INTERPOLATED 3-D DIGITAL WAVEGUIDE MESH FOR ROOM ACOUSTIC SIMULATIONS

PACS Reference: 43.55.Ka Computer simulation of acoustics in enclosures

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ABSTRACT

This paper reviews the basics of the 3-D digital waveguide mesh and the enhancements proposed to overcome the dispersion problems in the original method. The main attention is in interpolated structures that are utilized to cure the direction dependence of the wave-propagation characteristics in the rectangular mesh. In this paper we present some new sparse structures having reduced computational load and still being nearly as accurate as the full interpolated structure. Another applied technique is the frequency warping which helps to reduce the overall dispersion. Finally we discuss the applicability of the technique for practical room acoustic simulations.

INTRODUCTION

The digital waveguide mesh was originally developed for modeling of 2-D musical instruments such as membranes of drums and plates [1]. The technique is based on digital waveguides traditionally applied in physically based sound synthesis such as in simulation of vibrating strings [2]. The 3-D digital waveguide mesh (WGM) [3] originally presented in 1994 extends this approach to room acoustic simulations. This technique aims at providing a computationally efficient method for simulation of the low-frequency behavior of acoustically interesting spaces such as concert halls, churches, auditoria, and listening rooms.

The original 3-D WGM suffers from an error in the wave travel speed. The error depends on both the wave propagation direction and frequency [4], and it is called the direction-dependent dispersion error in the following. This error has been a severe drawback of the method, and several techniques have been developed to overcome it. In this paper we present some of those techniques including the interpolated WGM [5, 6] and frequency warping applied with WGMs [7, 5, 6]. In addition, we present a couple of new structures with reduced computational load, and in the last section we evaluate the current state of WGMs from the viewpoint of practical room acoustic simulations.

INTERPOLATED 3-D DIGITAL WAVEGUIDE MESH

In the original three-dimensional mesh there are digital waveguides in three orthogonal directions, and they are interconnected to each other. This structure illustrated in Fig. 1a forms a rectangular grid, in which each node has a neighbor at a unit distance in six directions: up, down, left, right, front, and back. The wave propagation in such a structure is governed by the following difference equation

$$p(n+1,x,y,z)=\frac{1}{3}\,\big[p(n,x+1,y,z)+p(n,x-1,y,z)+p(n,x,y+1,z)+ p(n,x,y-1,z)+p(n,x,y,z+1)+p(n,x,y,z-1)\big]-p(n-1,x,y,z) \tag{1}$$

where $p(n, x, y, z)$ represents the sound pressure at time step n at position (x, y, z) [4].

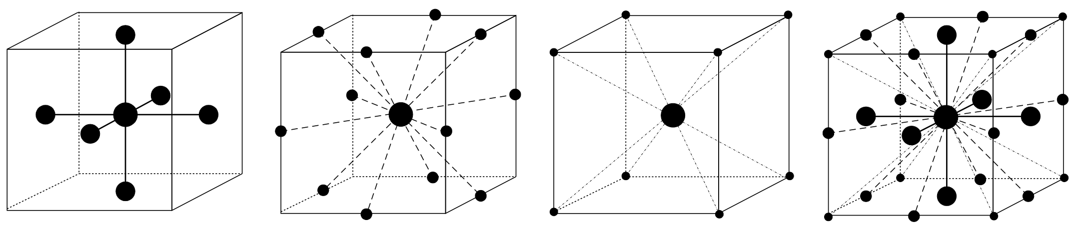


Fig 1. Construction of the interpolated digital waveguide mesh is based on a) the original WGM with 6 neighbors, b) 2-D diagonal neighbors, and c) 3-D diagonal neighbors. Finally, d) the full interpolated WGM structure with 26 neighbors is depicted.

Basically WGM method is a finite-difference time-domain (FDTD) method and it is easy to derive (1) from the wave equation. The difference between these methods comes from their backgrounds. The WGM method is based on digital signal processing (DSP) while FDTD is more mathematically oriented. The further developments of WGM originate strongly from the field of DSP as well, and it has made available some new techniques not applied with FDTD methods formerly.

All the waveguide mesh structures presented in this paper can be analyzed by Von Neumann analysis (see, e.g., [8]), which is based on a spatial Fourier transform performed to the difference scheme. The analysis of the 3-D case gives a dispersion factor as a function of three spatial frequencies. A detailed description of the results can be found from a previous article [6]. The main result is that in the 3-D diagonal directions (shown in Fig. 1c) there is no dispersion, and the maximal error occurs in the axial directions. Figure 2a illustrates the relative frequency errors (RFE) in the original mesh in three different directions. At the upper limit frequency $0.25 \times f_s$, the maximal RFE is as much as 23.5% whereas at the lowest frequencies the error is negligible. In practice this means that in simulations some of the mode frequencies are at correct locations, but most of them are distorted, the maximal displacement of mode frequencies being nearly one fourth of the actual frequency.
The main reason for the direction dependence of the dispersion error in the original WGM structure is the limited number of allowed wave propagation directions. Applying (1) it takes one time step for the excitation to reach an axial neighbor whereas the 3-D diagonal neighbors are at the distance of three time steps although their real distances are in the ratio of $1/\sqrt{3}$. Based on this observation the interpolated structure was invented. In that method the 2-D diagonal (see Fig. 1b) and 3-D diagonal (see Fig. 1c) neighbors of a node are connected to the center node. This results in a structure illustrated in Fig. 1d having connections to 26 neighbors plus one connection to itself. The difference equation to govern the new mesh structure is as follows

$$p(n+1,x,y,z)=\sum_{l=-1}^{1}\sum_{m=-1}^{1}\sum_{p=-1}^{1}h(l,m,p)\;\;p(n,x+l,y+m,z+p)-p(n-1,x,y,z) \tag{2}$$

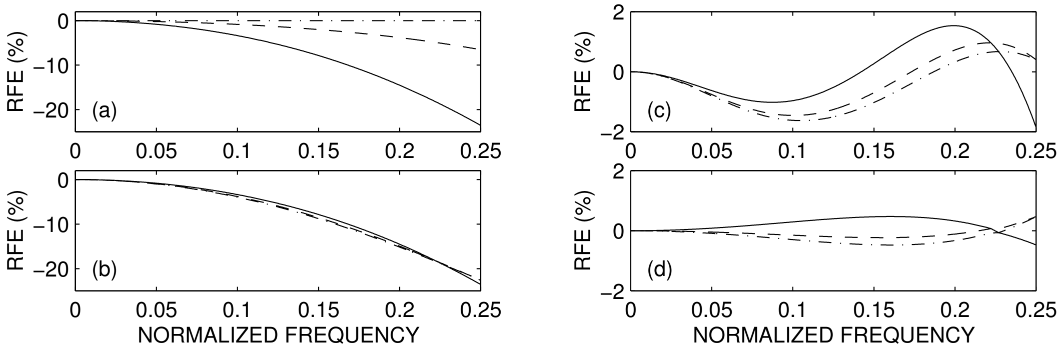


Fig 2. Relative frequency error in the digital waveguide mesh a) in the original structure, b) in the optimally interpolated mesh, c) in the interpolated structure with two-stage multiwarping, and d) in the interpolated mesh with warping in the frequency domain. The curves show RFE in axial (solid line), 2-D diagonal (dashed line), and 3-D diagonal (dash-dotted line) directions. Please note the different vertical scales in left and right columns.

where $h(l,m,p)$ are the weighting coefficients for different neighbor types. There are four different types of nodes, and each of them should have their own interpolation coefficient. The coefficients are denoted as follows:

$$h(l,m,p) = \begin{cases} h_a, \text{if } |l|+|m|+|p| = 1 \\ h_{2D}, \text{if } |l|+|m|+|p| = 2 \\ h_{3D}, \text{if } |l|+|m|+|p| = 3 \\ h_c, \text{if } |l|+|m|+|p| = 0 \end{cases} \tag{3}$$

For the original rectangular mesh the coefficient values are $h_a = 1/3$ and $h_{2D} = h_{3D} = h_c = 0$. The goal in the interpolated mesh is to obtain as uniform wave propagation characteristics as possible in all directions. By this structure it is not possible to cure the dispersion in the axial directions. Therefore we had to aim at making the behavior of other directions as similar as possible with respect to the behavior of axial directions, and finally correct the overall dispersion error with another method.

There are various strategies to find out the optimal interpolation coefficients [5, 6]. A practical method is to minimize the difference between the maximal and minimal RFE on a certain frequency band. For the frequency range $[0, 0.25 \times f_s]$, the following coefficient values are obtained

$$h_a = 0.12052 \quad h_{2D} = 0.03860 \quad h_{3D} = 0.01460 \quad h_c = 0.69686 \tag{4}$$

With the above-mentioned values the difference between the maximal and minimal RFEs is approximately 0.95%. This result is interesting but as such it does not solve the problem since there is still dispersion as illustrated in Fig. 2b. The benefit of the interpolation is that now the dispersion is nearly direction independent.

The interpolated rectangular mesh is not the only solution presented for this problem. Another interesting approach is utilization of different mesh topologies such as a tetrahedral mesh [4, 9, 10], which renders the dispersion nearly uniform in all directions as well. However, the tessellation of the space with tetrahedral elements is more complicated than usage of cube shaped elements. In addition, the required data structures are much simpler with rectangular mesh, and thus the implementation of the interpolated digital waveguide mesh is easier.

FREQUENCY WARPING WITH DIGITAL WAVEGUIDE MESH

The dispersion error obtained with the interpolated structure increases monotonically as a function of frequency. Therefore it is possible to compensate the error to a certain degree. A solution for this is a digital signal processing technique called frequency warping. It is possible to perform the warping either in the time domain or in the frequency domain [11]. Both ways are briefly discussed in the following.

Frequency warping in the time domain

The time-domain implementation of frequency warping is based on cascading digital allpass filters. For this purpose first-order filters are used. In practice this means that both the input and output signals of a WGM must be filtered with a warped FIR filter. The filter is an FIR filter in which all the unit-delay elements have been replaced by first-order allpass sections having the transfer function

$$A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}} \tag{5}$$

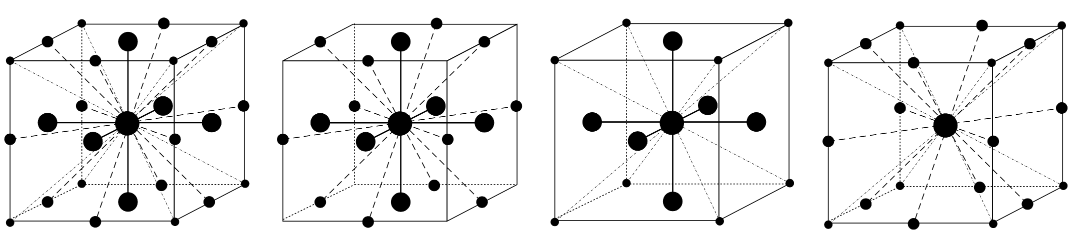
Applying this warping with the warping coefficient $\lambda = -0.252902$ yields a maximal RFE of 3.8% with the optimally interpolated rectangular mesh. The value of λ was optimized to reach the minimal maximum error. In the 2-D case even more accurate results were obtained, but for the 3-D case the accuracy should be still improved. One way to enhance the results is to cascade frequency warping and resampling operations, the technique called multiwarping [12]. With this technique having two consecutive warpings and resamplings the RFE can be reduced down to 2.0% as depicted in Fig 2c.

Frequency warping in the frequency domain

The most accurate results of 3-D WGMs are obtained when the frequency warping is performed in the frequency domain [11]. This technique is based on non-uniform resampling of the Fourier transformed signal [13]. The resampling intervals are obtained from the relative wave propagation speed curves given by the Von Neumann analysis. By using this technique with the optimally interpolated WGM the maximal RFE is only 0.47% as illustrated in Fig. 2d.

SPARSE MESH STRUCTURES

The full interpolated mesh with 27 connections is computationally more demanding than the original mesh. Therefore we have studied alternative solutions in which some of the interpolation coefficients have been set to zero thus effectively reducing the number of connections [11]. Various alternatives are illustrated in Fig. 3. The full structure is shown in Fig. 3a, and Figs. 3b and 3c depict cases where either 3-D diagonals or 2-D diagonals have been disconnected, and finally in Fig. 2d $h_a = 0$ resulting in disappearance of axial connections. Of those structures the one with $h_{3D} = 0$ has been shown to be the most effective. It is still quite accurate the maximal RFE being 1.4% with warping in the frequency domain, but the computational load is reduced by 30% [11].



(a) (b) (c) (d)

Fig 3. Sparse rectangular mesh structures based on pruning the a) full interpolated WGM structure. In b) the axial and 2-D diagonal neighbors are utilized, in c) axial and 3-D diagonals are effective, and in d) 2-D diagonals and 3-D diagonals are in use.

APPLICABILITY OF DIGITAL WAVEGUIDE MESH FOR ROOM ACOUSTIC SIMULATIONS

The digital waveguide mesh has been under active research since early 1990's, but it has not gained popularity as a tool for real room acoustic design. There are several reasons for that. One of them is surely the fact that the method has been under development all the time and no established version has been available. Although the implementation of the algorithms presented in this paper is quite straightforward, there should be a supported and stable release to make the method a practical tool for real design cases.

However, the most important defect lies in the method itself. There has been a lot of research on the behavior of the mesh and compensation of the direction-dependent dispersion error, and those issues are now in good control. The main problem currently is the behavior of the boundary of the mesh. Preliminary suggestions for setting the boundary conditions have been published [14], but no thorough analysis of the boundaries is available, yet.

With the current algorithms it is possible to make simulations to find out the mode frequencies of a room, but the Q values of the resonances are not yet reliable due to the boundary conditions. Therefore, the research should be continued to find out ways to set the desired material properties for boundaries. In the first phase, locally reacting surfaces should be covered, and after that non-local reactions should be investigated.

The WGM method is most suitable for studying the low-frequency behavior of large spaces. There are two reasons for this. The first one is the fact that the method is based on solving the wave equation, and thus it includes all the phenomena due to the wave nature of sound such as diffraction as illustrated in Fig. 4. The second reason is the computational simplicity of the method. If WGM is compared to traditional methods applied at low frequencies, such as FEM and BEM, this method is much lighter. On the other hand the modeling of boundaries has been studied much more with FEM and BEM, and there do exist methods and products applying the techniques.

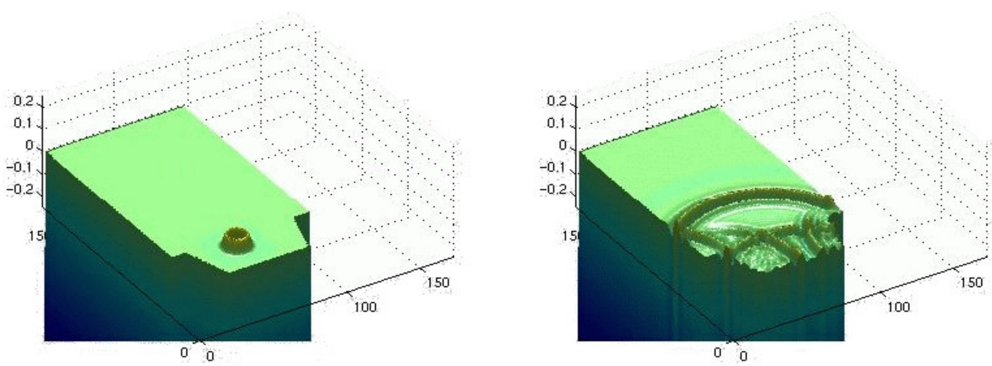


Fig 4. Simulation of a simple stagehouse. On the left the excitation is illustrated. On the right the primary wave front has reached halfway of the hall and diffractions from the edges are visible.

CONCLUSIONS

The interpolated 3-D digital waveguide mesh is a method suitable for simulation of low-frequency behavior of large spaces. It is a finite difference method operating in the time domain, and it has its origins in digital signal processing. The original method suffers from direction-dependent dispersion, but the effect of the error can be remarkably reduced by the use of more advanced mesh structures, such as the interpolated rectangular mesh or the tetrahedral mesh, combined with frequency warping. The method is computationally efficient, and if the accuracy is not crucial, even more effective but less accurate structures called sparse mesh structures can be utilized. The main unsolved problem with the digital waveguide mesh technique is the setting of boundary conditions. That research should be continued so that we get a new practical and efficient method for real room acoustic design.

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