

Prediction of the Sound Reduction Index: Application to Monomurs Walls

Thomas Buzzi, Cécile Courné, André Moulinier, Alain Tisseyre

*TISSEYRE & ASSOCIES, www.planete-acoustique.com
16 chemin de Manel, 31400 Toulouse, France
Paris branch, 16 Avenue de Friedland, 75008 Paris, France*

ABSTRACT: The calculation of the sound reduction index in modal analysis is presented in a general way; different approaches are described. These calculations are done in two steps: a vibratory study to determine the transverse displacement of the plate and a study of radiation. The specificity of orthotropic plates is presented. This study led to programming a calculation algorithm. Initial hypotheses are indicated, as well as results obtained for various plates or partitions. Modal analysis calculation results are then compared to the Cremer-Sewell approach results and to laboratory measurements, in particular for “Monomur” walls.

1. INTRODUCTION

The knowledge of the sound reduction index is essential in the acoustical engineering of buildings. Cremer and Sewell provided expressions yielding general tendencies for the infinite plate. Modal analysis takes reflections on the edges into account. Lesueur, Maidanik, Wallace developed calculation techniques.

The purpose of this paper is to present the PIAA Ta software and its applications, e.g. to Monomurs. Monomur walls are made of bricks of complex structure for good thermal properties (Figure 6). The different stages of the development of calculations in the analytic modal approach are presented. Mathematical expressions were programmed and examples of results are presented.

2. THEORETICAL APPROACH

The first work achieved on the prediction of sound reduction index are based on the study of an infinite plate and give general tendencies, like the mass law at low frequencies. The most important work on this subject was Cremer's in the middle of the last century. In the seventies, Sewell gave a correction term in the low frequencies to take account of the dimensions of the plate, but it is still an approximation [1][2].

On the contrary, the modal theory is based on the study of a finite plate and its modes. The dimensions are now taken into account. The principle is that the transmission coefficient of a plate is equal to the sum of the transmission coefficients of each of its modes.

The approach to the sound reduction index is done in two steps. First, the vibration of the plate is studied. The Rayleigh-Ritz method is used to expand the displacement w of the plate



over a basis $\Phi_m(x) \Phi_n(y)$

$$w(x, y, t) = \sum_m \sum_n a_{mn}(\omega) \phi_m(x) \phi_n(y) e^{i\omega t} \quad (1)$$

where a_{mn} is the amplitude of mode m, n . Two bases can generally be used: a sinusoidal or a polynomial basis. In order to calculate this displacement, we need the amplitude of the transversal displacement of the plate

$$\{a_{pq}\} = \{f_{mn}\} [A_{mnpq}]^{-1} \quad (2)$$

where $\{f_{mn}\}$ represents the generalised driving forces and A_{mnpq} the total impedance matrix of the plate.

In the second step, the radiation of the plate is studied. The radiation impedance is a pressure to velocity ratio and represents the effect of the fluid on the structure. In the Piasa Ta software, this impedance is calculated with the formulas of Maidanick [1][2], given for the case of a sinusoidal basis and a light fluid.

Vibration and radiation yield the sound reduction index for a diffuse field [1][2].

$$\tau_d = \frac{16\pi c^2}{S} \sum_m \sum_n \frac{Z_{mnmn}^2}{|A_{mnmn}|^2} \quad (3)$$

3. APPLICATION

The formulation of Cremer, and Sewell at low frequency, was the basis of earlier versions of our program. The study of the various approaches and the selection of hypotheses described below led to programming a modal calculation algorithm, for isotropic as well as orthotropic plates. Compound walls or ceilings can be calculated in the same manner as in the earlier versions, but using the results of the modal calculation of their components.

3.1. Calculation hypotheses of the Sound Reduction Index program.

The expansion of the transversal displacement is done over a sinusoidal basis (Fourier series). For this reason, a comparison is made of plates with simply supported boundary conditions. This leads to a diagonal matrix A where non-null terms are

$$A_{mnpq} = \rho_0 h (\omega_{mn}^2 - \omega^2) + 2j\omega Z_{mnpq} \quad (4)$$

with ω_{mn} the natural pulsation for an isotropic plate:

$$\omega_{mn} = \frac{D}{\rho_0 h} \left(\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right) \quad (5)$$



D is the stiffness of the plate. The fluid in which the plate is immersed is assumed to be light (air).

3.2. Case of orthotropic plates

In the case of orthotropic plates, the natural pulsations are given by

$$\omega_{mn} = \frac{1}{\rho_0 h} \left(D_x \left(\frac{m\pi}{a} \right)^4 + 2(D_1 + 2D_{xy}) \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_y \left(\frac{n\pi}{b} \right)^4 \right) \quad (6)$$

D_x , D_y and D_1 are the bending stiffnesses along the directions of orthotropy, D_{xy} the torsion stiffness. In the case of a parallelepipedic thin plate of thickness h , their expressions are

$$D_1 = \frac{E_x \nu_{yx} h^3}{12(1 - \nu_{xy} \nu_{yx})} \quad D_{xy} = \frac{G_{xy} h^3}{12} \quad D_x = \frac{E_x h^3}{12(1 - \nu_{xy} \nu_{yx})} \quad D_y = \frac{E_y h^3}{12(1 - \nu_{xy} \nu_{yx})} \quad (7)$$

For plates with a geometrical orthotropy, having therefore only one Young's modulus, the theoretical model changes. Indeed, at the beginning of the calculation, the presence of an integral on the volume can be noticed in the expressions of the kinetic and potential energies. The resolution of these integrals for complex geometries such as ribbed steel plates is rather difficult and requires heavy computational means. As our purpose is to develop a software that can be run on any standard computer, and with reasonable calculation times, the plate model was adapted to suit complex geometry panels.

To this end, an equivalent plate method was used, i.e. the calculation of Young's moduli of a thin plate such that its stiffnesses are equal to that of the plate to study. These moduli are a function of the modulus and the geometric characteristics of the original construction element. In that case, the expressions of stiffnesses are determined from the equations of the bending and torsion moments. Several authors handled this problem. Timoshenko [4] gives the stiffness of stiffened plates. For ribbed plates, the formulas by P. Cordonnier [3] were used.

3.3. Numerical implementation

The formulation was developed in standard C language and the programme runs on Windows machines equipped with Pentiums (500 MHz to 2 GHz). The sound reduction index is calculated over the frequency range 50-10,000 Hz, every ninth octave. Third octave bands results are then calculated and displayed. The calculation time is comparatively short (from negligible to a few seconds) and depends on the number of modes chosen for the calculation.

3.4. Inputs for isotropic plates

The software needs the following inputs: length (m), width (m), thickness (cm), density (kg/m³), stiffness (N/m), internal loss factor η (the internal damping loss of the plate), number of modes (the highest order for numbers m and n used in the calculation).

A really important problem for simulations is to determine Young's modulus and the internal loss factor of materials. Material properties data can be found in the literature or on the Internet. Many simulations for each plate were first made with different values. Then,

comparing results with measurements made it possible to determine an adequate value for each material. All these values were collected and make up our bank of materials.

Theoretically, there are an infinite number of modes. In practise, we must limit the computation to a certain row. The modal density determines this row and depends on the upper frequency limit and on material properties and dimensions of the plate.

3.5. Inputs for orthotropic plates

The same inputs as in the case of isotropic plate are necessary, except that stiffness is more difficult to calculate. The first two types of orthotropic plates studied are steel cladding and L-shape steel panel. As explained in 3.2, the formulas by Cordonnier-Cloarec [8] were used in order to calculate the stiffness of these types of plates.

Using the same principle, stiffnesses can be obtained for plates with more complex geometry. Stiffnesses yield equivalent Young's moduli, using the following expression:

$$D_i = \frac{E_i e^3}{12(1-\nu^2)} \quad (8)$$

4. EXAMPLES OF SIMULATIONS

Several comparisons between the Cremer-Sewell and modal approaches were carried out, using our experimental data bank. A representative selection of typical variations is presented here. The three selected orthotropic panels are:

1. Steel cladding, 0.7 mm thick.
2. Hollow brick wall.
3. Alveolar concrete floor.

1. Cladding

This is a corrugated steel plate (Figure 1). Its specific geometry makes it orthotropic. The formulas of Cordonnier-Cloarec [8] were used to calculate them from dimensions and material properties.

The modal calculation of the sound reduction index is close to the measured low frequency behaviour (Figure 2). The Cremer-Sewell formulation overestimates the sound reduction index by 5 dB and provides only a global behaviour. Accidents appear with modal calculations. The 200 Hz drop is due to modes (1,2), (2,2), (3,2) and (4,2), which have the same resonance frequency. At this frequency, the transmission loss is low for each of these modes.

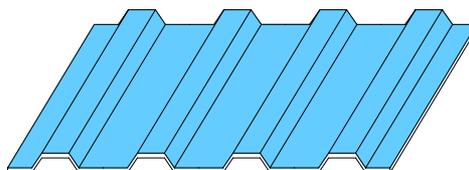


Figure 1- View of the cladding

The saw tooth aspect is due to the prevalence of one of the two bending stiffnesses over the other. This phenomenon implies that several modes of same row m , but of different rows n , have identical resonance frequencies. The accidents due to the various modes thus are no longer distributed over the entire frequency band, but are centred on certain frequencies. This phenomenon can be due to an over-estimated bending stiffness or an under-estimated internal loss factor, which would increase the dips due to each mode. The loss factor is actually a function of the frequency, but too little data obtained about it made it necessary to enter it as a single value.

2. Hollow brick wall

The wall is made of hollow baked clay bricks, 50 mm thick; each brick consists of six lines of horizontal cells; the bricks are mounted with plaster in an alternate way. The study was done in two steps.

First, the lack of influence of the layout and the jointing was demonstrated, using measurements of a solid brick wall, also mounted with plaster in an alternate way. The measured sound reduction index was compared with the calculated index of a wall made up of brick material of same characteristics. The results obtained being similar, it can be concluded that the alternate mounting or the presence of joints are not determining factors for the calculation of the sound reduction index.

Then, on the basis of these results, the hollow brick wall was simulated, by considering that the structure of a brick extends over the entire wall. Dimensions and density are identical to those of the real wall. The internal loss factor corresponds to that of material and is frequency dependant.

The calculation required the use of 160x160 modes. A very good agreement between simulation by modal theory and the results obtained in laboratory was found (Figure 3).

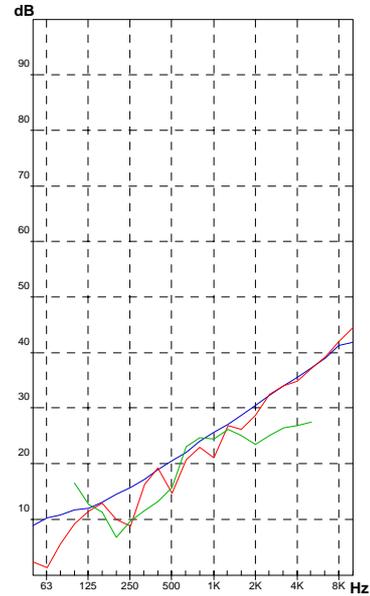


Figure 2 - 7/10th mm cladding,
— modal theory,
— Cremer-Sewell theory,
— measurement

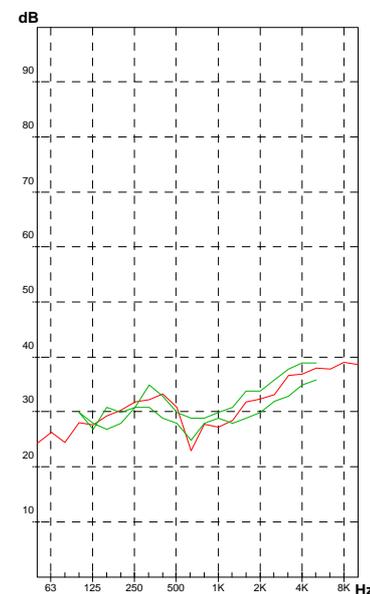


Figure 3 - Hollow brick wall
— modal theory, — measurements

3. Alveolar concrete floor

This plate has a geometry comparable to the hollow brick's (Figure 4). The same formula for calculating the bending stiffnesses was therefore used; to this end, the approximation was made that the cell was rectangular.

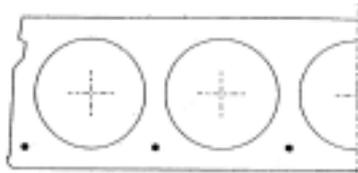


Figure 4 - Alveolar concrete floor

This calculation was made with about 250 modes, hence a calculation time of 3 s. A very good agreement between modal theory and laboratory measurement can again be observed.

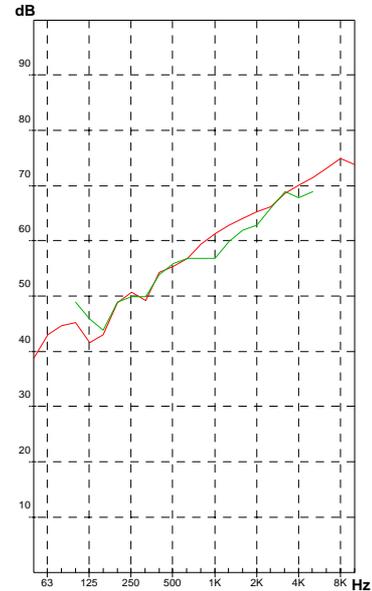


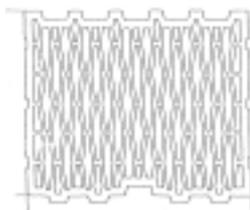
Figure 5 - Alveolar concrete floor ,
 — modal theory, — measurement

5. SOUND REDUCTION INDEX OF MONOMURS

5.1. Monomurs studied

“Monomurs” walls are made of bricks designed for good thermal properties (Figure 6).

Monomur 1
 220 kg/m²
 E = 20 Gpa



Monomur 2
 384 kg/m²
 E = 20 Gpa

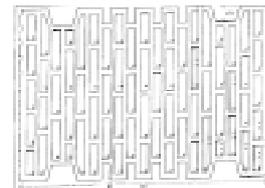


Figure 6 - The two monomurs studied

5.2. Analytical calculations of stiffnesses

The wall's stiffness is its Young's modulus times the moment of inertia. The moment of inertia of the brick can readily be calculated in the direction where the cross-section is constant. In that direction, the stiffness is calculated over the entire wall and can be considered close to reality.

In the second direction, the moment of inertia is more difficult to calculate. An approximation was therefore used: the brick is split into “layers” that can be calculated. The total stiffness is then considered equivalent to the sum of the stiffnesses of each layer, which implies that the layers are not mechanically connected. The resulting stiffness is therefore approximate.

5.3. Approach to the loss factor

A measurement of the internal loss factor was carried out in a laboratory (Figure 7). The loss factor varies strongly with frequency. Using a single value is therefore an inaccurate approach. It is then necessary that when measuring the sound reduction index, the loss factor as a function of frequency be also measured as this information is very important.

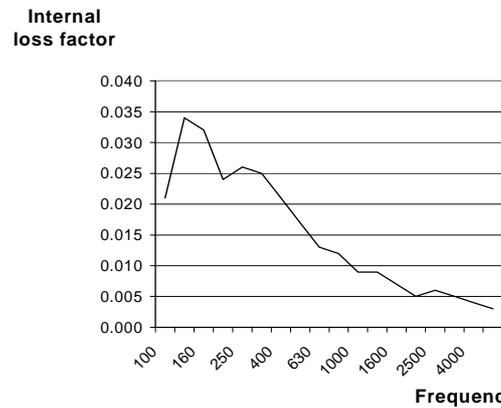


Figure 7 - Loss factor as a function of frequency

5.4. Results

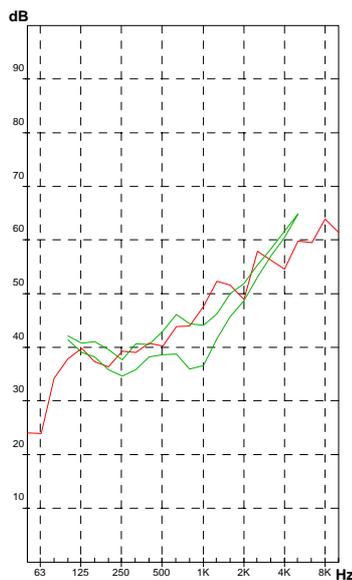


Figure 8 - Calculation for monomur 1
— modal theory, — measurement

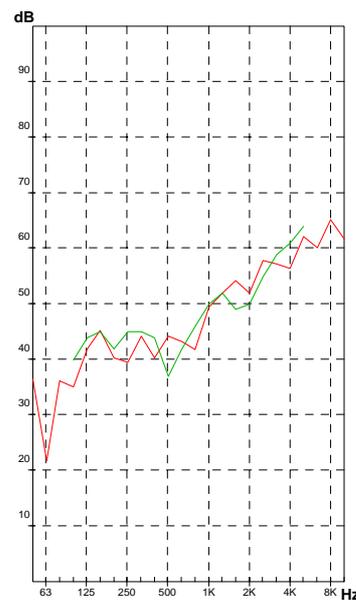


Figure 9 - Calculation for monomur 2
— modal theory, — measurement

The results obtained by this method are generally satisfactory. However, a precise knowledge of input data appears to be essential, particularly stiffnesses. The present method has the drawbacks of being partially approximate and to apply only to profiles that can be split into corrugated, flat, or L-shaped plates, which limits its possibilities.

6. EVOLUTION OF THE SOFTWARE

An algorithm using expansion over a polynomial basis is currently under development. The advantage of a polynomial calculation is to take into account various edge conditions and see its influence on the index. Currently, this algorithm allows calculations only at low and medium frequencies, as can be seen on Figure 10, which shows the case of 12-mm fibreboard. With identical edge conditions, drops at the resonance frequencies of the first modes are not so strong with the polynomial calculation, which is closest to reality. Calculation over the full frequency range, 50 to 10,000 Hz, is currently studied.

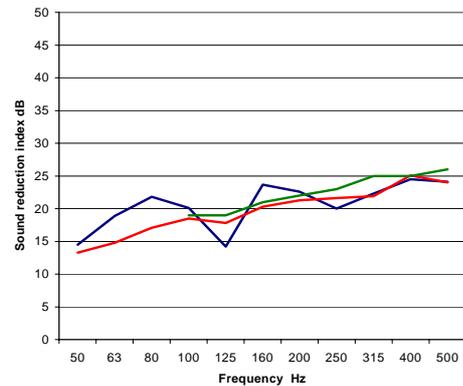


Figure 10 - Sound reduction index of a 12-mm fibreboard
 — polynomial calculation, — sinusoidal calculation — measurement

7. CONCLUSION

A module for the calculation of the sound reduction index was developed on the basis of a modal development, using a sinusoidal expansion (Fourier series). The results obtained were compared to laboratory measurements. Good agreement between experimental results and simulations is observed with the new calculation, especially in the lower frequencies where accidents due to modes are clearly visible. A new calculation module, making it possible to take account of various edge conditions, is currently under development. This should lessen the drops of the sound reduction index at the resonance frequencies of the first modes, as well as at the critical frequency. Finally, the early version of PIAA TA using the Cremer and Sewell formulas is available as freeware. Requests should be made by e-mail via our Internet site.

REFERENCES

1. A. Tisseyre, T. Buzzi, C. Courné, A. Moulinier, "Prediction of the sound reduction index: a modal approach", *Applied Acoustics*, Vol 64, No 8, pp 793-814, 2003
2. A. Tisseyre, T. Buzzi, C. Courné, A. Moulinier, "Prediction of the Sound Reduction Index: Application to the Orthotropic Plates». 147th meeting of the Acoustical Society of America, 24-28, May 2004.
3. P. Cordonnier-Cloarec, *Contribution à l'étude du rayonnement acoustique de parois métalliques nervurées*. Thèse de l'Université Paul Sabatier, Toulouse (1989).
4. Timoshenko, *Théorie des plaques et coques*. (1961)