STATISTICAL STUDY OF THE INSTANTANEOUS VALUES OF THE SOUND PRESSURE LEVEL, GIVEN SOUND EVENTS RELATED TO ENVIRONMENTAL ACOUSTICS.

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ABSTRACT

In environmental acoustics the knowledge of the time dependency of the sound level provides relevant information about a sound event. In this sense it may be mentioned that the conventional sound level metres have frequently implemented programs to calculate the fractiles (percentiles) of the distribution of instantaneous sound levels; and there are several indexes to evaluate the noise pollution, based on different statistical parameters. For further analysis of sound, and to obtain the commented indexes, it is accepted that this distribution is normal or gaussian. The questions we've tried to solve in this work are the following: First of all, if the time dependent distribution of noise shall be considered as Gaussian in general cases or only in same particular. In the other hand, we have studied how the frequency of the sampling, and the applied exponential time constant (fast slow), affects to the kind of the resulting distribution given a sound event. To get these aims, it has been studied a set of sound events such as the traffic noise in different points of a highly contaminated town, obtaining the conventional "statistics" (moments of n-th order and percentiles), studding the shape of different plots, etc.
After the analysis of the measured events, we can advance that in many cases it is not reasonable to consider the distribution as gaussian, but asymmetrical distributions are frequently found.

1.-INTRODUCTION

A gaussian distribution verifies:

$$\Phi\left(\frac{x_q - \mu(x)}{\sigma(x)}\right) = 1 - q\,, \qquad (1)$$

where $\Phi(z)$ stands the typified error function (with average 0 and standard deviation 1) , y x_q represents some value of x with a probability q ($q \in (0,1)$) to be surpassed. On the other hand, $\mu(x)$ y $\sigma(x)$ mean respectively the average and standard deviation of the distribution of x.
Concretely, for the measure of the statistical distribution of levels from a sound signal, stands:

$$\Phi\left(\frac{L_n - \mu(L)}{\sigma(L)}\right) = 1 - \frac{n}{100}\,, \qquad (2)$$

being Ln the percentile level surpassed during n% of the measurement time. Previous expression can be written in inverse form as:

$$\frac{L_n - \mu(L)}{\sigma(L)} = \Phi^{-1}\left(1 - \frac{n}{100}\right) \qquad (3)$$

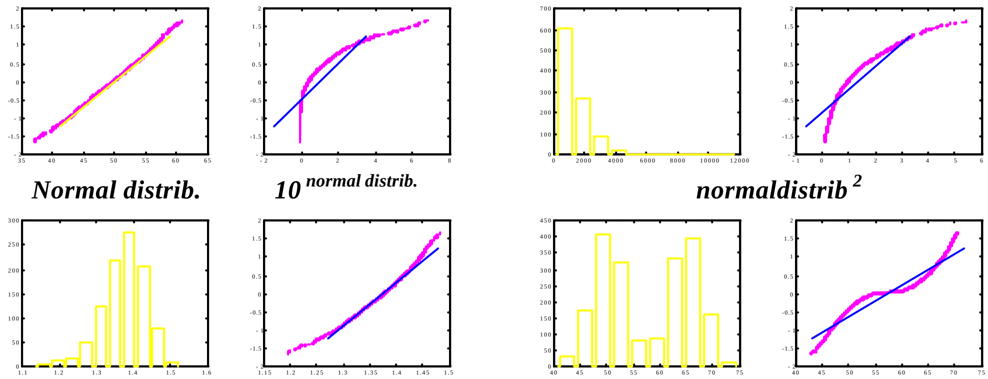
being $\Phi^{-1}(\beta)$ la inverse typified error function. Reordering

$$L_n = \sigma \cdot \Phi^{-1}\left(1 - \frac{n}{100}\right) + \mu \, , \qquad (4)$$

it can be observed a linear relationship between the percentile levels measured and the inverse typified error function. This relationship can be used to estimate the average and the standard deviation of a distribution given a set of values measured of the percentile levels. In fact, developing a linear regression, the average is obtained as the ordenade in the origin and the standard deviation as the slope of the adjustment.

This property is used in *normal probabilistic plots*. They are diagrams where vertical scale have being modified; therefore, representing on ordenade the percentile and representing on abscises the level associated, we get a perfect straight line if the datos are from a gaussian distribution. This plot is very used when statistical analysis from short series of datos is required. In this plots the axes of the expression (4) have been inverted; and so, the standard deviation is calculated as the inverse of the slope and the average is the abscise on the origin of ordenades.

Some examples of *probabilistic plots* from different distributions are presented:



Normal distrib. ***10 $^{\text{normal distrib.}}$*** ***normaldistrib ^{2}***

normaldistrib $^{(1/2)}$ ***Two normal distributions***

Figure 1.- Normal probabilistic plots from different kind of distributions

The coefficients of asymmetry and curtosis are respectively defined as the momentum of third and forth order respect to the average, divided by the standard deviation to the power of the order number:

$$M(n) = \frac{1}{\sigma^n} \int_{-\infty}^{\infty} (x - \bar{x})^n \, dx \qquad\qquad M(n) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^n}{\sigma^n (N-1)} \qquad (5)$$

- Asymmetry coefficient ($AC \equiv M_{(3)}$) gets a null value on a perfectly symmetric distribution, it is positive if the histogram has bigger tail in the right hand side; and negative on the contrary.
- Curtosis coefficient ($CC \equiv M_{(4)}$) provides information about if the extreme values have been "cut". Normal distribution is characterised by CC=3; lower values indicates that the extreme values have been eliminated (for example because they are out of measurement range) and bigger values indicates that there are more extreme values than those expected in a gaussian distribution with the same standard deviation.

In acoustic measurements, a particularity must be pointed out. The variable that is statistically analysed is the sound pressure level, and thus the average of this distribution will be calculated like this

$$Lp(mean) = \frac{1}{N} \sum_{i=1}^{N} Lp_i \, , \qquad (6)$$

however, this value is generally not coincident with the most used parameter in environmental acoustics. This is the equivalent sound pressure level that stands on the level of the average quadratic pressure. And given that this is calculated as $p^2/p_{ref}^2=10^{LP/10}$, the expression of the equivalent sound pressure level is:

$$Leq = 10 \log\left(\frac{1}{N}\sum_{i=1}^{N} 10^{\frac{Lp_i}{10}}\right). \qquad (7)$$

An average calculated like this gets usually bigger values than a arithmetical average, thus for any kind of distribution is to be expected:

$$Leq \geq Lp(\text{mean}) \qquad (8)$$

being strictly equal in the ideal case of a distribution with null standard deviation. In the results section, a study about the difference between these parameters related to the standard deviation of a normal distribution of sound pressure levels is shown.

2.- DISTRIBUTION OF INSTANTANEOUS LEVELS IN TRAFFIC NOISE.

2.1.- Measured Events

In environmental acoustics we can study many different kind of sound events (urban noise, industrial noise, traffic noise, ...) where the source and the propagation of sound are varied. In this work, we study one of the most extended noise sources in our urban environments: the road traffic. Concretely, measurements of noise caused by N332 passing by the town of Bellreguard have been effectuated. A open road close to this town has been also studied. The registered sound events are:

a) Traffic light into the town
b) Perpendicular street, 40m far from N332
c) N332 into the town
d) N332 into the town (II)
e) Open road, 10m far
f) Side of the road, close to the town
g) Side of the road, 100m far from town end
h) Side of the road, on open road

2.2.- Measurement Routines

The kind of statistical distribution of sound pressure levels will depend on the kind of sound event, but also on how these levels are obtained from the recordings. It is known that the time depending average of the acoustic pressure in any wave is null, so in brief instants we can find acoustic pressures positive negative, or even null. For this reason, to calculate the "instantaneous" sound pressure level, this must be evaluated during a time "short" but long enough to be representative of the energy associated to perturbation. In this sense, as we want to evaluate the human audible spectrum, it has been decided to calculate the "instantaneous" levels integrating the quadratic acoustic pressures on intervals of time bigger than the period corresponding to the lower audible frequency.

This frequency is considered 20 Hz, and its corresponding period is 50 ms. The instantaneous levels are calculated integrating the acoustic pressure in intervals of 20ms 200ms 2s y 8s. Thus, from each acoustic event, we have four different "populations" of instantaneous levels. The expression applied to get these levels is:

$$Lp_{t_i} = 10 \cdot \log\left(\frac{1}{\Delta t \cdot p_{ref}^2} \int_{t_i-\frac{\Delta t}{2}}^{t_i+\frac{\Delta t}{2}} p(t)^2\, dt\right) \qquad (9)$$

The numb of "habitants *Lp*" of each population is calculated as the duration of the measure divided by the integration interval Δt. The duration of the recordings is 7 min and so:

Integration time (ms)	20	200	2000	8000
N de "samples"	21.000	2.100	210	53

The duration of the measure also affects to the representativity of the source. To determine it, empirical criteria, dependent on the density and kind of traffic, have been used. Finally, it must be commented that the dynamic range of the events is included into the dynamic range of the configuration used for the measurements.

3.-RESULTS

3.1.- Measured Sound Events

3.1.1.- Dependency of distribution with the kind of sound event

In next figure the normal probabilistic plots from the events a, b, c, and d are shown. It con be observed that the supposition of normal distribution is completely acceptable. Then the average of the distribution *Lp(mean)* will be coincident with the percentile *L50*, and both can be obtained from the knowledge of other percentiles. To obtain the equivalent sound pressure level, the expression (10) can be used.

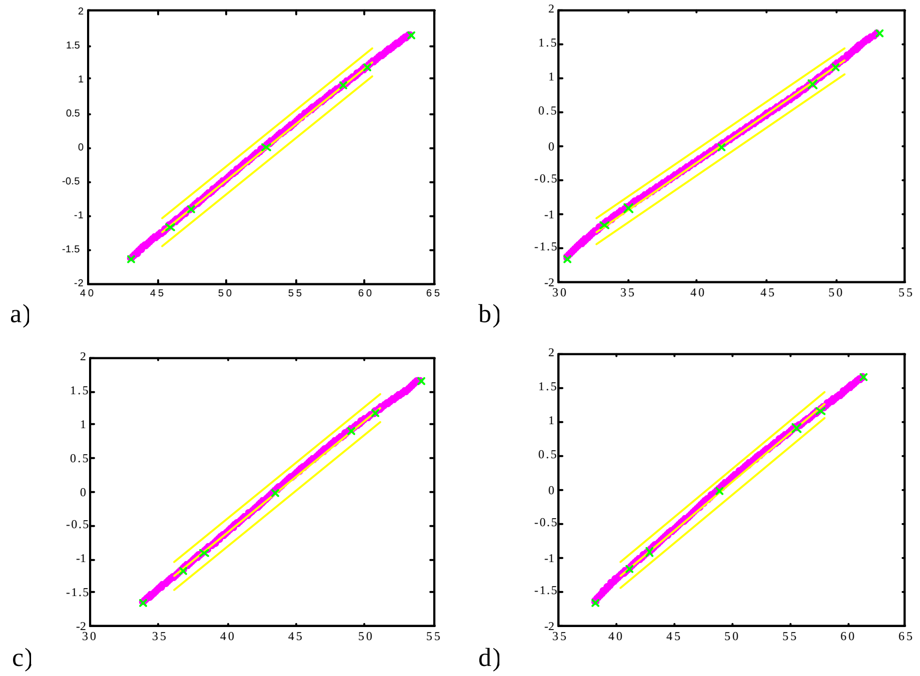


Figure 2.- Normal probabilistic plots of sound events a b c d with 20 ms integration time.

In figures 3a and 3b its shown how decreasing the distance to the sound source the distribution increases its positive asymmetry. It must be pointed out that this kind of positive asymmetry is found on distributions such as $10^{\text{normal distrib}}$ and so, this kind of asymmetry is not due to the logarithm used to obtain the level of a physical magnitude. On the contrary, if we study the distribution of the quadratic pressures, they will be more asymmetrical than these.

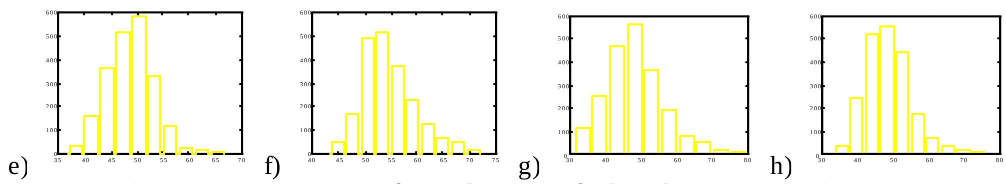
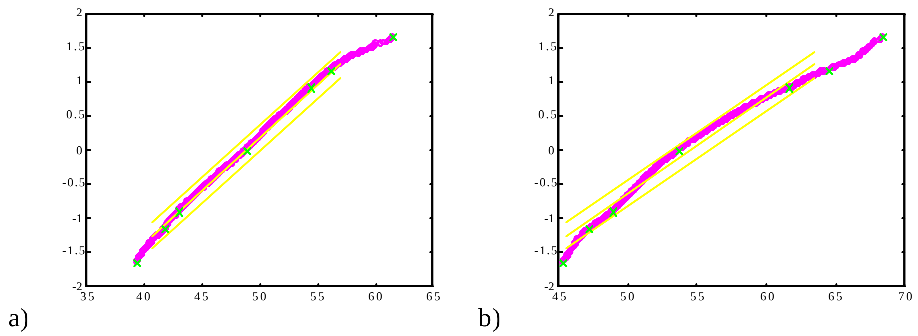
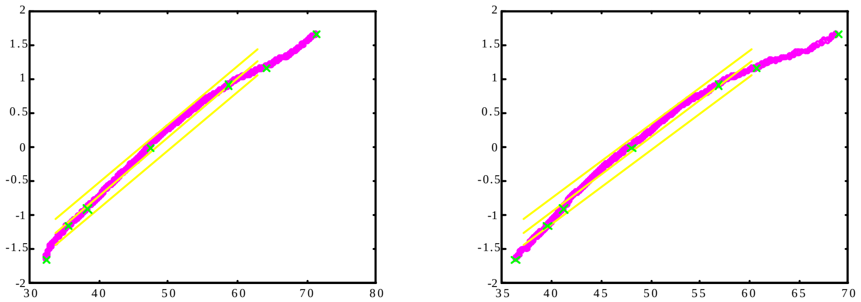


Figure 3a.- Histograms of sound events e f g h with 0.2s integration time.





c) d)

Figure 3b.- Normal probabilistic plots of sound events a b c d with 0.2s integration time.

The dynamic range of the measurement configuration (48 dB) seems to be large enough to study the measured events, after visual inspection of the histograms. However, the curtosis coefficient of the series of datos evaluated is shown. Asymmetry coefficient is also pointed out in the next table:

	A	**b**	**c**	**D**	**E**	**f**	**g**	**H**
AC	0.0657	-0.0198	0.1848	0.1383	0.2610	0.5818	0.5067	0.6645
CC	2.7892	2.8015	2.8633	2.6390	2.9116	2.9622	3.1303	3.4418

Notice that *CC* in any case is close to 3, that indicates that the configuration employed is adequate and do not eliminate extreme values. On the other hand, asymmetry coefficient shows that last three events are the most asymmetrical .

3.1.2.- Dependency of kind of distribution with integration time.

We have developed this analysis in order to study the concept of "instantaneous" sound pressure levels in subsequent works with psichoacoustical point of view

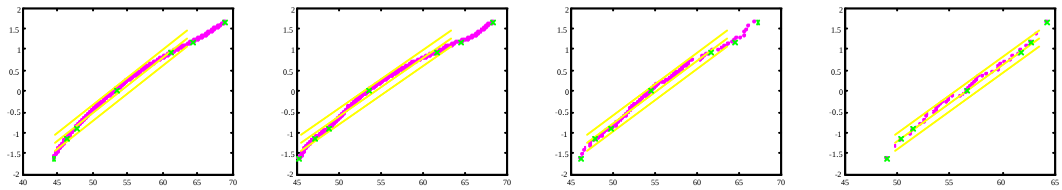


Figure 4.- Normal probabilistic plots for integration times of 20ms, 0.2s, 2s y 8s. Sound event f

It can be observed that, increasing the integration time we get interesting effects.

- The dispersion of the datos decreases (this is a trivial effect)
- The distributions tend to be "more gaussian"
- The Lp(mean) increases, but not Leq that is independent of integration time

This results points out that the percentile levels -and concretely L_{50}, that is coincident with *Lp(mean)* if normal distribution- are considerably dependent on the linear integration time o the instruments, or, if the case, on the sampling frequency and the time constant of the exponential averaging.

3.2.- Comparision Between The Mean Of The Levels And The Equivalent Level

As commented above, these parameters are calculated in different way. And the value of the difference between both depends on the kind of distribution evaluated. It has been shown that normal distribution appears frequently, and for this reason we have try to find the relationship between the standard deviation and the difference *Leq-Lp(mean)* in this kind of distribution.

Next plots have been obtained from normal distributions of 10.000 values generated by computer. The standard deviations get 150 stepped values in the interval [0.1dB, 15 dB]

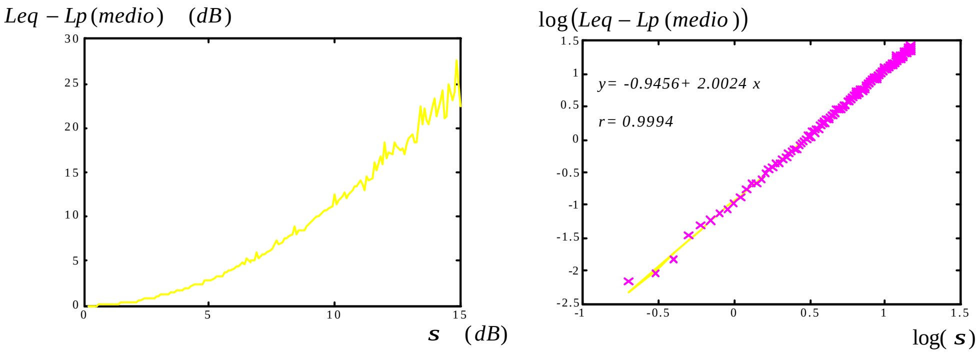


Figure 5.- Relationship between standard deviation of a gaussian distribution of sound pressure levels and the difference Leq-Lp(mean)

It is found a parabolic relationship in the figure 5a with a exponent obtained as the slope of the linear regression obtained from the logarithm of both variables shown in figure 5b. It is to be pointed out the high correlation obtained, and also the fact of been the slope a entire number. Definitively, if the sound pressure level can be considered a normal distribution, next expression is proposed:

$$Leq = Lp(medio) + 0'11\,\sigma^2 \qquad (10)$$

4.- CONCLUSSIONS

After the analysis of the results obtained and commented in the previous sections, the following aspects of the present work, must be pointed out:

- The hypothesis of considering traffic noise as a gaussian distribution seems to be more acceptable whereas more parameters affect to the propagation of sound from source to receiver. Thus, into the town we found quasi-perfect gaussian distributions, due to the sound field comes to many sources (cross of roads, engine, brake) and the propagation depends on multiple parameters (reflection on fronts of different shapes, etc). This result is in accordance to the central limit theorem
- Close to roads in free field, distributions get a positive asymmetry that increases as the distance from receiver to road decreases. In this situations the value of L_{50} and *Lp(mean)* are not clearly related to equivalent sound pressure level.
- The integration time of the measurement instrument affects relevantly to parameters of distribution , such as its mean, symmetry and standard deviation. Generally asymmetry coefficient and dispersion decreases and the *Lp(mean)* increases when increasing the interval of integration.
- Finally, in general cases where the sound pressure level can be considered a normal distribution, next expression to predict, knowing the standard deviation, the difference between the equivalent level and the mean of the instantaneous levels, is proposed en (10).

In subsequent works, we will evaluate different acoustic environments; and, on the other hand, in the ambit of Psichoacoustics, we will try to find the adequate interval of integration, to obtain results with good accuracy with the subjective perception of the events.

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