Use of SEA to Model the Sound Field in Large Acoustic Spaces

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ABSTRACT: The problem of predicting the sound pressure level distribution inside a large acoustic space due to one or more sound sources is examined using a development of Statistical Energy Analysis (SEA). The resulting model may be applied to predict the sound field inside an industrial workroom.
The prediction method is based on a separate treatment of the direct and reverberant sound fields. The direct sound field is dealt with by assuming free-field propagation, with the application of corrections such as barrier theory as required. The reverberant field is analysed using the SEA framework with the power input corrected for energy lost at the first reflection. The total sound pressure level at any receiver point is given as the sum of the direct and reverberant fields.
Measurements made in a workroom using experimental SEA methods, with the acoustic space considered as being divided into three distinct subsystems, demonstrated that the presence of a direct field can also significantly affect the experimental SEA procedure.
Comparing measured and theoretically predicted SEA parameters, it is found that the coupling loss factors are in good agreement, but the damping loss factors estimated from published absorption coefficients for the surfaces and fittings agreed less well with the measured data.

1. INTRODUCTION

This paper presents a method of modelling the sound field in large acoustic cavities using Statistical Energy Analysis (*SEA*) techniques. The immediate aim is a method that could be used to determine the sound distribution inside factories, but the resultant model is applicable to many other problems where there is a balance between direct and reverberant fields, for example the sound field inside motor cars.
A number of methods for predicting the noise distribution inside factories have previously been developed; generally there is a trade off between the complexity of the model and data input and the accuracy of the final predictions.
Simplified models derived from experimental results have been applied. Hodgson [1] compares different empirical models with the sound-propagation curves measured in 30

different workrooms. More recently Heerema [2] has developed a new model where the fittings density is included as a new parameter.
Scale models provide reasonably accurate results for the predictions of sound propagation inside big acoustic spaces with no reduced complexity of measurements [3]. However, the limitations of cost and time needed to build a model prevail.

Comentário: Hodgson, M. and Orlowski, R. J. (1987). "Acoustic scale modelling of factories. Part II: 1:50 scale model investigations of factory sound fields," J. Sound Vib. **113** (2), 257-271.
Hodgson, M. and Orlowski, R. J. (1988). "Acoustic scale modelling of factories. Part III: the potential of scale models for predicting the effect of noise control measures," J. Sound Vib. **121** (3), 525-545.

Computer models based on mathematical methods developed according the principles of the Image Source Method [4], or Ray Acoustics techniques [5] or even radiosity method [6] have proven to be more accurate – [7] makes a comparative study between the Complete Image Source Method (*CISM*) and two other empirical methods: Hodgson and Heerema models where it is shown that *CISM* generally provides better results that the other two models tested. Ray tracing technique has the advantage of easily representing enclosed spaces of arbitrary shape, facilitating the modelling of internal walls, barriers or pitched roofs. Particularly interesting are the works developed by Hodgson, [8] and [9], applying this technique to some practical situations.
The aim of this study is to investigate ways of determining the sound distribution in acoustic cavities using an energetic approach, applying the techniques of *Statistical Energy Analysis* (*SEA*). The prediction method is based on a separate treatment of the direct and reverberant sound fields with the total sound pressure level at any receiver point being given as the sum of the direct and reverberant fields.
It is expected that the resulting model could be used both for acoustic optimisation during the design stage and also for evaluating noise control measures in existing rooms. An additional advantage of the SEA approach is that, once the interior sound field of the workshop modelled, the extension of the model to sound transmission through the walls of the building or to other locations inside the building is expected to be straightforward as *SEA* models have already proved to be suitable for this particular range of problems.

2. SEPARATION OF DIRECT AND REVERBERANT FIELDS

In an enclosed space at any given point the sound field is the result of the addiction of the direct field radiated by the source and the reverberant field, i.e. in energy terms, the total sound energy measured is the sum of the sound energy due to direct and reverberant field.
Thus for a correct study of the sound field in enclosed spaces both direct and reverberant components have to be modelled. As the SEA framework is applied in reverberant spaces it was thought to separate both components in the analysis. The direct component is taken into account using the wide spread theory of point, line or area sources propagation in free field while the reverberant part is dealt into the SEA model.
It is important to note that all the reflections contributions are considered in the SEA model and that the power input in the SEA model is also corrected to exclude the direct contributions.

Assuming incoherent direct and reverberant fields, the spatial averaged total mean square pressure is given by:

$$\left\langle p_{tot}{}^{2} \right\rangle = \left\langle p_{rev}{}^{2} \right\rangle + \left\langle p_{dir}{}^{2} \right\rangle \qquad (2.1)$$

2.1 Direct Field

The direct field contributions at any point inside the enclosed space are determined either by:

i. assuming spherical propagation away from the source and inside the workroom if no obstacles/barriers are located between the source and receiver;
ii. barriers theory otherwise.

While in the first case, when there is no obstacle between source and receiver point free field sound propagation might be applied assuming spherical propagation of point source. In cases where the dimensions or characteristics of the source can not be included as a point source, line or even area sources theory might be applied. In the latter case, assuming no contribution from side reflections, it is simply a question of determining the insertion loss of the barrier or obstacles.

The direct field contribution is mainly a function of the distance between source and receiver and the sound source directivity. For an omni-directional point source, in free field conditions (direct component), the mean square pressure is given by:

$$\left\langle p_{dir}{}^{2} \right\rangle = \frac{W_M \rho c}{4\pi r^2} \qquad (2.2)$$

Where W_M is the radiated sound power of a monopole.

2.2 The SEA power balance equation as a model of the reverberant sound field

Statistical Energy Analysis *(SEA)* is a framework based on energy conservation principles. Its starting point is the division of a built up system into a number of smaller approximately homogeneous subsystems assuming diffuse flow of energy between subsystems [10], and then subsystem dissipated energy lost by subsystem internal damping effects, and energy flow as a function of coupling between subsystems can be determined. From conservation of energy this reverberant power input is equal to the sum of the locally dissipated power and the power transmitted to the reverberant field of other subsystems via the coupling paths:

$$P_{rev,i} = P_{diss,i} + \sum P_{ij} \qquad (2.3)$$

where $P_{diss,i}$ is the power dissipated in reverberant field of subsystem i , given by:

$$P_{diss,i} = \omega \eta_i E_i \quad [\mathrm{W}] \qquad (2.4)$$

and P_{ij} is the power flow from the reverberant field of subsystem i to that of subsystem j, assuming that the coupling power flow is proportional to the difference in modal energies.

$$P_{ij} = \omega\eta_{ij}E_i - \omega\eta_{ji}E_j \quad [W] \tag{2.5}$$

Where by reciprocity:

$$n_i\eta_{ij} = n_j\eta_{ji} \tag{2.6}$$

Combining equations 2.3 – 2.5 give the power balance equation for the *i'th* subsystem:

$$P_{rev,i} = \left(\omega\eta_i + \sum\omega\eta_{ij}\right)E_i - \sum\omega\eta_{ji}E_j \tag{2.7}$$

Thus for the whole system,

$$\begin{Bmatrix} P_{rev,1} \\ P_{rev,2} \\ \ldots \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} + \ldots + \eta_{1n} & -\eta_{21} & \ldots \\ -\eta_{12} & \eta_2 + \eta_{21} + \ldots + \eta_{2n} & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ \ldots \end{Bmatrix} \tag{2.8}$$

which may be written:

$$\hat{P}_{rev} = \omega\overline{L}\,\hat{E} \tag{2.9}$$

The reverberant field input power $P_{rev,i}$, from the external sources in subsystem i, is given by

$$P_{rev,i} = P_{in,i}\left(1 - \overline{\alpha_i}\right)$$

The factor $\left(1 - \overline{\alpha_i}\right)$, where $\overline{\alpha_i}$ is the mean absorption coefficient of the surfaces, accounts for the loss of the power at the first reflection. As it has been previously stated no direct contribution is here included. At every subsystem (excited or not) only the reverberant component is taken into account. Thus the corrected form of the power balance equation is as follows,

$$P_{in,i}\left(1 - \overline{\alpha_i}\right) = \left(\omega\eta_i + \sum\omega\eta_{ij}\right)E_{rev,i} - \sum\omega\eta_{ji}E_{rev,j} \quad [\mathrm{W}] \tag{2.10}$$

The mean square pressure of the reverberant field contribution in subsystem i ($p_{rev,i}^{2}$) is given by:

$$\left\langle p_{rev,i}^{\;2} \right\rangle = \frac{E_{rev,i}}{\rho c^2}V_i \tag{2.11}$$

3. SEA PARAMETERS FOR THE REVERBERANT FIELD MODEL

3.1 Loss Factor Matrix

To define the *loss factor matrix* (*L*) a wave approach of the *SEA* technique is used to determine the coupling and damping loss factors.

3.2 Coupling Loss Factor

The coupling loss factor between subsystem *i* and *j*, $\eta_{i,j}$, controls the power flow from subsystem *i* to subsystem *j*. It is a function of the diffuse field wave power transmission coefficient τ_{ij}. It is assumed that the response of the receiving subsystem does not influence the transmission coefficient. τ_{ij} is an average value over all the angles of incidence assuming diffuse incident field. The coupling loss factor is defined as follows,

$$\eta_{ij} = \frac{\tau_{ij} c_i A}{4\omega V_i} \tag{3.1}$$

The transmission coefficient ($\tau_{i,j}$) is determined by calculating an area weighted spatial average of the transmission coefficients on each portion of the boundary. For this, it was assumed that a surface area corresponding to the aperture area would have a transmission coefficient (τ_A) of unity while the fittings or any other obstacle would have a transmission coefficient (τ_B) equal to zero. Thus,

$$\tau_{ij} = \frac{\tau_A A_A + \tau_B A_B}{A_A + A_B} \tag{3.2}$$

Shorter [11] shows that, for the case of a simple partition, if the incident power is assumed to be associated with a diffuse field in the definition of the coupling loss factor the SEA reciprocity (equation 2.6) is only guaranteed if the modal density is a function of the volume (and not of the area or the perimeter). The modal density of subsystem i (Ni) is then determined only in terms of its volume.

Comentário: Shorter, P. (2001). “Rigid walled cavities and SEA,” Vibro-Acoustic Sciences Technical Memorandum, Document number PJS-0177-01.Page 8 Part III 2nd sentence from the 1st paragraph.

$$N_i = \frac{w^2}{2c_i^3 \pi^2} V_i \tag{3.3}$$

From the modal densities and *CLF's* of each pair of subsystem it is then possible to check that reciprocity holds equation (2.6).

3.3 Damping Loss Factor

The *DLF* defines the amount of energy dissipated within a given subsystem due to damping effects. The *DLF* of each subsystem is based on the average absorption coefficient of each subsystem. The effect of fittings in the mean average absorption coefficient of the workroom can also be included according to published works. Since general industrial rooms have large volumes, air absorption effects should also be added. Even for smaller subsystem its volume may still play a significantly role at high frequencies. Air absorption can be added as follows:

$$\eta_i = \frac{c_i A_i}{4\omega V_i} \tag{3.4}$$

Where,

$$A_i = \left(\sum S_k \alpha_k\right) + 9.21 \times 10^{-4} m V_i \tag{3.5}$$

The air absorption term is dependent upon temperature and relative humidity. In current literature it can be found tables relating both quantities.
It is clear from equations 2.10, 3.4 and 3.5 that the accuracy of the *SEA* model is likely to be strongly dependent on the correct estimation of the average absorption coefficient as it plays a double role: correcting the power input and calculating the *DLF*.

4. APPLICATION TO WORKSHOP PREDICTIONS

4.1 Validation Measurements

A set of measurements inside a typical Workshop was made to validate the previous assumptions against actual data, measured *in situ.* The place chosen was the Mechanical Workshops of Southampton University which is a representative medium size industrial room almost parallelepiped (12 metres wide x 36 metre long x 5 metres height with several metal working machines and benches). The floor of the building was concrete, the walls were unpainted block-work and its ceiling was a typical steel deck construction of corrugated metal supported by metal truss work. On the ceiling there was also the lighting system and some hanging cables and pipes.
The standard procedure for determining *SEA* parameters (*CLF* and *DLF*) on the given acoustic cavity was the *Power Injection Method* (*PIM*). The workroom was divided into three different subsystems and twenty measurement points for each source location were considered; eight points for subsystem 1 and six for each subsystem 2 and 3. The energy values, converted to level, for each of the measuring conditions were:

$$\overline{E} = \begin{bmatrix} E_{1(1)} & E_{1(2)} & E_{1(3)} \\ E_{2(1)} & E_{2(2)} & E_{2(3)} \\ E_{3(1)} & E_{3(2)} & E_{2(3)} \end{bmatrix} = \begin{bmatrix} 74.5 & 68.2 & 65.1 \\ 66.6 & 75.0 & 72.2 \\ 63.8 & 72.1 & 75.5 \end{bmatrix} \left(\text{dB re.}10^{-5}\ \text{Pa}\right) \tag{4.1}$$

$$E_{i(j)} - \textit{total energy in subsystem i when subsystem j is excited}.$$

Once the energy values were corrected to account only for the reverberation contribution and the mean absorption coefficient at each subsystem determined, predicted and measured SEA parameters could be compared.

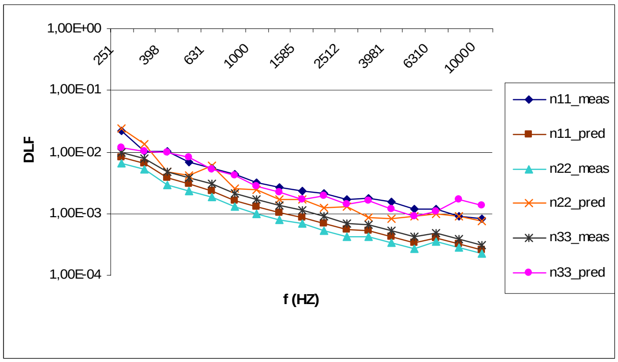

Figure 1 – *Comparison between measured and predicted values for the DLF.*

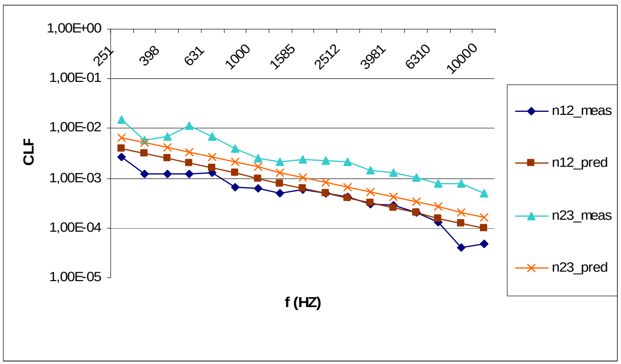

Figure 2 – *Comparison between measured and predicted values for the CLF.*

5. CONCLUSIONS

A new approach using *SEA* modelling techniques has been used to solve the problem of sound propagation inside industrial workrooms. The model separates the direct and reverberant sound field contributions and only the latter are analysed by the *SEA* framework using a correction to the traditional *SEA* equation.
Measurements in a typical workroom were made so that some of the parameters of the model could be measured and then tested against theoretical predictions. The *CLF* predictions

proved to be accurate, but *DLF* estimates based on published values for typical absorption coefficients of surfaces, and fittings were less accurate.
The model proved to be quite sensitive to the assumed values for the absorption coefficient due to its double influence via the correction of the power input and also on the *DLF* estimation.

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