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OBERST BEAM AS A TOOL FOR COMPLEX YOUNG'S MODULUS MEASUREMENT OF POROUS MATERIALS.

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ABSTRACT

The prediction of the vibroacoustic performance of systems including porous materials requires a correct estimation of the mechanical parameters of porous materials. This paper investigates the adaptation of Oberst beam method developed for viscoelastic materials to the determination of complex Young's modulus of porous materials. Analytical and numerical results based on a finite element model show that there are some special conditions including dimensions of test sample and boundary conditions where porous materials behave as viscoelastic ones. It is then possible to determine the complex Young's modulus according to this method. For these configurations, acceptable experimental results are obtained in the case of different foamed and fibrous materials.

1. INTRODUCTION

Porous materials are often used as passive elements of noise reduction systems. Mainly used for sound absorbing purposes porous materials have also an ability to damp vibrations of mechanical structures. Being integrated into a multifunctional noiseproofing part, a porous material can reduce appreciably the vibration levels of, for example, steel car body panels. Among the parameters of porous materials, the complex Young's modulus of skeleton influences strongly the damping behaviour of the porous media, and several methods exist for its determination [1], [2]. However, considering the porous material mainly as a damper, it is interesting to examine the application to porous material of a test method especially developed for damping material, such as Oberst's beam method.

Oberst's beam method [3] is well-known and widely used [4], [5] for characterising viscoelastic damping materials. Porous materials being normally described by Biot poroelastic model can however be considered as viscoelastic ones at low frequencies [1] or in the case of finite-dimension sample with small shape factor and open faces [6], that is in the conditions where the influence of the air in the pores on the total dynamic behaviour of the porous sample is negligible. To apply Oberst's method to the determination of complex Young's modulus of porous materials, it is necessary to adapt the measurement routine, taking into account the features of porous material, such as low magnitudes of density and Young's modulus. This paper contains an adaptation of Oberst's beam to the assessment of the determination of Young's modulus and loss factor of porous materials. The limits of the proposed method are investigated through the analysis of experimental and numerical results.

2. THEORY: COMPLEX YOUNG’S MODULUS DETERMINATION

Oberst’s beam consists of a material to be tested bonded onto a metal slice, which is clamped at one end and excited at the other end (see figure 1).

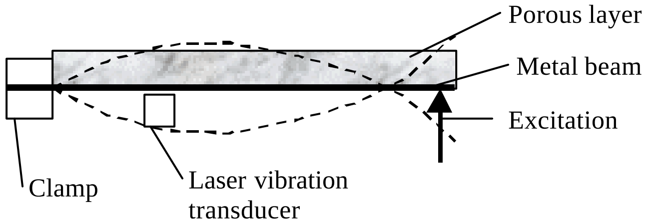

Figure 1: Oberst beam

Before dealing with the theoretical background of the proposed method it is important to recall the main assumptions made:

- the test sample is considered as a rod-like one, i.e. its deformations are extensional-compressional and are controlled by complex Young’s modulus without any influence of Poisson ratio (a);
- the dimensions of the test sample and the frequency range of measurements suppose a viscoelastic dynamic behaviour of a porous material (b);
- the porous material behaves in a linear way in the frequency range and deformation amplitudes of interest (c);
- angle of deformation of porous layer does not change with thickness that means the angle is the same in lower layer and in upper layer of porous material as it is shown on figure 2 (d);
- in the resonance zone, the variation of Young’s modulus and loss factor vs. frequency is negligible (e).

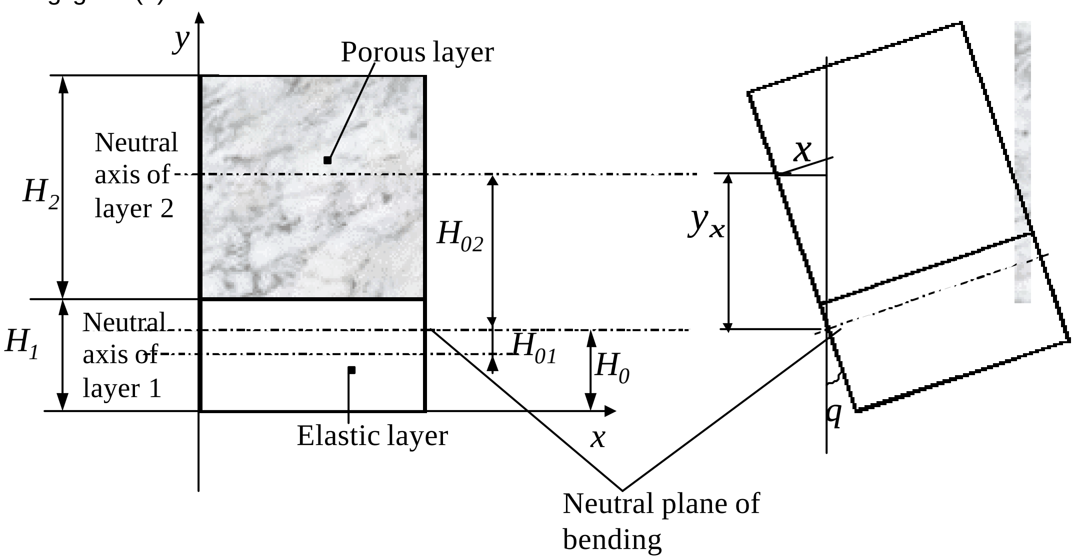

Figure 2: Deformed and undeformed composite beam

Briefly, the Young’s modulus determination by this method is based on the comparison of the natural frequencies of the bare beam and the composite beam (metal beam with porous layer).
The natural frequency of the homogenous beam is given by

$$f_{0n} = \xi_n^2 \sqrt{\frac{B_b}{m_b L_b^4}}\,, \qquad (1)$$

where

n is the mode index,

ξ_n is the mode coefficient,

B_b is the bending stiffness of the beam,

m_b is the mass per unit length per unit width, $m_b = \rho_b H_b$, where ρ_b is the beam density, and H_b is the thickness of beam,

L_b is the length of the beam.

It can be shown [4], after comparison of the bending moments with respect to neutral plane of bending of composite beam with the bending moment with respect to horizontal axis *x* (see figure 2), that the Young's moduli of the substrate and of the material are linked to each other by the following equation

$$E_2 = E_1 \cdot \frac{\left(\frac{\omega_c}{\omega_1}\right)^2 \frac{m_c}{m_1} \cdot I_1 - I_{01}}{I_{02}} , \qquad (2)$$

where

E_1, E_2 are the Young's moduli of steel substrate and porous layer, respectively,

ω_c is the natural frequency of the two-layer beam,

ω_1 is the natural frequency of the steel substrate alone,

I_1 is the moment of inertia per unit length of the steel substrate $I_1 = \frac{H_1^3}{12}$,

I_{01}, I_{02} are the moments of inertia per unit length relative to the neutral plane of bending of the composite beam,

m_1 is the mass per unit length per unit width of the steel substrate, $m_1 = \rho_1 H_1$, where ρ_1 denotes the density of steel, and H_1 denotes its thickness,

m_c is the mass per unit length per unit width of the composite beam, $m_c = \rho_1 H_1 + \rho_2 H_2$, where ρ_2 denotes the density of the porous layer, and H_2 denotes its thickness.

It is necessary to note that equation (2) supposes a linear deformation of the porous layer as it is supposed by assumption (d).

Although the Young's modulus of the porous material can be obtained directly, from the classical Oberst's beam approach, the assessment of the loss factor is not as straight founded. Indeed, classical Oberst's beam theory supposes the loss factor of the bare steel slice is negligible in comparison with the loss factor of the composite beam. In the case of the composite beam involving a porous layer, the loss factor of this composite beam may not be very large. The porous layer does not change strongly the dynamic behaviour of the beam due to a low specific weight and a low Young's modulus of the skeleton. This feature requires to a modification the technique in order to take in account the loss factor of the metal substrate.

The length of the paper does not allow one to present a complete derivation of the loss factor expression. Therefore only the most important steps and assumptions will be presented. The loss factor is defined as the ratio of the energy dissipated per cycle of oscillations (*D*) to the potential energy (*U*) of the system (or structure). In general, the energy of the two- (or multi) layer structures can be described as a sum of the energies stored and dissipated in each layer. The damping action of a multi-layer structure is considered as a set of springs, where each spring represents a storing energy mechanism. The displacement of the first layer (bending deformation of a beam to be damped) causes a displacement of the second one. The energy stored in one spring is $W_i = K_i \cdot x_i^2$, where K_i is the stiffness of the spring, and x_i its strain. In case of periodic displacement, the energy dissipated per cycle of oscillation, is $D_i = 2\pi\eta_i W_i$, so that the loss factor of the multilayer structure is given by:

$$\eta = \frac{D}{2\pi \cdot U} = \frac{\sum_i D_i}{2\pi \sum_i W_i} = \frac{\sum_i \eta_i W_i}{\sum_i W_i} = \frac{\sum_i \eta_i K_i x_i^2}{\sum_i K_i x_i^2} , \qquad (3)$$

Taking in account that the structure of interest is a two-layer beam in which the porous layer behaves viscoelastically, one may assume extensional-compression deformations takes place into this system and each layer acts as a "spring" ($K_i = K_{ext}$). Considering the extensional mechanism one may conclude that the value of K_{ext} is determined by the bending stiffness of each layer, and by the extensional deformations of both layers. On the basis of energy equations, shown in [7], it is possible to write for one layer

$$K_{ext} x_{ext}^2 = w_{ext} + w_{flex} = \frac{1}{2} K \cdot e^2 + \frac{1}{2} B \left| dq/dx \right|^2 \tag{4}$$

where $K = E \cdot A$ is an extensional stiffness, A is the cross-section area; $e = \frac{dx}{dx}$, where x is the mean axial displacement of an element of material, x is the horizontal coordinate, B is the bending stiffness of the layer relatively to the neutral plane of bending, q is the bending angle.

Figure 2 shows the displacements in the system. In order to simplify the model, it is assumed that because of the small value of q the axial displacement x is taken as equal to the displacement along the *x*-axis. In other words, one assumes the absence any shear deformation into the porous layer. Thus, the following relation holds:

$$x = y_x \cdot \tan q \equiv y_x \cdot q \,, \tag{5}$$

where y_x denotes on figure 2. Thus, taking into account that $e = \frac{dx}{dx} = \frac{d}{dx}\left(y_x \cdot q\right) = y_x \cdot \frac{dq}{dx}$, equation (3) can be rewritten as

$$h_{c\ ext} = \frac{\frac{1}{2}\left(\frac{dq}{dx}\right)^2 \sum_{i=1}^{2}\left(h_i\left(E_i h_i y_{xi}^2 + B_i\right)\right)}{\frac{1}{2}\left(\frac{dq}{dx}\right)^2 \sum_{i=1}^{2}\left(E_i h_i y_{xi}^2 + B_i\right)} = \frac{\sum_{i=1}^{2}\left(h_i\left(E_i h_i y_{xi}^2 + B_i\right)\right)}{\sum_{i=1}^{2}\left(E_i h_i y_{xi}^2 + B_i\right)} \,, \tag{6}$$

where B_i is the bending stiffness of the layer *i* relatively to the neutral plane of bending, E_i, h_i, h_i are the Young's modulus, thickness, and loss factor of layer i, respectively, and y_{xi} are determined from geometrical relationships shown on figure 2.

Equation (6) establishes the relationship between the loss factor of the composite beam (h_{cext}), the loss factor of the bare steel slice (h_1), which need to be measured, and the loss factor (h_2) of the porous layer, which is the objective of the measurement. The loss factor h_2 can then be calculated from the following:

$$h_2 = \frac{h_{cext} \sum_{i=1}^{2}\left(E_i h_i y_{xi}^2 + B_i\right) - h_1\left(E_1 h_1 y_{x1}^2 + B_1\right)}{E_2 h_2 y_{x2}^2 + B_2} \,, \tag{7}$$

Thus, equations (2) and (7) allow one to calculate the Young's modulus and the loss factor of porous layer.

3. EXPERIMENTAL RESULTS AND FINITE-ELEMENT MODELLING: DISCUSSION OF THE LIMITATION OF THE PRESENTED APPROACH

Measurements of complex Young's moduli by this method have been carried out by using a steel substrate of thickness 0.55 mm and length 130 mm. Figure 3 presents one example of the frequency response function (FRF) measured for one of the different tested porous materials. The resonance frequencies are clearly identified. The loss factor can easily be determined by the thickness of the resonance curve or with the help of a curve fitting procedure on the basis of these measured FRF. To assess the performance of this method three porous materials of different thickness have been tested.

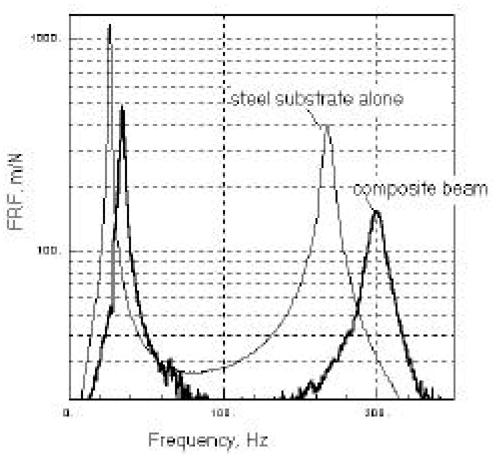

Figure 3: Example of measured frequency response function

Table 1 shows E_a and η_a (apparent Young's modulus and loss factor), which have been determined using equations (2) and (7) for the second natural mode of the beam. The values of Young's moduli indicated on the first line have been obtained by numerous measurements based on the resonance test method and are considered as a reference.

Table 1.

Foam 1: ρ=28 kg/m^3; E_{mod}=118 kPa, η=0.125				Foam 2: ρ=8.4 kg/m^3; E_{mod}=530 kPa, η=0.09				Fibrous: ρ=63 kg/m^3; E_{mod}=116 kPa, η=0.30			
T, mm	F_{res}, Hz	E_a, kPa	η_a	T, mm	F_{res}, Hz	E_a, kPa	η_a	T, mm	F_{res}, Hz	E_a, kPa	η_a
27	168.1	70	0.14	27	199.3	188	0.09	10	158.3	120	0.35
15	164.4	113	0.13	20	189	310	0.09				
10	164.16	115	0.12	10	172.4	535	0.08				
5	165.7	119	0.12								

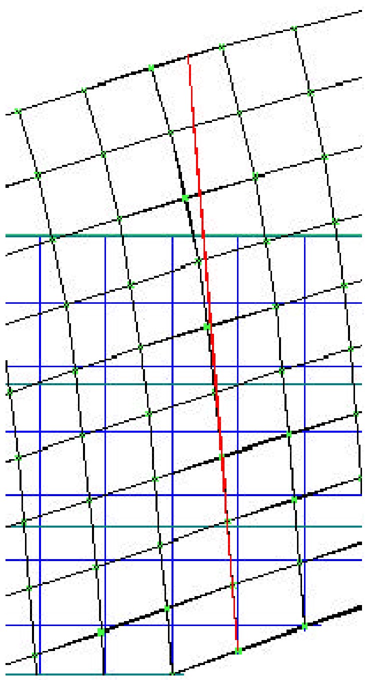

Figure 4: Finite-element modelling: displacements of cross-section of composite beam

The indicated results show a good agreement between loss factors determined by the present method and the resonance one, but also exhibits big differences between Young's modulus values obtained in the case of maximal and minimum thickness of porous materials. After examination of different probable causes of this effect the response was found by detailed consideration of assumption (d), which supposes a linear bending of the side of the porous material. The finite element simulations allow one to analyse the displacement field inside the porous material. Figure 4 displays the deformation shape of beam with foam 1 (thickness of foam makes 27 mm) at the second natural mode of bending. The steel substrate has been meshed by shell elements, the porous layer have been modelled by solid elements and has been divided onto nine elements along the thickness. Totally, the sample of porous material (130*20*27 mm) has been meshed by 2730 solid elements.

The picture illustrates the change of the bending angle along the thickness. It shows that upper layers of porous material bend more strongly than lower ones. In the case of low thicknesses of porous layer (5…15 mm), finite-element (FE) simulations show that the bending angle remains constant in the thickness of the porous material. It is confirmed by finite-element simulations shown in table 2.

Table 2.

	Foam 1: ρ=28 kg/m^3; E_{mod}=118 kPa		Foam 2: ρ=8.4 kg/m^3; E_{mod}=530 kPa	
Thickness of porous layer, mm	Resonance frequency, FE simulation	Resonance frequency, analytical prediction by equation (2)	Resonance frequency, FE simulation	Resonance frequency, analytical prediction by equation (2)
0	168.1	168.2	168.1	168.2
5	165.7	165.9	167.9	168.1
10	163.6	164.3	170.8	172.2
15	162.1	164.5	176.1	183.8
20	161.1	167.0	185.3	205.0
27	160.6	175.5	201.7	251.9

One can conclude that in the case of both materials a good agreement between analytically and numerically predicted frequencies is observed at thicknesses of porous materials equal to 5 and 10 mm. Very similar results have been obtained experimentally (see table 1). Thus, one can say that for porous materials with Young's modulus ranging from 100 to 500 kPa, the thickness of the porous layer should be limited to 10 mm if Oberst's beam is to be used. On the other hand, it is necessary to point out that this limitation is related to the assumptions made in the derivation of the analytical formula for Young's modulus calculation. This is not a limitation inherent to Oberst's beam method, which could use numerical FE calculation.

4. CONCLUSIONS

Oberst's beam method has been adapted to determine complex Young's moduli of porous materials. The modified equations of loss factor determination and one principal limit of the method have been established. The experimental results obtained by the proposed method for three different materials showed a good agreement with the ones received by an other test method.

5. ACKNOWLEDGEMENTS

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