholtz equation (1) using the MFS, the solution of the problem is estimated by a linear combination of fundamental solutions. For that purpose, as the proposed numerical model is intended to be applicable considering scatterers that are closed or open (to induce resonance

phenomena), different boundary conditions need to be considered, and imposed on the scatterers border, namely Dirichlet or Neumann conditions, defined by, respectively:

$$p(\mathbf{x},k)=\bar{p} \tag{4}$$

and

$$-\frac{1}{\mathrm{i}\rho\omega}\cdot\frac{\partial}{\partial\vec{n}}\cdot p(\mathbf{x},k)=\bar{v} \tag{5}$$

where $\bar{p}$ and $\bar{v}$ are pressure and particle velocity values at the border of a given scatterer.
In Figure 1 the general representation of the problem is presented. The bi-dimensional domain, $\mathbf{\Omega}$, corresponds to the xy-plane, and the Sonic Crystal is materialized by cylindrical scatterers, whose axes are parallel to the z-axis. When closed scatterers are used, their borders are defined as a whole and will be denoted as Γ_1. If the scatterers are open then the borders will result from the addition of two parts, $\Gamma=\Gamma_1\cup\Gamma_2$, where $\Gamma_1$is the actual physical border of the scatterer and Γ_2 represents a "virtual" part of the border (as described ahead).

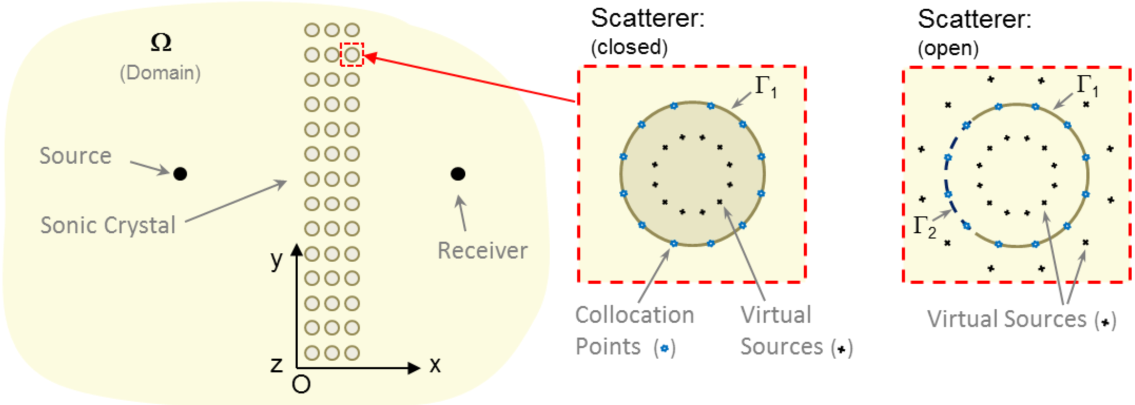


Figure 1 - General schematic representation of the problem.

To obtain a solution that simultaneously satisfies equation (1) with the boundary conditions (4) and (5), a set of NS virtual sources is considered, and it is assumed that the pressure field at any point of the domain, $\mathbf{x}$, can be represented by a linear combination of the effects of the NS sources, positioned at points $\mathbf{x}_j$, so that:

$$p(\mathbf{x},k)=\sum_{j=1}^{NS}Q_jG(\mathbf{x},\mathbf{x}_j,k) \tag{6}$$

where Q_j is an amplitude factor associated with each virtual source, which is, a-priori, unknown.
Because the fundamental solution presents a singularity at its point of application, it should be noted that the virtual sources will necessarily have to be placed outside the domain, otherwise those singularities would occur within that domain.
For that reason, if the scatterers are closed the virtual sources will be located inside them. If open scatterers are assumed two sets of virtual sources need to be considered, as shown in Figure 1. In this last case a virtual interface, Γ_2, is used for modelling purposes only, letting the domain, Ω, be considered as divided in two sub-domains, Ω_1 and Ω_2, one "outside" and the other "inside" the border defined by $\Gamma_1\cup\Gamma_2$. In such case the set of NS virtual sources placed in Ω_1 will determine the fields of pressure and particle velocity in Ω_2, and *vice-versa*, (thus totaling 2×NS virtual sources).
For the problem under study, taking into account the representation of the pressure fields and velocities defined above, it is also necessary to consider a set of NC collocation points, at coordinates $\mathbf{x}_i$, distributed along the boundary (Γ_1 or $\Gamma_1\cup\Gamma_2$), as shown in Figure 1. Now the boundary conditions (4) and (5), will assume, depending on the type of border (real or virtual) where each collocation point is placed, the generic form:

$$\sum_{j=1}^{NS}Q_jG(\mathbf{x}_i,\mathbf{x}_j,k)=\bar{p}_i \tag{7}$$

and,

$$\sum_{j=1}^{NS} Q_j H(\mathbf{x}_i, \mathbf{x}_j, k, \vec{n}_i) = \bar{v}_i \tag{8}$$

where $\bar{p}_i$ and $\bar{v}_i$ are the values of sound pressure and normal particle velocity (in the direction $\vec{n}_i$) to be prescribed at each collocation point i.
Based on these last equations, it is possible to establish a system with NC equations for NS (or 2×NS) unknowns, allowing the calculation of the unknown amplitude factors Q_j (or $Q_{1,j}$ and $Q_{2,j}$, related to Ω_1 and Ω_2, respectively if open scatterers are used).
In this work it was chosen to adopt NC to be the same as NS, as it led (as demonstrated ahead) to good results, but also because this will result in a square system, which is solvable by common Gaussian elimination.
The above mentioned equations result from imposing null velocity along the boundary Γ_1 (on the outside of closed scatters, or, if open scatterers are considered, on both sides), or continuity of pressure and velocity along the virtual interface Γ_2.
If acoustic absorption is to be ascribed to the scatterers' surface, it is possible to impose a standard impedance boundary condition (Robin) by combining equations (7) and (8).
When open scatterers are assumed, to mimic the possibility that its interior may be filled with a material that may significantly modify the sound propagation behavior, with respect to a situation in which the medium is air, Miki's model [12] for equivalent fluid properties was adopted. In such case, the propagation velocity within the acoustic domain, c, and its density, ρ, are adjusted for the wave number, k, and characteristic impedance, Z_c, of that medium, given by:

$$k = \frac{\omega}{c_0}\left[1 + 7{,}81\left(10^3 \cdot \frac{f}{\sigma}\right)^{-0{,}618} - \mathrm{i} \cdot 11{,}41\left(10^3 \cdot \frac{f}{\sigma}\right)^{-0{,}618}\right] \tag{9}$$

and

$$Z_c = \rho_0 c_0\left[1 + 5{,}50\left(10^3 \cdot \frac{f}{\sigma}\right)^{-0{,}632} - \mathrm{i} \cdot 8{,}43\left(10^3 \cdot \frac{f}{\sigma}\right)^{-0{,}632}\right] \tag{10}$$

which are obtained from a single parameter, namely the flow resistivity, σ, for that material.
Finally, being able to compute the pressure field throughout the domain, it will be possible to evaluate the sound attenuation on a given position of the receiver, by calculating (at each frequency or a frequency band) the difference between the values of the sound pressure levels, with and without the barrier.

Validation of the proposed model

The proposed MFS model's accuracy was assessed both by experimental measurements and by comparison with the results provided by another numerical method, used as reference.
For this purpose a reduced-scale physical model of a two-dimensional sonic crystal was built, allowing different geometrical configurations. The guiding concept was to compare, for several different lattice arrangements, the sound attenuations obtained through experimental measurements with the results estimated from numerical calculations. Six different geometrical configurations of the SC, including the positions of the source and grid of receivers, were analyzed and the results clearly revealed that the predictions provided by the numerical model are very close to those measured experimentally.
As an example Figure 2 shows the geometrical configuration of one of the tested setups as well as the Insertion Loss (IL) curves, disclosing the difference between sound levels with and without the barrier, obtained experimentally and numerically.
Additionally to gauge the correct formulation and implementation of the proposed MFS model to problems with two distinct regions, when open scatterers are considered, its results were compared with those obtained from a reference solution based on another numerical method.
Such approach was developed considering a single scatterer arrangement, shown in Figure 3, exposed to the incidence of acoustic waves generated by a line source, and computing the pressure fields in two locations by using the MFS model as well as implementing a Boundary Element Method (BEM) based model. More specifically, to adequately allow describing such case with elements of very small thickness, a Dual-BEM variant was employed, using a total of 140 elements to discretize the problem. The outcomes from both numerical models show a very close proximity between the results, in both receiver locations, as can be seen in Figure 4, suggesting a correct implementation and good behavior of the MFS model, also for this case.

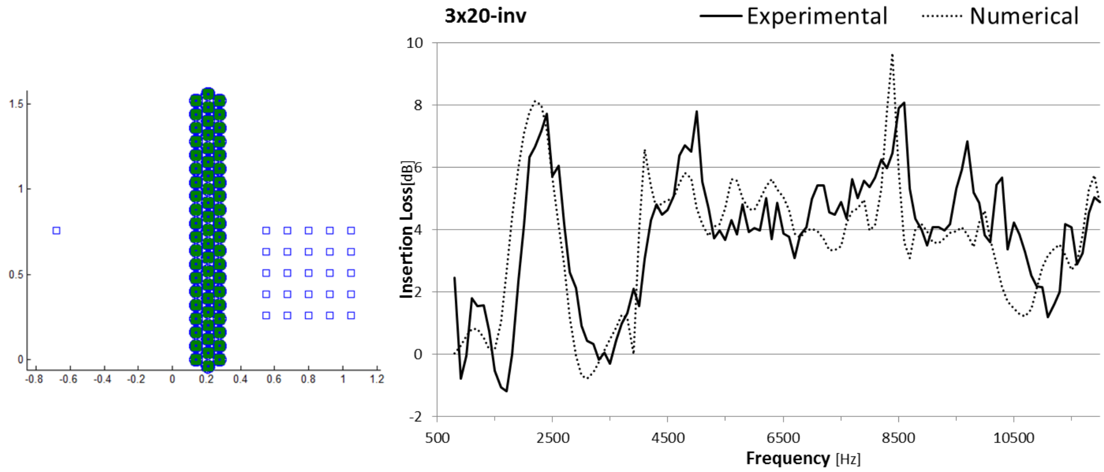


Figure 2 – Geometrical setup and sound attenuations (experimental vs. numerical)

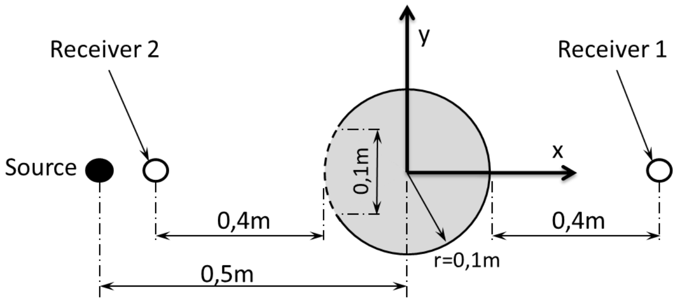
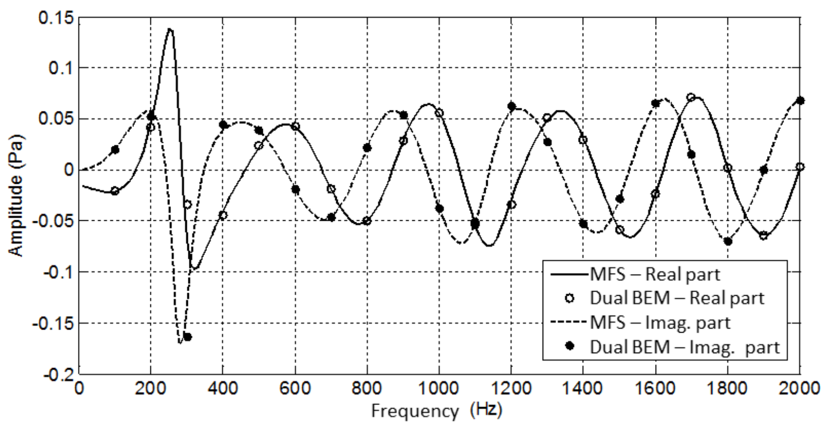


Figure 3 – Geometrical setup for MFS and BEM based numerical models analysis

i)



ii)

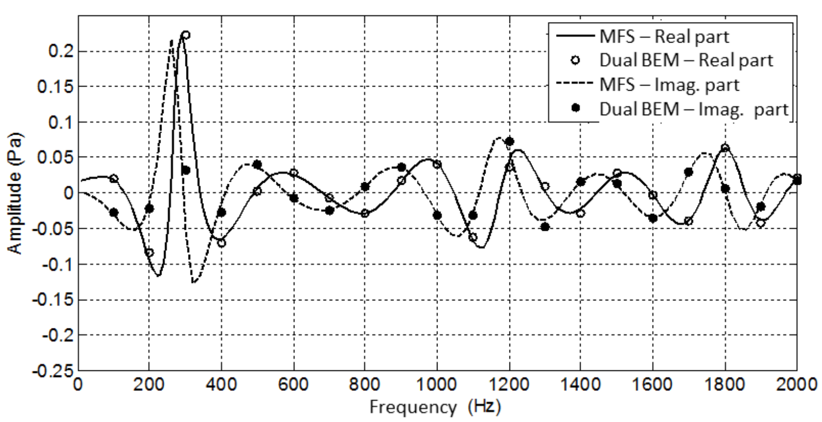


Figure 4 – Results comparison at Receiver 1 (i) and Receiver 2 (ii)

DISCUSSION OF RESULTS

To assess the versatility of the suggested numerical model, based on the use of the MFS, in evaluating the performance of a SC when used as road noise barriers, some simulations were carried out to analyze the consequences of certain aspects of such barriers:

- Geometry of the periodic arrangement of the scatterers;
- Use of either rigid scatterers or ones with some level of acoustic absorption;
- Whether scatterers are closed or open (split-ring resonator – SRR – type);
- If the (open) scatterers' interior is filled with some type of absorbent material or air.

Implementing the proposed model, a vehicle will act as the sound source, a nearby house will relate to the receiver and a SC noise barrier will be located between them. It will be materialized by a set of vertical cylinders, considered to be arranged in two distinct lattice configurations, typical of Sonic Crystals, namely square or triangular, as seen in Figure 5.

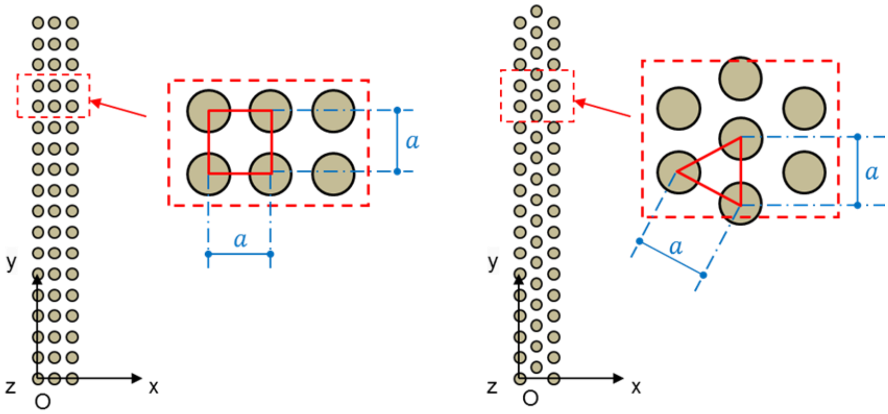


Figure 5 – Square and triangular lattice configurations of the Sonic Crystal barriers.

Attempting to simulate a realistic setting, based on usual dimensions from a typical cross section of a road, the source and receivers positions will correspond, respectively, to values of x=-6.5 m and x=7.5 m, in the axis system shown in Figure 5. As for the y-axis values, both source and receiver were considering at the center of the barrier along that axis.

An interesting prospect on how to materialize a SC noise barrier is to use tree logs as scatterer elements, which may be judged as an environmentally friendly option, if trees are to be farmed for that purpose. Considering that the scatterers should have a plausible dimension if obtained from trees, the diameter of each scatterer will be assumed to be 0.10 m.

Regarding the distance between the centers of the scatterers, or lattice constant a (see Figure 5), its value will result from the desired sound attenuation. It has been documented [13] that, in a SC barrier, the central frequency of the Bragg interference, f_B, on sound wave propagation is related to the speed of sound, c, and the SC's lattice constant by the relationship:

$$f_B = \frac{c}{2.a} \tag{11}$$

Attending to the chosen diameter of the scatterers, to prevent very dense layouts, and because road traffic noise typically presents a noise frequency spectrum with a maximum near 1000 Hz, as often mentioned in the literature [14], to achieve the highest insertion loss, around that peak the value of a will be set at 0.20 m (rounded from 0.17m).

In trying to keep the structures as economical as possible, the "width" of the SC, (in the x-axis) will be set to two or three rows of scatterers. The number of scatterers along the y-axis resulted from the smallest "length" along that direction for which the diffraction effect near the extremities of the structure is negligible, when source and receivers are at the center of that "length". An earlier study [15] showed that, for those positions of source and receivers, a length of 16m would be enough, in both lattice configurations, with either two or three rows of scatterers. For the abovementioned value of a, that will result in a total of 80 scatterers along the y-axis.

Once this main setup was established, the proposed model was used to run four distinct configurations, considering different features of the scatterers. The computations carried out focused on the values of Insertion Loss (IL) provided by SC barriers corresponding to a range of frequencies nearby the 1000 Hz peak, namely between 100 and 1500 Hz.

The computed results for the several cases that were studied are summarized in Figure 6.

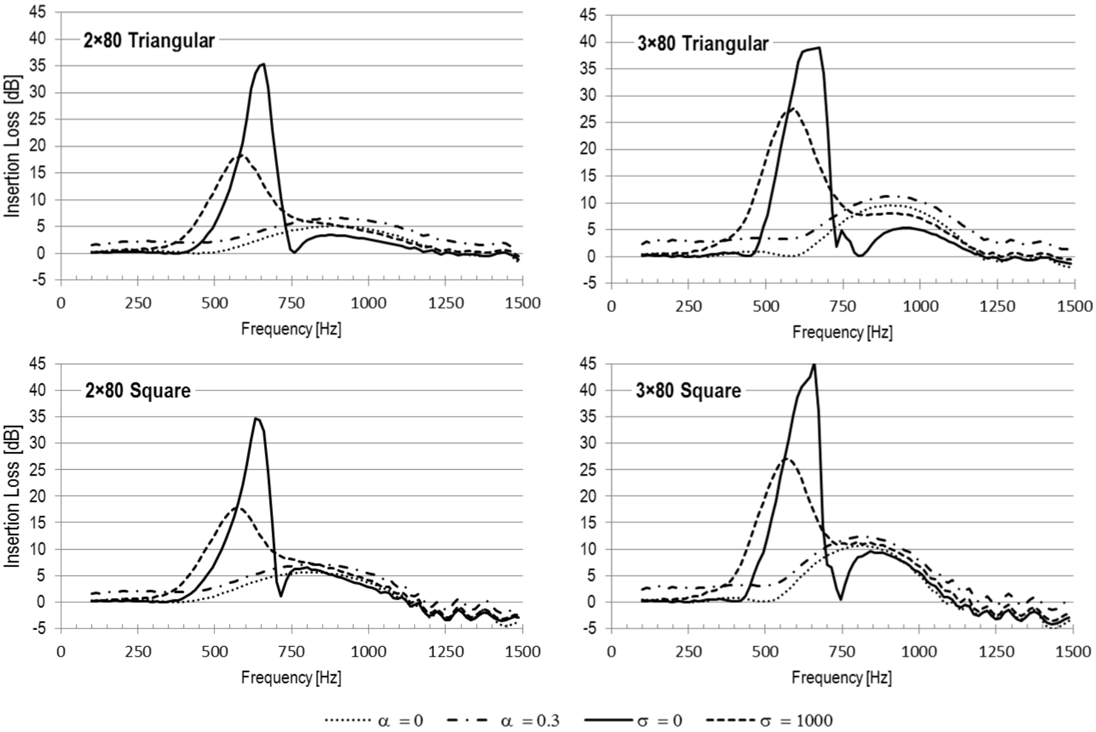


Figure 6 – Sound attenuations [dB] for different SC geometries and types of scatterers.

Each set of results depicted refers to the use of two types of closed and two of open scatterers. In the former they are considered either rigid or as having some degree of sound absorption, in which case, the real valued impedance incorporated in the numerical analysis, depends on the sound absorption coefficient, α: Therefore the IL related to closed scatterers are denoted by α=0 (the rigid) and α=0.3 (as that was the value chosen to illustrate this case).

When open scatterers were assumed, they were defined by an opening, towards the sound source, with a centered angle of $\pi/3$ radians. Two distinct situations were assumed: empty scatterers, indicated by σ=0, and σ=1000 for scatterers filled with some sound absorbing material, where σ represents the flow resistivity of the material (thus σ=0 if air is present).

From those findings some preliminary conclusions may be forwarded. For example, regarding the most critical traffic noise frequencies, on all four geometrical setups the best sound attenuation is achieved when closed scatterers with sound absorption are used. Additionally, by increasing the ‘width’ of the SC, from 2 to 3 rows, an almost duplication of IL is achieved.

When SRR type elements are used, the sound attenuations around the central frequency of the Bragg interference are comparable to those obtained by the use of rigid closed scatterers, particularly when the SRR is filled with an acoustically absorbent material (such as mineral wool, typically presenting a flow resistivity above 1000 N.s/m^4).

Although the SC with open scatterers presents very high IL values due to the resonance phenomena they induce, these happen at lower frequencies than desired (1000 Hz). For such resonances to occur at higher frequencies, without changing the position of the band gap associated with the Bragg interference, slight changes would need to be made, namely in what concerns the dimensions of the opening in the SRR scatterer. This specific issue was not treated here, since this was not the primary focus of the present work; instead, the authors mostly intended to demonstrate the presence of the relevant IL peaks occurring in the case of sonic crystals built using SRR elements.

CONCLUSIONS

The use of Sonic Crystals as road noise barriers was discussed in this work with, based on a numerical approach using the Method of Fundamental Solutions. The accuracy of the numerical model is examined by comparing the results against those obtained by experimental measurements as well as against results from another numerical method. The very good matching between the results being compared yielded very favorable indications in favor of the proposed model’s capabilities in evaluating a sound pressure field.

Several arrangements were studied, covering different combinations of geometrical and acoustical features: two different lattice configurations of the SCs, but also the use of either closed or open scatterers and the possibility that those elements possess some degree of acoustic absorption was analyzed. The influence of those aspects in the sound attenuations provided by the SCs was evaluated, allowing some broad indications to be established.

More notably, the use of closed scatterers with sound absorbing attributes was found to be associated with the highest levels of sound attenuation, for all different SC structures studied.

On the other hand using open scatterers only provides some sort of benefit for lower frequencies, well under the target range where sound attenuation was intended to occur.

Future developments will predictably include analysis of more complex features, like those related with three-dimensional effects of the Sonic Crystal, which, in a more realistic configuration, has a limited height and thus may also be affected by diffraction effects occurring over its top. The proposed model is only applicable in a 2D domain, so significant adaptations will be needed, as well as the need of further validation procedures.

Other relevant topics are the study of technical and economic aspects related to the use of SC, in which not only its acoustical performance is analyzed, but also regarding other aspects like the use of certain materials, mechanical stability, durability or associated costs. Research on what type of acoustically absorbing material can be adopted seems very relevant.

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