THEORETICAL AND PRACTICAL REVIEW OF REVERBERATION FORMULAE FOR ROOMS WITH NON HOMOGENYC ABSORPTION DISTRIBUTION

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ABSTRACT

This paper shows the results of a preliminary part of a study about the behaviour of reverberation time equations. We have focused our work on the non-uniformly absorption distribution case, because this situation is not studied by classical formulations.
The first part of the paper includes a theoretical introduction about equations for predicting reverberation time from Sabine's formula to some of the newer ones.
The second part shows some results from measurements, simulations and analytical calculations for a real case.

INTRODUCCION

1- Classic Theories:

The first theory developed was Sabine's formula (1902) and is the base of the so-called classic theories. Also Eyring and Norris (1930-1932) and Millington and Sette (1932 – 1933) or Kuttruff formulae are included in Sabine's theory assumption.

Reverberation time defined by Sabine is inversely proportional to the averaged absorption coefficient and is calculated as the arithmetic average of each absorption coefficient for all surfaces in the room. This equation is valid for live rooms and diffuse acoustic field.

$$\boxed{RT60_{Sabine} = \frac{0.163 \cdot V}{S \cdot \bar{a}_{SAB.}}} \qquad \boxed{\bar{a}_{Sab.} = \frac{1}{S}\sum_{i=1}^{n} a_i \cdot S_i}$$

Eyring and Norris used the same prediction formula introducing a logaritmic dependence to Sabine's reflection coefficient. The equation considers a binominal distribution for the number of reflections (Sabine used a Poisson distribution).

$$\boxed{RT\,60_{Eyring} = \frac{0.163 \cdot V}{S \cdot \bar{a}_{E\&N.}} = \frac{0.163 \cdot V}{-S \cdot \ln\left(1 - \bar{a}_{Sab.}\right)}}$$

Results obtained with this equation are similar to Sabine's results (they are equal for coefficient values smaller than 0.2). Eyring's absorption coefficient is always less than 1.

Millington and Sette defined a new averaged absorption coefficient as is shown in the formula:

$$\boxed{TR60_{M\&S} = \frac{0.163V}{S\cdot a_{M\&S.}} = \frac{0.163V}{-\sum_{i=1}^{n} S_i \cdot \ln(1-a_i)}}$$

Kuttruff proposed a statistical distribution of sound considering random gaussian variables following Rayleigh probability. Variance of the probability is defined as a function of the mean free path: $g = \frac{\overline{l^2} - \bar{l}^2}{\bar{l}^2}$; Absorption defined by Kuttruff is like Eyring and Norris, but corrected by the variance factor.

$$\boxed{RT60_{Kuttruff} = \frac{0.163\cdot V}{S\cdot \hat{a}_{Kutt}}} \quad \boxed{\hat{a}_{Kutt} = -\ln(1-\hat{a})\cdot\left(1+\frac{g^2}{2}\cdot\ln(1-\hat{a})\right)}$$

All these theories assume that the acoustic field in the room is diffuse.

<u>2- Directional theories:</u>

We have chosen three equations to evaluate the behaviour of the directional theories. Fitzroy (1959), Arau (1988) and Neubauer (2000).

Fitzroy was the first in defining the reverberation time in three directions. He was the first in considering the placenebt of the material in the room, not only the occupied surface.

For this reason Fitzroy's equation can be used to predict reverberation time in non-uniform sound fields.
This formula uses the arithmetic average of the reverberation periods in each direction. The absorption coefficients used are Eyring's coefficients.

$$\boxed{RT60_{Fitzroy} = \left(\frac{S_x}{S}\right)\cdot T_x + \left(\frac{S_y}{S}\right)\cdot T_y + \left(\frac{S_z}{S}\right)\cdot T_z} \quad \boxed{T_x = \frac{0.163\cdot V}{S\cdot a_{Fitz,x}}} \boxed{T_y = \frac{0.163\cdot V}{S\cdot a_{Fitz,y}}} \boxed{T_z = \frac{0.163V}{S\cdot a_{Fitz,z}}}$$

$$\hat{a}_{Fitz,x} = -\ln(1-\hat{a}_x) \quad \hat{a}_{Fitz,y} = -\ln(1-\hat{a}_y) \quad \hat{a}_{Fitz,z} = -\ln(1-\hat{a}_z)$$

X, y, z, are the three directions corresponding to ceiling and floor, lateral walls and back and front walls. Accordingly, an averaged absorption coefficient in each direction is calculated.

Arau uses the geometric average of the three reverberation periods in each direction.

$$\boxed{RT60_{Arau} = \left[\frac{0.163\cdot V}{S\cdot a_{Arau,x}}\right]^{\frac{S_x}{S}} \cdot \left[\frac{0.163\cdot V}{S\cdot a_{Arau,y}}\right]^{\frac{S_y}{S}} \cdot \left[\frac{0.163\cdot V}{S\cdot a_{Arau,z}}\right]^{\frac{S_z}{S}}}$$

$$a_{Arau,x} = -\ln(1-a_x) \quad a_{Arau,y} = -\ln(1-a_y) \quad \hat{a}_{Arau,z} = -\ln(1-\hat{a}_z)$$

Arau also defined three energy decays (early, middle, late). The equation presented above is for the middle time decay, to obtain early and late decays he calculates a dispersion factor of normal-logaritmic distribution (for rectangular rooms).

The last formula evaluated in our work is Neubauer's equation. He applies a correction in Fitzroy's formula finding an expression in two directions: side and back and front walls and floor and ceiling surfaces. He gives special importance in this last direction because contains the audience, wich is the largest absorption area in the room.

$$\boxed{RT60_{Neubauer} = \left(\frac{0.32 \cdot V}{S^2}\right) \cdot \left(\frac{h \cdot (l+w)}{\bar{\alpha}_{WW}} + \frac{l \cdot w}{\bar{\alpha}_{CF}}\right)}$$

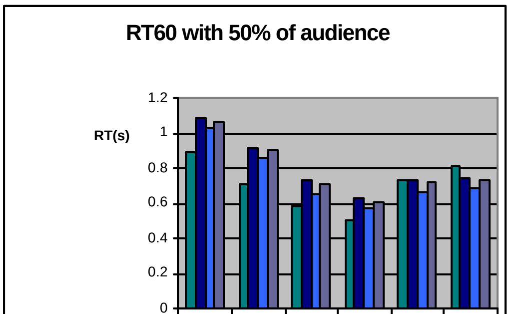
$$\alpha_{WW} = -\ln(1-\alpha_{Sab}) + \left[\frac{(1-\bar{\alpha}_{WW,Sab}) \cdot (\bar{\alpha}_{Sab.} - \bar{\alpha}_{WW,Sab}) \cdot S_{WW}{}^2}{(1-\bar{\alpha}_{Sab.})^2 \cdot S^2}\right]$$

$$\bar{\alpha}_{CF} = -\ln(1-\bar{\alpha}_{Sab}) + \left[\frac{(1-\alpha_{CF,Sab.}) \cdot (\alpha_{Sab.} - \alpha_{CF,Sab.}) \cdot S_{CF}{}^2}{(1-\alpha_{Sab.})^2 \cdot S^2}\right]$$

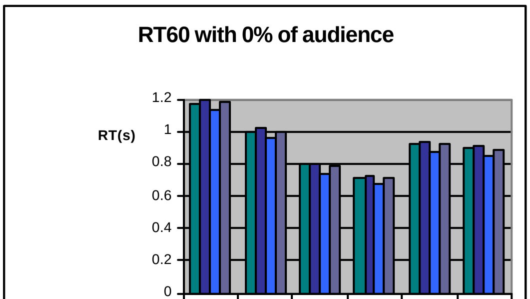
COMPARISON OF ANALITIC AND MEASURED RESULTS

For experimental results we made measurements in a classroom of our university. We measured (in different points) the reverberation time in three conditions: empty class, with 50% of audience and with 100% of audience.
For analytic calculations and for simulations, we used absorption coefficients of audience seated in wooden chairs wich we measured in the reverberation chamber of our laboratory.

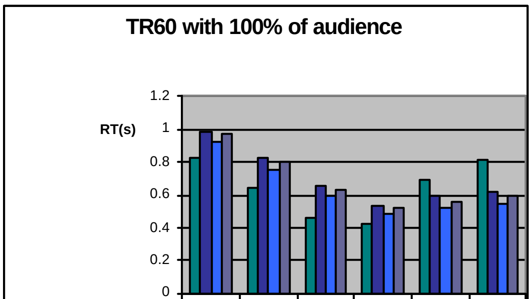
The next figures show the results output by the simulator software compared to the measured reverberation time.



	125	250	500	1000	2000	4000
Experimental	0.89	0.71	0.58	0.5	0.735	0.82
Sabine	1.09	0.92	0.73	0.63	0.74	0.75
Eyring	1.03	0.86	0.66	0.57	0.67	0.69
Kuttruff	1.07	0.9	0.71	0.61	0.72	0.73



	125	250	500	1000	2000	4000
Experimental	1.17	1	0.8	0.72	0.92	0.9
Sabine	1.2	1.02	0.8	0.73	0.94	0.91
Eyring	1.13	0.96	0.74	0.67	0.87	0.85
Kuttruff	1.18	1	0.78	0.71	0.92	0.89

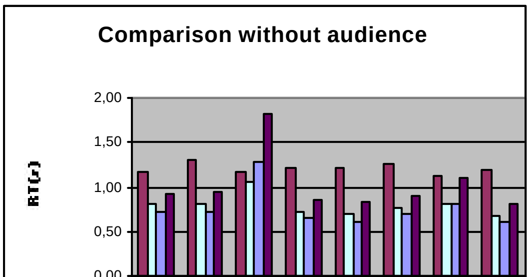


	125	250	500	1000	2000	4000
Experimental	0.832	0.64	0.464	0.428	0.692	0.816
Sabine	0.99	0.83	0.66	0.54	0.59	0.62
Eyring	0.92	0.76	0.59	0.48	0.52	0.55
Kuttruff	0.97	0.8	0.63	0.52	0.56	0.6

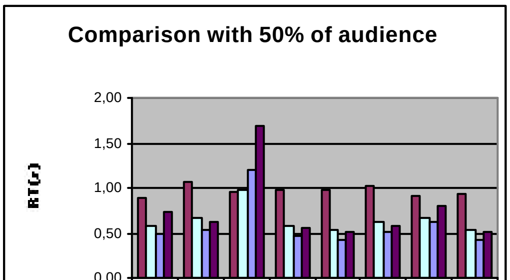
We can see how classic theories fail as the audience in the room is increased. This fact is due to the non-uniform sound field created in the room. With 50% of audience discrepancies are found in low frequencies. On the other hand, when the classroom is full, of people discrepancies or deviations appear at low and high frequency bands. These deviations are bigger than the measurement results, for low frequencies, and smaller for high frequencies.

We have compared the results using Sabine's, Eyring's and Kuttruff's equations because they are the reverberation time formulas offered by our acoustic simulation software.

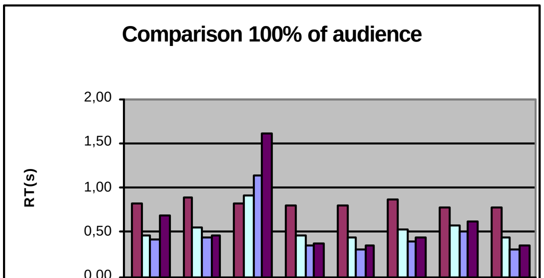
We have made another comparison using analytical data obtained with all the reverberation time equations explained in the theoretical introduction part. We show three graphics to see all the results of the 7 equations, in 3 octave bands for the 3 absorption situations



	Exp.	Sab.	Fitz.	Eyr.	Mill.	Kutt.	Arau	Neu.
125Hz	1,17	1,30	1,16	1,22	1,22	1,26	1,13	1,19
500Hz	0,80	0,81	1,05	0,73	0,70	0,77	0,80	0,68
1000Hz	0,72	0,73	1,28	0,64	0,61	0,69	0,80	0,60
2000Hz	0,92	0,94	1,83	0,86	0,83	0,90	1,10	0,81



	Exp.	Sab.	Fitz.	Eyr.	Mill.	Kutt.	Arau	Neu.
125Hz	0,89	1,06	0,95	0,98	0,97	1,02	0,92	0,94
500Hz	0,58	0,66	0,97	0,57	0,54	0,62	0,67	0,53
1000Hz	0,50	0,54	1,20	0,46	0,42	0,50	0,62	0,42
2000Hz	0,74	0,63	1,69	0,55	0,50	0,59	0,79	0,50



	Exp.	Sab.	Fitz.	Eyr.	Mill.	Kutt.	Arau	Neu.
125Hz	0,83	0,89	0,83	0,81	0,80	0,88	0,78	0,77
500Hz	0,46	0,55	0,92	0,47	0,44	0,54	0,58	0,43
1000Hz	0,43	0,43	1,15	0,35	0,30	0,39	0,50	0,31
2000Hz	0,69	0,47	1,62	0,39	0,34	0,44	0,61	0,35

The first situation, without audience, shows how, approximately, all theories fit the values, except Fitzroy's equation. For this case of quite uniform sound field, classic theories, give good results. Sabine's equation results are very close to the real ones, not only in values but also in tonal curve shape.

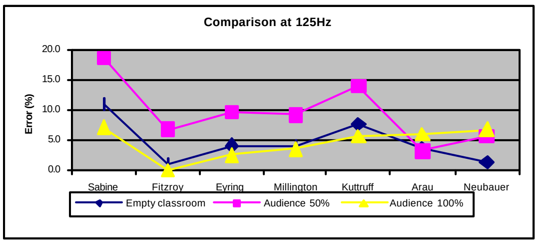
In the second graphic, with 50% of audience, we can see how classical theories differ from real values. This behaviour is present at low and high frequencies. The theory that fits best the measured tonal curve shape is Arau's theory, giving, also, good reverberation time values.

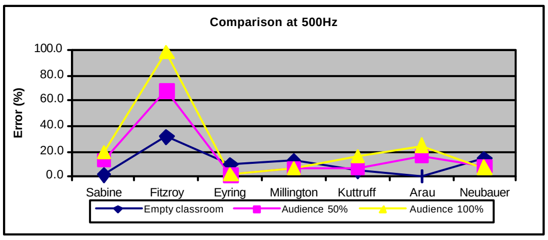
The tonal curve shape becomes very important, because the calculated values depend on the absorption coefficients assigned to the audience, so the value is relative to them. This is an initial error source we can't forget.

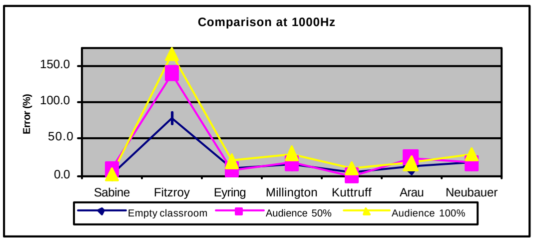
In the third situation, with 100% of audience, Arau's formula gives good results and classical formulation differs from real values, specially as frequency rises.

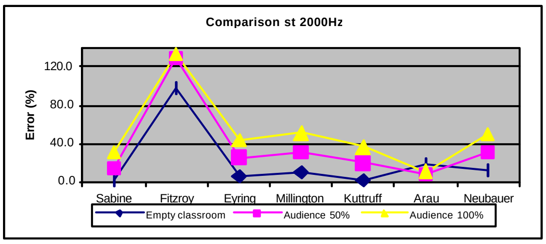
Neubauer's theory reaches its goal improving Fitzroy's equation behaviour, but the tonal shape obtained by this equation is similar to the classic theories shape, an a little far from real results.

Now we present 3 graphics showing the relative error of each theory. The results are presented at 4 octave bands in the three absorption situations.









CONCLUSIONS

First of all, we must remark that validity of this work can only be assured for the studied particular case. We have to analyse and study a lot of real cases in order to get some generalised conclusions. At this moment, we can only talk about our short experience.

Directional theories have give, in most cases, given good results for non-uniform field situations. Our work has pointed Arau's formula as one of the best, for the obtained values and de tonal shape prediction.

This work has helped us to point out the importance of the formulation used in simulation software and analytic calculations. The equations used in acoustical design are as important as the absorption values required in the formulae. The absorption coefficient values are obtained from measurements in a test chamber using non-appropriate equations, because ISO specifications for absorption coefficients measurements in reverberation chamber use Sabine's formula in non-uniform sound field situation. This fact introduces an error in predicting reverberation time independent of the chosen theory because initial values may be wrong.

In conclusion, we can only say that we have a lot of work to do, we have to analyse a lot of real situations in order to get generalised conclusions. Maybe we could find different equations for different situations and, in the future, a new formula for predicting reverberation time or measuring absorption coefficients.

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