ACCELEROMETERS WITH BE NDING ELEMENTS

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ABSTRACT

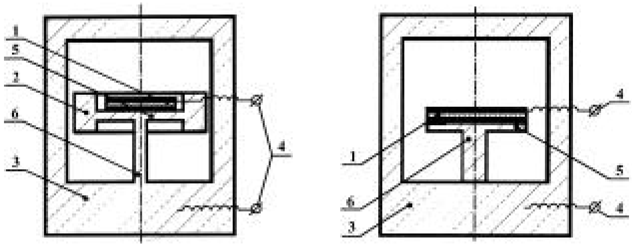
Piezo-accelerometers are widely used for measurements of elastic wave parameters. In this paper the basic requirements and principles of designing of high-sensitive piezo-accelerometers for acoustic diagnostics having bending bimorph elements and central cylindrical support are considered.

INTRODUCTION

Piezo-accelerometers are the most widely used for measurements of elastic wave parameters. In this paper the basic requirements and principles of designing of high-sensitive piezo-accelerometers for acoustic diagnostics having bending bimorph elements and central cylindrical support are considered. Such a sensors have many advantages by their parameters over accelerometers using other forms of piezo-element deformation (compression - tension, shift). The development of new methods of seismoacoustic diagnostics of soil and nondestructive testing of materials requires precise experimental information on parameters of elastic (acoustic) waves in a medium in wide amplitude and frequency range: amplitude range from (10^{-5} – 10^{-3}) g to several g, and frequency range from 0.1 Hz up to 100-1000 Hz. To satisfy these requirements it is necessary to use high-sensitive accelerometers with low dispersions in amplitude and phase characteristics, as well as with high degree of stability of their parameters.

DESIGN OF PIEZOELECTRIC HIGH-SENSITIVE ACCELEROMETERS

A piezoelectric sensor is a mechanic-electrical transducer intended for transformation of mechanical energy of oscillations of testing object surface (or liquid medium particles) into the energy of electrical signal for further processing. This definition differs from widely used explanation of the term “sensor of physical values” in technical literature [1, 2] since includes parts, without which the transformation function losses its completeness and definiteness, for example, preamplifier providing matching of output electrical impedance with input impedance of amplifier, filters, etc. Below we will use the term “sensor” as a piezoelectric transducer with a cable and input circuits of amplifying-transforming equipment (R_n, C_n). Figure 1 shows the structure schemes of high-sensitive piezoaccelerometers AP23 and AP36 using bending deformations of their sensitive bimorph elements.



a **b**

Figure 1. Principal design scheme of sensors AP23 (**a**) and AP36 (**b**). 1 - piezoelement; 2 - inertial element; 3 **-** casing; 4 **-** electrical outputs; 5 - elastic membrane; 6 - supporting rod.

Sensor is mounted on testing object with a glue, thus the bottom surface of casing 3 plays a role of a perceiving link. The bottom part of casing 3 and supporting rod 6 are a link of transfer of input effect, and the piezoelement 2 together with the elastic membrane 5 is a sensing element. Exact reproduction of measured parameter in time $U(t)$ depends on the performance of sensing element and its mode of operation. At the same time both casing 3 and supporting rod 6 can introduce essential changes in transmitted mechanical signal. Piezoaccelerometers with bending deformations are characterized by high sensitivity at rather small dimensions and weight, by minimum sensitivity to strain and to transverse oscillations. From the other hand the use of membrane makes sensor beeing insufficiently impact-resistant and having rather low resonance frequency. One of schemes improving these characteristics is shown in Fig. 1b. It is based on bimorph sensing element (SE) of “mushroom” shape without inertial elements and is used for different applications [3 – 5] (high-sensitive accelerometers, resonant-type knock sensors for automobile engines, etc.), and where a wide dynamic range is required. This scheme is the most preferable in comparison with analogs containing inertial elements fastened along contour of the bimorph plate. Sensors of such type provide wide dynamic (up to 120 dB) and frequency ranges (up to several kHz) of measurements. Values of effecting mechanical stresses, forces or moments at corresponding contour of plate should be specified, when calculating any elements as a thin plate subjected to bending strains. For small deflections the three-dimensional problem for plate can be reduced to the two-dimensional one basing on the hypothesis of flat stressed state. Taking into account assumptions for thin piezoceramic plate ($s_{zz} \approx 0$, $S_{rr} = -z \cdot \frac{\partial^2 u_z}{\partial r^2}$, $S_{qq} = -\frac{z}{r} \cdot \frac{\partial u_z}{\partial r}$) the equations of mechanical stresses and electric induction of piezoceramic material become as follows [3 – 5]:

$$s_{rr} = \frac{s_{11}^E}{\left(s_{11}^E\right)^2 - \left(s_{12}^E\right)^2} \cdot \left(\frac{\partial^2 u_z}{\partial r^2} - \frac{s_{12}^E}{s_{11}^E \cdot r} \cdot \frac{\partial u_z}{\partial r} \right) - \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot E_z \ ,$$

$$s_{qq} = \frac{s_{11}^E}{\left(s_{11}^E\right)^2 - \left(s_{12}^E\right)^2} \cdot \left(\frac{1}{r} \cdot \frac{\partial u_z}{\partial r} - \frac{s_{12}^E}{s_{11}^E} \cdot \frac{\partial^2 u_z}{\partial r^2} \right) - \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot E_z \ , \qquad (1)$$

$$D_z = \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u_z}{\partial r} \right) + \left(e_{33}^S - \frac{2 \cdot d_{31}^2}{s_{11}^E + s_{12}^E} \right) \cdot E_z \ .$$

The mechanical boundary equations can be obtained in supposition of absence of energy flow through corresponding contour of plate, when the contour either is fastened or is free. In the latter case, ($s_{rr} = 0$) the forces of electromechanical nature work and electrical energy imparted to piezoelement (PE) is transformed into mechanical energy. At fastened contour the PE is electromechanically passive.

For round plate of sensing element from isotropic material with specified edge forces or bending moments, the boundary conditions are reduced to equations relating the bending moment M_r

and cross force Q_r with derivatives of the deflection $u_z(r)$ and cylindrical rigidity C:

$$M_r = -C \cdot \left(\frac{d^2 u_z}{dr^2} + \frac{m}{r} \cdot \frac{du_z}{dr} \right) \quad , \tag{2}$$

$$Q_r = -C \cdot \frac{d}{dr} \left(\frac{d^2 u_z}{dr^2} + \frac{1}{r} \cdot \frac{du_z}{dr} \right) . \tag{3}$$

For edge free of fastening and forces, it is necessary to assume: $M_r = 0$ and $Q_r = 0$. If displacements on contour are specified, the boundary conditions should be written as:

$$u_z = u_{z0} \ , \ \frac{du_z}{dr} = y_0 \quad . \tag{4}$$

For a rigidly jammed contour the bounlary equations are reduced to the form:

$$u_z = 0 \ , \ \frac{du_z}{dr} = 0 \quad . \tag{5}$$

For a plate freely laying on a rigid support, it is necessary to use equations:

$$u_z = 0 \ , \ \frac{d^2 u_z}{dr^2} + \frac{m}{r} \cdot \frac{du_z}{dr} = 0 \quad . \tag{6}$$

At formulation of boundary conditions the multilayer SE should be changed for an equivalent plate having thickness h and density r, Poisson ratio m, and cylindrical rigidity C. On the outside contour R_0 of the plate the bending moment in radial direction M_r and cross force Q_r are equal to 0, and on the interior contour r_0 it should be assumed that deflection u_z is equal to zero, and the turn angle is $y = \frac{\partial u_z}{\partial r}$:

$$\begin{aligned}
&\frac{\partial^2 u_z}{\partial r^2} + \frac{m}{r} \cdot \frac{\partial u_z}{\partial r} = 0\Big|_{r=R_0} \ , \\
&\frac{\partial}{\partial r}\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u_z}{\partial r} \right) = 0\Big|_{r=R_0} \ , \\
&u_z = 0\Big|_{r=r_0} \ , \\
&\frac{\partial u_z}{\partial r} = 0\Big|_{r=r_0} \ .
\end{aligned} \tag{7}$$

The problem of calculation of multilayer SE is reduced to determination of location of a neutral surface and value of equivalent bending rigidity. Mechanical stresses in sections of *n*-th layer of a plate can be calculated as for one-layer plate. The obtained dependencies of SE displacements and mechanical stresses of PE are used for calculation of the basic characteristics of the transducer. As an example we present the system of the boundary mechanical equations for SE as a round three-layer plate rigidly fastened on the central cylindrical support and consisting of piezoceramics layers, glue and metal (see Figure 2). Density r and thickness h of the effective plate:

$$\begin{aligned}
&r = \frac{r_p \cdot h_p + r_m \cdot h_m + r_k \cdot h_k}{h} \ , \\
&h = h_p + h_k + h_m
\end{aligned} \tag{8}$$

where r_p is the piezoceramic density; h_p - thickness of piezoceramic disk; r_m - density of material of elastic element; h_m - thickness of elastic element; r_k - glue density; h_k - thickness of glue layer.

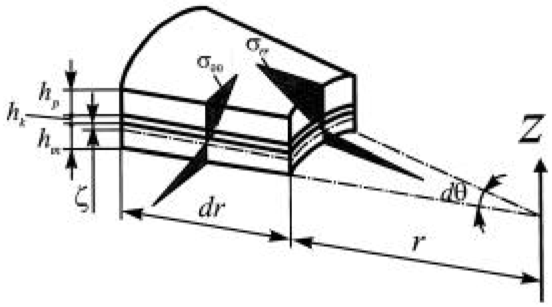


Figure 2. Stressed state of multilayer plate.

Equivalent rigidity of multilayer SE, as well as equivalent elasticity modulus E and Poisson ratio m are determined from the condition that the sum of bending moments in the side section of the multilayer plate is equal to the bending moment of equivalent one-layer plate M_r [3 – 5]:

$$D = \frac{s_{11}}{s_{11}^2 - s_{12}^2} \cdot \int_{h_k}^{h_p+h_k} y \cdot (y + z)dy + \frac{E_k}{1-m_k^2} \cdot \int_0^{h_k} y \cdot (y+z)dy + \frac{E_m}{1-m_m^2} \cdot \int_{-h_m}^{0} y \cdot (y+z)dy \ , \quad (9)$$

$$M_r = \left(\frac{dj}{dr} + m \cdot \frac{j}{r} \right) \cdot \left[\frac{s_{11}}{s_{11}^2 - s_{12}^2} \cdot \int_{h_k}^{h_p+h_k} y^2 dy + \frac{E_k}{1-m_k^2} \cdot \int_0^{h_k} y^2 dy + \frac{E_m}{1-m_m^2} \cdot \int_{-h_m}^{0} y^2 dy \right] = \\ = \left(\frac{dj}{dr} + m \cdot \frac{j}{r} \right) \cdot \frac{E \cdot h^3}{12 \cdot (1-m^2)} \ , \quad (10)$$

where j - angle of turning of the neutral surface of the plate; z - distance from the neutral surface to the considering plane of plate; z - distance from the neutral surface to the axis beginning. It should be noted that the neutral surface of multilayer plate incontrast to the one-layer plate, does not generally coincide with the median surface and, hence, the neutral axes of sections do not pass through geometrical centers of plate. Distance z from the neutral surface to the axis of measurement beginning (located in the plane where piezoelement is glued to elastic element) can be found from the condition of absence of normal stresses and forces at the neutral surface ($s_r\big|_{y=z} = 0$, $N_r\big|_{y=z} = 0$) [3 – 5]:

$$z = \frac{\frac{E_m}{1-m_m^2} \cdot h_m^2 - \frac{s_{11}}{s_{11}^2 - s_{12}^2} \cdot h_p^2 + \frac{E_k}{1-m_k^2} \cdot h_k^2 + 2 \cdot \frac{E_k}{1-m_k^2} \cdot h_k \cdot h_m}{2 \cdot \left(\frac{s_{11}}{s_{11}^2 - s_{12}^2} \cdot h_p + \frac{E_m}{1-m_m^2} \cdot h_m + \frac{E_k}{1-m_k^2} \cdot h_k \right)} \ . \quad (11)$$

Stresses in sections of *n*-th layer of bending multilayer plate can be calculated similar as for the one-layer plate having the same rigidity. Mechanical stresses and electric induction in piezoceramic layer are determined using the system of equations (1), and mechanical stresses in layers of glue and in the metal membrane are described as follows:

$$s_{rr} = \frac{-E_{k,m} \cdot (z + z)}{1 - m_{k,m}^2} \cdot \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{m_{k,m}}{r} \cdot \frac{\partial u_z}{\partial r} \right) \ ,$$

$$s_{qq} = \frac{-E_{k,m} \cdot (z + z)}{1 - m_{k,m}^2} \cdot \left(\frac{1}{r} \cdot \frac{\partial u_z}{\partial r} + m_{k,m} \cdot \frac{\partial^2 u_z}{\partial r^2} \right) \ . \quad (12)$$

In the electroelasticity problems the boundary conditions can be divided into two groups of conditions of conjugate field on the surface of a piezoceramic body: mechanical and electrical. The boundary conditions for the mechanical component are reduced to usual relations of the elasticity theory. Thus, if on surface of the body the vector of mechanical stresses is specified, it is necessary to take advantage of dependencies of the generalized Hooke's law and to equate stresses on the surface of body to their specified values. The obtained three equations relate components of displacements vector and electrostatic potential. The electrical boundary condition is formulated for the electrical component of conjugate field, and it depends on how electrical energy is picked off PE. On PE surface, where there are no electrodes, the approximated equality is valid [3 – 6]:

$$\vec{n} \cdot \vec{D} \approx 0 \quad . \tag{13}$$

On separated electrodes of area A_n from which the electrical energy is not removed, the condition is satisfied [3 – 6]:

$$\iint_{A_n} D_n dA = 0 \quad . \tag{14}$$

If the electrodes are closed by electrical loading with known complex conductivity $Y = Y_{Re} + jY_{Im}$, the electrilcal condition should he written as [3 – 6]:

$$\iint_{A_n} \frac{\partial D_n}{\partial t} dA = Y \cdot U \quad . \tag{15}$$

The integral in equation (15) is taken over the PE electrode surface and represents current $I(t)$ in the external circuit, and U is the required difference of potentials on PE electrodes. It should be noted that generally in PE a non-homogeneous mechanical field is formed, and only in the limits of elementary layers (volumes) mechanical stresses are homogeneous. To exclude ambiguity of the mechanical conditions, it is necessary to use equations for forces affecting on PE edges or for mechanical energy entering PE through corresponding edge.

The system of equations (7) gives sufficient number of relations for calculation of design-assembly parameters of SE. The electrical characteristics can be determined with use of formula (15) relating electric induction D_z with the current in the external circuit. Solution of the system of equations in the form of standing waves along r is:

$$u(r) = B_1 \cdot J_0(\lambda(f) \cdot r) + B_2 \cdot Y_0(\lambda(f) \cdot r) + B_3 \cdot I_0(\lambda(f) \cdot r) + B_4 \cdot K_0(\lambda(f) \cdot r) \quad , \tag{16}$$

where $B_{1,2,3,4}$ - coefficients depending on the boundary conditions; $J_0(\lambda(f) \cdot r)$, $Y_0(\lambda(f) \cdot r)$, $I_0(\lambda(f) \cdot r)$, $K_0(\lambda(f) \cdot r)$ - Bessel functions;

$$\lambda(f) = R_0 \cdot \sqrt[4]{\frac{\rho \cdot h \cdot \omega^2}{D}} \quad . \tag{17}$$

If the plate is solid and there is no support in the center, deflection at $r = 0$ should be finite, while functions $Y_0(\lambda(f) \cdot r)$ and $K_0(\lambda(f) \cdot r)$ tend to infinity. However it does not follow from here that coefficients B_2, and B_4 are equal to zero. These functions in vicinity of point $r = 0$ can be reduces as follows:

$$Y_0(\lambda(f) \cdot r) = \frac{2}{\pi} \cdot \left(1 - \frac{(\lambda(f) \cdot r)^2}{4}\right) \cdot \ln(\lambda(f) \cdot r) + \frac{2 \cdot \gamma}{\pi} + \ldots ,$$

$$K_0(\lambda(f) \cdot r) = -\left(1 + \frac{(\lambda(f) \cdot r)^2}{4}\right) \cdot \ln(\lambda(f) \cdot r) - \gamma + \ldots \quad . \tag{18}$$

If the plate has a support in the center, function *u(r)* should be written as:

$$u(r) = B_1 \cdot J_0(\lambda(f) \cdot r) + B_2 \cdot I_0(\lambda(f) \cdot r) - \frac{P}{8 \cdot D \cdot \lambda_1^2} \cdot \left[Y_0(\lambda(f) \cdot r) + \frac{2}{\pi} \cdot K_0(\lambda(f) \cdot r) \right] \quad , \tag{19}$$

where P - support response.

Fig. 3 shows calculated amplitude-frequency characteristics (AFC) for sensor AP36 in the mode of electrical no-load operation.

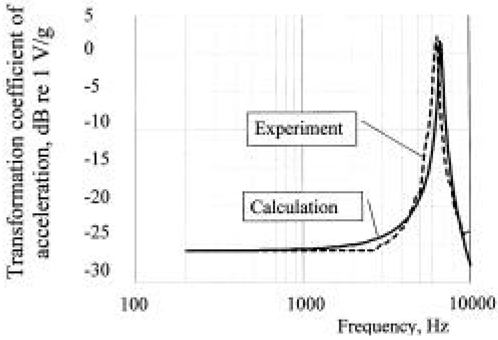


Figure 3. Calculated AFC for sensor AP36 in the mode of electrical no-load operation.

In the calculation of multilayer step plates, the parts of the plate constant in thickness should be considered independently. In this case, equality of shifts, rotation angles and radially directed moments present boundary conditions. So, for the sensing element having centered inertial element at its external circuit (Fig.2a), boundary conditions are summed up for two parts of the sensing element [3 – 5].

CONCLUSION

The present technique for calculation of piezoelectric sensors with sensitive element subjected to bending deformations allows one to optimize the design-assembly parameters of sensors. Results of calculations are in good agreement with experimental data. The technique has wide universality and physical illustrative feature for designing not only sensors of mechanical values, but also other type of piezoelectric transducers.

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