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Solving New Poroelastic Models by Finite Element Method

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RESUMEN: Este trabajo se enfrenta con la resolución numérica del comportamiento acústico de materiales porosos con matriz sólida elástica. Suponiendo una estructura periódica, usamos nuevos modelos poroelásticos obtenidos mediante técnicas de homogeneización. Con el objetivo de calcular los coeficientes en estos nuevos modelos, se resuelven problemas de contorno en la celda elemental del medio poroso. Finalmente, centramos nuestra atención en los materiales poroelásticos no disipativos con poros abiertos y proponemos un método de elementos finitos para calcular la respuesta a una excitación harmónica de una cavidad tridimensional que contiene un fluido y un material poroelástico. El elemento finito usado para el fluido es el elemento de orden más bajo de la familia introducida por Raviart y Thomas que evita los modos espúreos mientras que, para el campo de desplazamientos en el medio poroso, se usa el "mini elemento" con el objetivo de obtener un método estable.

ABSTRACT: This communication deals with the numerical solution of the acoustical behavior of elastic porous materials. Assuming a periodic structure, we use new poroelastic models obtained by homogenization techniques. In order to compute the coefficients in these new models, we solve boundary-value problems in the unitary cell. Finally, we focus our attention on non-dissipative poroelastic materials with open pore and propose a finite element method in order to compute the response to a harmonic excitation of a three-dimensional enclosure containing a free fluid and a poroelastic material. The finite element used for the fluid is the lowest order face element introduced by Raviart and Thomas that avoids the spurious modes whereas, for displacements in porous medium, the "mini element" is used in order to achieve stability of the method.

1. INTRODUCTION

Theory for mechanical behavior of poroelastic materials was established by Biot [5], when the porous elastic solid is saturated by a viscous fluid. However, when analyzing Biot's model, coefficients are not properly defined and, in general, their determination is not clear although several experimental procedures have been provided, as it can be seen in Biot and Willis [6]. Derivation of macroscopic models for poroelastic materials depends strongly on connectivity of the fluid part. Fundamental references are papers by Gilbert and Mikelić [9] and by Clopeau *et al* [7] where the classical dissipative Biot's model was derived by homogenization using two-scale convergence methods. They also contain a number of references to papers on dissipative Biot's law. Moreover, the same procedure has been applied, for the first time, in Ferrín and Mikelić [8] to derive macroscopic models for non-dissipative poroelastic material with open or closed pore.

Concerning numerical simulation, an increasing number of papers can be found for the two cases of rigid and elastic skeleton. The lowest order finite method introduced by Raviart and Thomas has been applied in Bermúdez *et al* [3] to the case of porous media with rigid skeleton to solve, in particular, the response problem when using both a Darcy's like model and the Allard-Champoux model. With respect to the case of elastic skeleton, papers by Panneton and Atalla [10], or Atalla *et al* [2], among others, are examples of application of finite element methods to sound propagation in poroelastic media by using Biot's model.
We only consider the case of elastic frame porous material. The non-dissipative model that we shall take into account has been derived in Ferrín and Mikelić [8] for open and closed pores. The advantages exhibited by this model with respect to classical Biot's model lies in that we know mathematical expressions allowing us to compute their coefficients.

2. STATEMENT OF THE PROBLEM

In the rest of the paper we consider a coupled system consisting of an acoustic fluid (i.e. inviscid compressible barotropic) in contact with an elastic porous medium. Both are enclosed in a three-dimensional cavity with rigid walls except one on which a harmonic excitation is applied. Let Ω_F and Ω_A be the domains occupied by the fluid and the porous medium, respectively (see Figure 1).

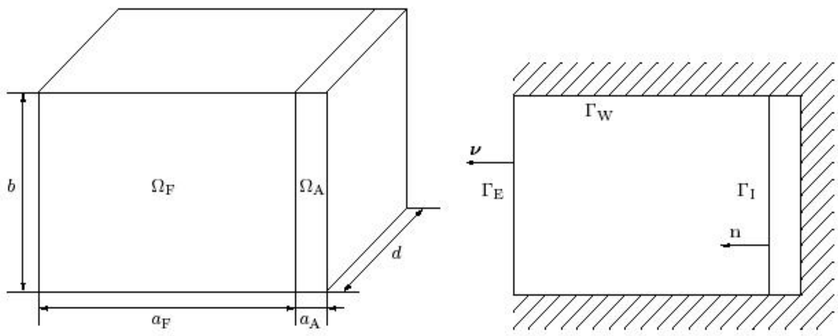

Figure 1 – *3D domain and vertical cut.*

The boundary of $\bar{\Omega}_F \cup \bar{\Omega}_A$, denoted by Γ, is the union of two parts, Γ_W and Γ_E, where Γ_W denotes the rigid walls of the cavity. Let $\mathbf{v}$ be the outward unit normal vector to Γ. The interface between the fluid and the porous medium is denoted by Γ_I and $\mathbf{n}$ is the unit normal vector to this interface pointing outwards Ω_A. In order to study the response of the fluid-porous coupled system subject to an harmonic excitation acting on Γ_E, we consider the model for open pore non-dissipative poroelastic media. Firstly, we recall that governing equations for free small amplitude motions of an acoustic fluid filling Ω_F are given, in terms of displacement and pressure fields, by

$$\rho_{\mathrm{F}} \frac{\partial^2 \mathbf{U}_{\mathrm{F}}}{\partial t^2} + \operatorname{grad} P_{\mathrm{F}} = 0 \text{ in } \Omega_{\mathrm{F}}, \tag{1}$$

$$P_{\mathrm{F}} = -\rho_{\mathrm{F}} c^2 \operatorname{div} \mathbf{U}_{\mathrm{F}} \text{ in } \Omega_{\mathrm{F}}, \tag{2}$$

where P_{F} is pressure, $\mathbf{U}_{\mathrm{F}}$ is displacement field, ρ_{F} is density and c is acoustic speed.
Secondly, if we denote by $\mathbf{U}_{\mathrm{A}}$ and P_{A} the macroscopic displacement and pressure fields in the porous medium, the equations describing small motions are,

$$\left(\rho \mathbf{I} - \rho_{\mathrm{F}} A\right) \frac{\partial^2 \mathbf{U}_{\mathrm{A}}}{\partial t^2} - \operatorname{div}\left(A^H \left[D\left(\mathbf{U}_{\mathrm{A}}\right)\right]\right) - \left(A - B^H - \phi \mathbf{I}\right) \operatorname{grad} P_{\mathrm{A}} = 0 \text{ in } \Omega_{\mathrm{A}}, \tag{3}$$

$$\hat{c} \frac{\partial^2 P_{\mathrm{A}}}{\partial t^2} + \frac{1}{\rho_{\mathrm{F}}} \operatorname{div}\left(A \operatorname{grad} P_{\mathrm{A}}\right) = -\operatorname{div}\left(\left(A + B^H - \phi \mathbf{I}\right) \frac{\partial^2 \mathbf{U}_{\mathrm{A}}}{\partial t^2}\right) \text{ in } \Omega_{\mathrm{A}}, \tag{4}$$

where $D\left(\mathbf{U}\right) = \frac{1}{2}\left(\operatorname{grad} \mathbf{U} + \operatorname{grad} \mathbf{U}^t\right)$ and coefficient $\hat{c}$, tensors A, B^H and linear operator A^H depend on geometry of cells composing the poroelastic material and also on physical properties of its solid and fluid parts. In fact, one can check that B^H is a symmetric linear operator and tensor A^H, such that $\left(A^H \left[D\right]\right)_{kl} = A_{klij} D_{ij}$, satisfies $A^H_{klij} = A^H_{lkij} = A^H_{lkji}$.
Since the fluid is supposed to be inviscid, only the normal component of displacements vanishes on Γ_{W}, namely, $\mathbf{U}_{\mathrm{F}} \cdot \nu = 0$ on $\Gamma_{\mathrm{W}} \cap \partial\Omega_{\mathrm{F}}$, whereas for boundary displacement of porous medium we suppose $\mathbf{U}_{\mathrm{A}} = \mathbf{0}$ on $\Gamma_{\mathrm{W}} \cap \partial\Omega_{\mathrm{A}}$. Similarly, on interface Γ_{I} between fluid and porous medium we consider the usual interface conditions of continuity of forces and normal displacements, that is,

$$-P_{\mathrm{F}} \mathbf{n} = A^H \left[D\left(\mathbf{U}_{\mathrm{A}}\right)\right] \mathbf{n} + P_A \left(A + B - \phi \mathbf{I}\right) \mathbf{n}, \;\; \mathbf{U}_{\mathrm{F}} \cdot \mathbf{n} = \mathbf{U}_{\mathrm{A}} \cdot \mathbf{n} \text{ on } \Gamma_{\mathrm{I}}. \tag{5}$$

If a normal displacement U_0 is imposed on Γ_{E}, the above equations describing the motion of coupled system (1)-(5) must be completed with boundary condition $\mathbf{U}_{\mathrm{F}} \cdot \nu = U_0$ on Γ_{E}. Finally, in order to close the model (see [2]), we are going to assume that

$$A \frac{\partial P_{\mathrm{A}}}{\partial \nu} = \mathbf{0} \text{ on } \Gamma_{\mathrm{W}} \cap \partial\Omega_{\mathrm{A}}, \;\; A \frac{\partial P_{\mathrm{A}}}{\partial \nu} = \mathbf{0} \text{ on } \Gamma_{\mathrm{I}}. \tag{7}$$

We are interested in harmonic vibrations so let us suppose excitation U_0 to be harmonic, i.e., $U_0\left(x, y, z, t\right) = \operatorname{Re}\left(e^{i\omega t} u_0\left(x, y, z\right)\right)$; then all fields are also harmonic. By replacing these expressions in equations (1)-(7), we can define a harmonic source problem associated with the unsteady source problem, namely,

$$-\omega^2 \rho_{\mathrm{F}} \mathbf{u}_{\mathrm{F}} + \operatorname{grad} p_{\mathrm{F}} = 0 \text{ in } \Omega_{\mathrm{F}}, \tag{8}$$

$$p_{\mathrm{F}} = -\rho_{\mathrm{F}} c^2 \operatorname{div} \mathbf{u}_{\mathrm{F}} \text{ in } \Omega_{\mathrm{F}}, \tag{9}$$

$$-\omega^2 \left(\rho \mathbf{I} - \rho_{\mathrm{F}} A\right) \mathbf{u}_{\mathrm{A}} - \operatorname{div}\left(A^H \left[D\left(\mathbf{u}_{\mathrm{A}}\right)\right]\right) - \left(A - B^H - \phi \mathbf{I}\right) \operatorname{grad} p_{\mathrm{A}} = 0 \text{ in } \Omega_{\mathrm{A}}, \tag{10}$$

$$-\omega^2 \hat{c} p_{\mathrm{A}} + \frac{1}{\rho_{\mathrm{F}}} \operatorname{div}\left(A \operatorname{grad} p_{\mathrm{A}}\right) = \omega^2 \operatorname{div}\left(\left(A + B^H - \phi \mathbf{I}\right) \mathbf{u}_{\mathrm{A}}\right) \text{ in } \Omega_{\mathrm{A}}, \tag{11}$$

$$-p_{\mathrm{F}} \mathbf{n} = A^H \left[D\left(\mathbf{u}_{\mathrm{A}}\right)\right] \mathbf{n} + p_A \left(A + B - \phi \mathbf{I}\right) \mathbf{n} \text{ on } \Gamma_{\mathrm{I}}, \tag{12}$$

$$\mathbf{u}_{\mathrm{F}} \cdot \mathbf{n} = \mathbf{u}_{\mathrm{A}} \cdot \mathbf{n} \text{ on } \Gamma_{\mathrm{I}}, \tag{13}$$

$$\mathbf{u}_{\mathrm{F}} \cdot \nu = 0, \ \mathbf{u}_{\mathrm{A}} = \mathbf{0} \text{ on } \Gamma_{\mathrm{W}} \cap \partial \Omega_{\mathrm{A}}. \tag{14}$$

$$\mathbf{u}_{\mathrm{F}} \cdot \nu = U_0 \text{ on } \Gamma_{\mathrm{E}}. \tag{15}$$

$$A \frac{\partial p_{\mathrm{A}}}{\partial \mathbf{v}} = \mathbf{0} \text{ on } \Gamma_{\mathrm{W}} \cap \partial \Omega_{\mathrm{A}}, \quad A \frac{\partial p_{\mathrm{A}}}{\partial \mathbf{v}} = \mathbf{0} \text{ on } \Gamma_{\mathrm{I}}. \tag{16}$$

3. WEAK FORMUALTION

In order to use finite element methods for numerical solution of (8)-(16), we write a weak formulation. For this purpose, we first introduce appropriate functional spaces. Let $\mathbf{V}$ be the Hilbert space $\mathbf{V} = \mathrm{H}(\mathrm{div}, \Omega_{\mathrm{F}}) \times \mathrm{L}^2(\Gamma_{\mathrm{I}}) \times \mathrm{H}^1(\Omega_{\mathrm{A}})^3 \times \mathrm{H}^1(\Omega_{\mathrm{A}})$ and $\mathbf{V}_0$ its closed subspace:

$$\mathbf{V}_0 = \left\{(\mathbf{v}_{\mathrm{F}}, q_{\mathrm{F}}, \mathbf{v}_{\mathrm{A}}, q_{\mathrm{A}}) \in \mathbf{V} : \mathbf{v}_{\mathrm{F}} \cdot \nu = 0 \text{ on } (\Gamma_{\mathrm{W}} \cup \Gamma_{\mathrm{E}}) \cap \partial \Omega_{\mathrm{F}}, \mathbf{v}_{\mathrm{A}} = 0 \text{ on } \Gamma_{\mathrm{W}} \cap \partial \Omega_{\mathrm{A}}\right\}. \tag{17}$$

Kinematic constraint in (13) is weakly imposed on the interface between the fluid and the porous medium by integrating this equation multiplied by a test function q_{F} defined on Γ_{I}. In conclusion, we can write the following source hybrid problem:

For fixed angular frequency ω*, find* $(\mathbf{u}_{\mathrm{F}}, p_{\mathrm{F}}, \mathbf{u}_{\mathrm{A}}, p_{\mathrm{A}}) \in \mathbf{V}$ *satisfying (14), (15) and furthermore,*

$$\begin{gathered}
\int_{\Omega_{\mathrm{F}}} \rho_{\mathrm{F}} c^2 \operatorname{div} \mathbf{u}_{\mathrm{F}} \operatorname{div} \overline{\mathbf{v}}_{\mathrm{F}} - \omega^2 \int_{\Omega_{\mathrm{F}}} \rho_{\mathrm{F}} \mathbf{u}_{\mathrm{F}} \cdot \overline{\mathbf{v}}_{\mathrm{F}} - \omega^2 \int_{\Omega_{\mathrm{A}}} \left(\rho \mathbf{I} - \rho_{\mathrm{F}} A\right) \mathbf{u}_{\mathrm{A}} \cdot \overline{\mathbf{v}}_{\mathrm{A}} + \\
\int_{\Omega_{\mathrm{A}}} A^H [D(\mathbf{u}_{\mathrm{A}})] : D(\overline{\mathbf{v}}_{\mathrm{A}}) + \int_{\Omega_{\mathrm{A}}} \operatorname{div}\left((A + B^H - \phi \mathbf{I})^t \overline{\mathbf{v}}_{\mathrm{A}}\right) p_{\mathrm{A}} + \int_{\Omega_{\mathrm{A}}} \hat{c} p_{\mathrm{A}} \overline{q}_{\mathrm{A}} + \qquad (17) \\
\int_{\Omega_{\mathrm{A}}} \frac{1}{\rho_{\mathrm{F}} \omega^2} A \operatorname{grad} p_{\mathrm{A}} \cdot \operatorname{grad} \overline{q}_{\mathrm{A}} + \int_{\Omega_{\mathrm{A}}} \operatorname{div}\left((A + B^H - \phi \mathbf{I}) \mathbf{u}_{\mathrm{A}}\right) \overline{q}_{\mathrm{A}} = \int_{\Gamma_{\mathrm{I}}} p_{\mathrm{F}} (\overline{\mathbf{v}}_{\mathrm{F}} \cdot \mathbf{n} - \overline{\mathbf{v}}_{\mathrm{A}} \cdot \mathbf{n}),
\end{gathered}$$

$$\int_{\Gamma_{\mathrm{I}}} \overline{q}_{\mathrm{F}} (\mathbf{u}_{\mathrm{A}} \cdot \mathbf{n} - \mathbf{u}_{\mathrm{F}} \cdot \mathbf{n}) = 0, \tag{18}$$

for all $(\mathbf{v}_{\mathrm{F}}, q_{\mathrm{F}}, \mathbf{v}_{\mathrm{F}}, q_{\mathrm{A}}) \in \mathbf{V}_0$.

4. FINITE ELEMENT DISCRETIZATION

Fluid and porous displacement fields belong to different functional spaces, $\mathrm{H}(\mathrm{div},\Omega_{\mathrm{F}})$ and $\mathrm{H}^1(\Omega_{\mathrm{A}})^3$, respectively, hence different types of finite elements should be used for each of them in order to discretize weak problem (17)-(18). Let T_h be a regular family of tetrahedral partitions of $\Omega_{\mathrm{F}} \cup \Omega_{\mathrm{A}}$ compatible with the different domains and boundaries.

To approximate fluid displacements, the lowest order Raviart-Thomas finite element (see [4]) is used in order to avoid spurious modes typical of displacement formulations when they are discretized by standard Lagrange finite elements.

They consist of vector valued functions which, when restricted to each tetrahedron, are incomplete linear polynomials of the form $\mathbf{u}^h(x,y,z)=(a+dx,b+dy,c+dz), a,b,c,d \in \mathbb{R}$. These vector fields have constant normal components on each of the four faces of a tetrahedron which define a unique polynomial function of this type. Moreover, the global discrete displacement field $\mathbf{u}^h$ is allowed to have discontinuous tangential components on the faces of tetrahedra of partition T_h. Instead, its constant normal components must be continuous through these faces (these constant values being the degrees of freedom defining $\mathbf{u}^h$). Because of this, $\operatorname{div}\mathbf{u}^h$ is globally well defined in Ω_{F}. Thus, for fluid displacements we use the Raviart-Thomas space

$$\mathbf{R}_h(\Omega_{\mathrm{F}}) := \left\{\mathbf{u} \in \mathrm{H}(\mathrm{div},\Omega_{\mathrm{F}}) : \mathbf{u}|_T \in R_0(T), \forall T \in T_h, T \subset \Omega_{\mathrm{F}}\right\}, \tag{19}$$

where $R_0(T) := \left\{\mathbf{u} \in P_1(T)^2 : \mathbf{u}(x,y,z)=(a+dx,b+dy,c+dz), a,b,c,d \in \mathbb{R}\right\}$.

To approximate displacements in the porous medium, we use the so called "MINI element" in order to achieve stability in the discrete problem (see [1]). We recall definition of the corresponding discrete space by first defining bubble functions. For fixed $T \in T_h$, we denote by $\lambda_1^T,\ldots,\lambda_4^T$ barycentric coordinates in tetrahedron T. Then bubble function α, associated with T, is defined by the product $\alpha = 256\lambda_1^T\lambda_2^T\lambda_3^T\lambda_4^T$. This bubble function is a polynomial of degree four, null on surface of tetrahedron T and taking value one at barycenter of T. The approximating space associated with the MINI element consists of continuous vector valued functions whose components, restricted to each tetrahedron, are sum of a bubble function and a polynomial of degree one, i.e., $\mathbf{u}_i^h(x,y,z)|_T = ax+by+cz+d+e\alpha(x,y,z), a,b,c,d,e \in \mathbb{R}$. The degrees of freedom for functions in this space are the values of the vector field at vertices and barycenters of tetrahedra. Then, for porous displacements, we use the MINI space

$$\mathbf{M}_h(\Omega_{\mathrm{A}}) := \left\{\mathbf{u} \in \mathrm{H}^1(\Omega_{\mathrm{A}})^3 : \mathbf{u}|_T \in (P_1(T) \oplus P^b(T))^3, \forall T \in T_h, T \subset \Omega_{\mathrm{A}}\right\}, \tag{20}$$

where $P^b(T) = \{a\alpha : a \in \mathbb{R}\}$.

To approximate porous medium pressure, continuous piecewise linear finite elements are used. They consist of scalar valued functions which, when restricted to each tetrahedron, are polynomials of the form $p^h(x,y,z)|_T = ax+by+cz+d, a,b,c,d \in \mathbb{R}$. Thus, porous medium pressure is approximated in the finite-dimensional space,

$$\mathbf{L}_h(\Omega_\mathrm{A}) := \left\{ p \in \mathrm{H}^1(\Omega_\mathrm{A}) : p|_T \in P_1(T), \forall T \in T_h, T \subset \Omega_\mathrm{A} \right\}. \tag{21}$$

We recall that the degrees of freedom defining p^h are its values at vertices of tetrahedra. Finally, in order to approximate the interface pressure we use piecewise constant functions on the triangles of the mesh lying on the interface Γ_I. In other words, for interface pressure we use the space

$$\mathbf{C}_h(\Gamma_\mathrm{I}) := \left\{ p \in \mathrm{L}^2(\Gamma_\mathrm{I}) : p|_{\partial T} \in P_0(\partial T), \forall T \in T_h, \partial T \cap \Gamma_\mathrm{I} \neq \emptyset \right\}. \tag{22}$$

The degrees of freedom of this finite element space are the (constant) values on triangles in Γ_I. Consequently, the discrete analogue to $\mathbf{V}$ is $\mathbf{V}_h = \mathbf{R}_h(\Omega_\mathrm{F}) \times \mathbf{M}_h(\Omega_\mathrm{A}) \times \mathbf{L}_h(\Omega_\mathrm{A}) \times \mathbf{C}_h(\Gamma_\mathrm{I})$ while the corresponding to $\mathbf{V}_0$ is

$$\mathbf{V}_{0h} = \left\{ (\mathbf{v}_\mathrm{F}, q_\mathrm{F}, \mathbf{v}_\mathrm{A}, q_\mathrm{A}) \in \mathbf{V}_h : \mathbf{v}_\mathrm{F} \cdot \nu = 0 \text{ on } (\Gamma_\mathrm{D} \cup \Gamma_\mathrm{N}) \cap \partial\Omega_\mathrm{F}, \mathbf{v}_\mathrm{A} = 0 \text{ on } \Gamma_\mathrm{D} \cap \partial\Omega_\mathrm{A} \right\}. \tag{23}$$

With these finite element spaces we can define the approximate problem to (17)-(18) searching the discrete solution $(\mathbf{u}_\mathrm{F}^h, p_\mathrm{F}^h, \mathbf{u}_\mathrm{A}^h, p_\mathrm{A}^h) \in \mathbf{V}_h$.

5. NUMERICAL RESULTS

In order to validate our method, we are going to build a simple example which can be reduced to a one-dimensional problem and then solved exactly. If we assume that every linear operator is a multiple of identity operator, we can find a solution of the form $p_\mathrm{A}(x,y,z) = p_\mathrm{A}(z)$, $\mathbf{u}_\mathrm{A}(x,y,z) = u_\mathrm{A}(z)\mathbf{e}_3$, and rewrite the above three-dimensional problem as an one-dimensional problem where the prime denotes derivative with respect to *z* and we have supposed that $A^H\left[D(\mathbf{u}_\mathrm{A})\right]\mathbf{e}_3 = s u'_\mathrm{A}\mathbf{e}_3$, $A = a\mathbf{I}$, $B^H = b\mathbf{I}$. Let us assume a similar assumption for fluid displacement and interface pressure, i.e., $\mathbf{u}_\mathrm{F}(x,y,z) = u_\mathrm{F}(z)\mathbf{e}_3$ and $p_\mathrm{F}(x,y,z) = p_\mathrm{F}(z)$. We also suppose that $\Omega_\mathrm{F} = (0,b)\times(0,d)\times(-a_\mathrm{F},0)$ and $\Omega_\mathrm{A} = (0,b)\times(0,d)\times(0,a_\mathrm{A})$. We have considered that fluid is air with $\rho_\mathrm{F} = 1.225\,\mathrm{kg/m^3}$ and $c = 343\,\mathrm{m/s}$, whereas properties of the porous material are summarized in $s = 9.18633\times10^{10}\,\mathrm{N/m^2}$, $\phi = 0.95$, a=0.67857, b=-0.05, $\hat{c} = -6.59172\times10^{-6}\,\mathrm{ms^2/kg}$ and $\rho = 1.26163\times10^2\,\mathrm{kg/m^3}$. With respect to dimensions of the enclosure, length and width are

1m while height is 1m for the first layer of free fluid and 1 m for the second layer of porous material whereas the normal displacement on Γ_E is $u_0 = 60$.

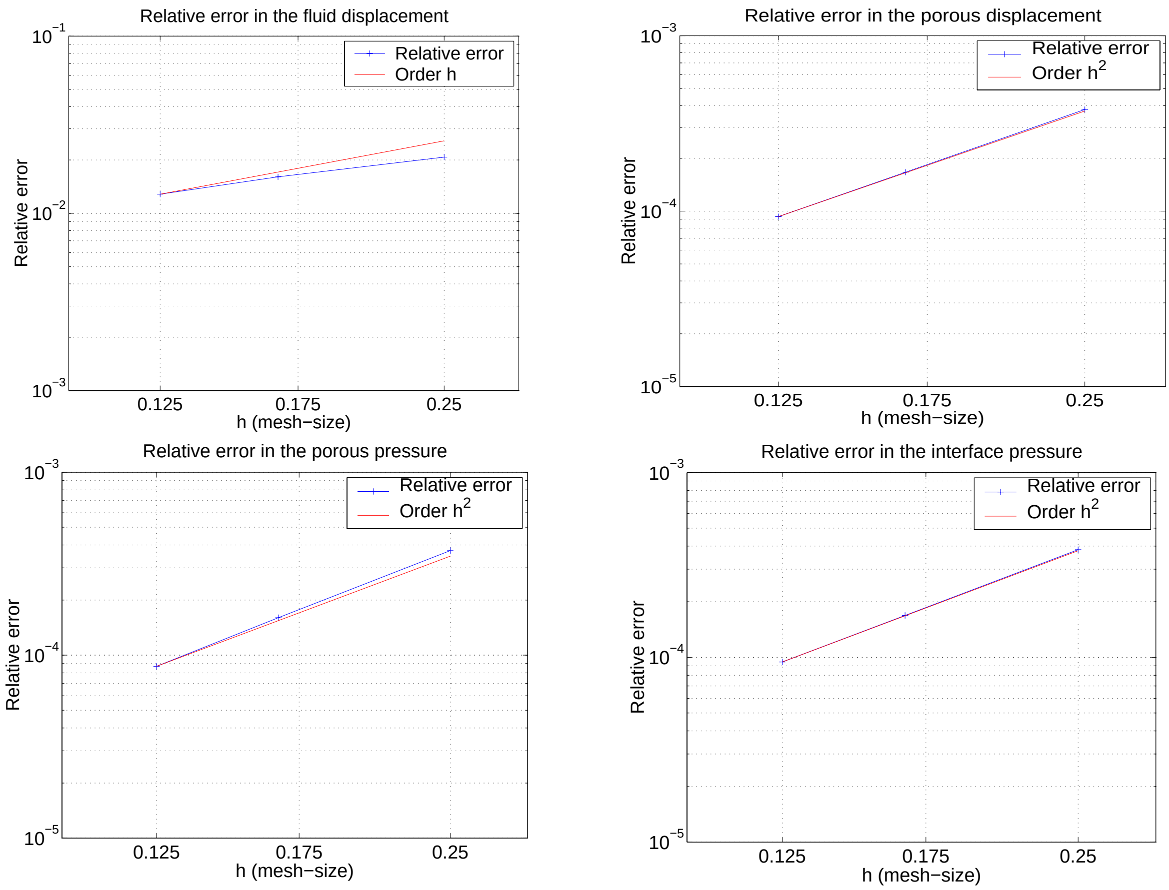

Figure 1 – *Curves of convergence for fluid and porous fields.*

We have computed the solution to this problem with three different uniform meshes, named mesh 1, mesh 2 and mesh 3 of 2548, 8140 and 18788 degrees of freedom, respectively. In Figure 1 we show the L^2-norm of the relative errors for fluid and porous displacement and pressure, against mesh-size, h. As it can be seen, convergence of order 2 is achieved for poroelastic fields and interface pressure. In addition, convergence of order 1 is achieved for fluid displacement.

6. CONCLUSION

We have considered a mathematical model for acoustic propagation in periodic non-dissipative porous media with elastic solid frame and open pore. Parameters of this model have been computed by solving some partial differential equations in the unit cell obtained by homogenization methods. Then a three-dimensional finite element method has been proposed and implemented for numerical solution of the coupling between a fluid and the above porous medium. In order to validate the proposed methodology and to assess convergence properties, the computer code has been used for a test example having analytical solution.

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