ABSTRACT

This paper describes work done in the EC-project NABUCCO (GRD1-1999-10785) aiming at noise synthesis at the design stage. The NST methodology is based on splitting a system into different sources and corresponding transmission paths. This paper present the result of applying this methodology for noise prediction on a small axial fan mounted inside a cabinet. The airborne source strength of the fan is given by its measured sound power converted into an equivalent dipole force. The transmission path is described by a transfer function from this dipole force to external sound pressure. The transfer function is measured using reciprocal methods.

INTRODUCTION

A fan generates noise through a complex interaction (vibro+aero-acoustic) with its surrounding environment. The Noise Synthesis Technology (NST) is based on splitting this complex interaction into different sources and corresponding transmission paths. This paper presents the result of applying the NST for noise prediction on a single small axial cooling fan mounted inside a cabinet. As vibrations from the fan have been established to be of less importance only airborne sound is considered.

A SMALL AXIAL FAN AS A SOURCE OF SOUND

Small axial fans (blade diameter of approx. 15 cm) exist in two types. Pure axial fans and fans of mixed flow type. Here both types are considered.

From mechanical point of view a force from the fan blades excites the fluid particles. Because the blade curvature the resulting force isn’t parallel to the fan axis instead it’s rotating with a certain angle to it. Since the rotating force will fluctuate in time due to flow disturbances, unsteady forces are created which can be seen as acoustic sources of dipole type. For strictly in-duct fans previous research [1, 2] show that a single dipole, oriented in the axial direction of the fan, serve as a good model in the plane wave range of the duct.

In order to quantify the difference between the real fan and the single dipole model of it, the acoustic field generated by a freely mounted fan is examined using an expansion into spherical harmonics.
Coherent Power Output

For axial symmetry the complex sound pressure can be written, omitting the harmonic time dependence $e^{i\omega t}$, as a sum of spherical harmonics

$$\hat{p}_n = \sum_{n=0}^{N} r^n A_n(\theta),$$

where $A_n(\theta)$ is a complex valued and frequency dependent amplitude factor of order $n = 0, 1, 2, ..., N$, and $P_n(\cos \theta)$ the real valued Legendre polynomial of the same order $n$. The zero order harmonic, $n = 0$, corresponds to a radiation pattern identical to the radiation pattern of a monopole. The first order harmonic corresponds to the radiation pattern of a dipole.

In order to determine the amplitudes, $A_n(\theta)$, the freely mounted fan is placed in an anechoic environment. The orientation of the first order harmonic, i.e., the dipole, is assumed to be in the axial direction of the fan. By positioning a microphone at a fixed reference point along that direction and measure its autospectrum $\hat{G}$ and also measure the transfer functions $\hat{H}$ the complex pressures are given by: $\hat{p}_n = \sqrt{\hat{G}_n} \cdot \hat{H}_n$. Using this technique only the coherent part of the pressure is present in $\hat{p}_n$. To reduce the influence of errors the amplitudes are determined by solving an overdetermined equation system $A_n(\theta) = P_n^{-1}(\cos \theta_n) \hat{p}_n$ in a least square sense.

The coherent power of each harmonic is given by

$$\overline{W}_n = \int \overline{I}_n \cdot dS = \int I_{r,n} dS$$

where $S_0$ is the surface of a sphere surrounding the source. Assuming a far field approximation the time averaged intensity in radial direction, $I_{r,n}$, is

$$I_{r,n} = \frac{1}{2} \Re \left( \frac{\hat{p}_n^2}{\rho \alpha} \right)$$

where $\rho_s$ is the static fluid density and $c$ is the speed of sound. For the zero order harmonic this approximation introduces no error in the intensity. For the first order harmonic it gives an error of less than 0.5 dB in the whole frequency range of interest using a measurement radius of 2 meter.

Combining the equations above and carrying out the integration gives the final expression for the coherent power of each order

$$\overline{W}_n = \frac{2\pi r^2 |A_n(\theta)|^2}{\rho \alpha (2n+1)}.$$

From measurements on a fan of mixed flow type the coherent sound power levels shown in figure 1 were obtained. As contributions from higher orders are negligible only the zero and first order coherent sound power levels are shown. The broadband part of the coherent power is mainly of the first order, i.e., of dipole character. However, some of the tones are zero order dominated. Take the bladepassing frequency at 340 Hz for example.
Directivity

Based on the third-octave sound pressure levels at each measurement point, the directivity is plotted in figure 2. In the frequency range $100 \text{Hz} \leq f \leq 1 \text{kHz}$ where tones dominate the spectrum, the directivity is almost omnidirectional for all third-octaves. Looking at higher frequencies an even more omnidirectional pattern will occur. The fan show a directivity similar to that of a single monopole.

Total And Coherent Power

As different orders are orthogonal to each other the total coherent power is calculated from

$$\tilde{P}_{\text{coh}} = \sum_{n=0}^{N} \tilde{P}_n.$$  

Summing over $n = 0, 1$ the total coherent power is compared to the true total power in figure 3. The total coherent power is always less than the true total power.

Source Model

In order to interpret these results knowledge of the source mechanisms is essential. Because the rotation of the resulting force the dipole changes its direction continuously. Measuring at stationary conditions, i.e., time averaged quantities, the field will therefore appear to be that of a combination of monopole and dipole contribution. But the source mechanism isn’t that of a monopole and therefore no monopole shall be included in the model. Instead, the fan can be modelled by several fixed and partly coherent dipoles by dividing the resulting rotating dipole into orthogonal components. Unfortunately the above analysis doesn’t give sufficient information to establish this model.

Restricted to in-duct fans and below the cut-on frequency of the first higher order mode of the duct, a single dipole oriented in the axial direction of the fan will serve as a good model. In order to capture the total power of the fan, all of its power will be put into this single dipole. The strength of a dipole is given by its dipole force, $\tilde{F}^2 [N^2]$, therefore the total sound power of the fan will be converted into the force of a dipole in free space using the ordinary textbook formula

$$\tilde{F}^2 = \frac{12 \pi \rho c}{k^2}, \quad k = \frac{\omega}{c}. \quad (1)$$

Using the NST terminology dipole force is the Air-Borne Component Source Strength, $CSS_{AB}$, of an axial fan.
AIRBORNE TRANSMISSION PATH

A harmonic force acting on a fluid at the source point gives rise to a harmonic pressure at the receiver point. This can be written as

\[ \ddot{p}_s = H_{F,p} \ddot{F}_s, \]

where \( H_{F,p} \) is the transfer function between force at the source point to pressure at the receiver point. For a single source an energy formulation can be used. Using the NST terminology the expression above is then transferred into

\[ \ddot{p}_s = |H_{F,p}|^2 \ddot{F}_s = ACCF_{AB} \cdot CSS_{AB} \]

where \( ACCF_{AB} = |H_{F,p}|^2 \) is the airborne assembly conductivity function. Assuming linear conditions different \( CSS_{AB} \) can be multiplied by the \( ACCF_{AB} \) in order to predict the sound pressure at different running conditions.

Reciprocal Measurement Of The Transfer Function

Forward measurement of the \( ACCF_{AB} \) on a real structure requires a source of dipole type, e.g., a loudspeaker. Mounting a loudspeaker large enough to deliver power in the whole frequency range of interest is sometimes not possible or at least time-consuming, especially if several source positions shall be tested. Instead microphones can be used by taking use of Lyamshevs reciprocity relation [3] which read

\[ \frac{\ddot{p}_r}{\dot{F}_s} = \frac{\ddot{v}_r}{Q}. \]

For reciprocity to hold there must be no flow. However, in applications using small axial fans the Mach number is normally small and the no flow condition is therefore approximately satisfied. The \( ACCF_{AB} \) is now measured reciprocally by positioning a source of monopole type, delivering acoustic volume flow \( \dot{Q} \), at the receiver point and then measure the particle velocity \( \ddot{v} \) at the source point.

From the equation of motion the particle velocity can be approximated from the pressure difference between two points symmetrically positioned at a distance \( \Delta r/2 \) from the geometrical center of the fan blades, which is defined to be the source point. The distance between these two points determines the upper frequency limit of the velocity approximation.

The final expression for the airborne assembly conductivity function becomes

\[ ACCF_{AB} = \frac{\ddot{p}_s^2}{F_s^2} = \left( \frac{\ddot{v}_s}{Q_s} \right)^2 = \left| \frac{H_i - H_j}{\rho_0 \omega^2 (\Delta r)^2} \right|^2, \quad H_i = \frac{\ddot{p}_r}{Q_r}, \quad i, j = 1, 2. \]

Because of the small dimensions of the fan relative to the wavelengths of the frequencies of interest, there is no need for it to be present when measuring the airborne \( ACCF \) reciprocally. Its absence will not change the reflection of sound waves dramatically. The absence of flow when measuring the \( ACCF \) will result in a more resonant \( ACCF \) compared to the "true" \( ACCF \) since normally flow introduces damping.

DETERMINATION OF SOURCE STRENGTH

The source strength of an axial fan depends on: Rotation speed, Load and In/Outlet flow conditions. In order to measure the right source strength, the status of these parameters must be the same in the test-rig as in the real structure. A test-rig for small fans already exists and is described in ISO 10302. This rig allows us to load the fan as it blows into a wooden-frame box covered with a thin "acoustically transparent" plastic film.
Baffling Effect

The fan is mounted to the box on a thick rubber-cloth in order to support it and avoid vibrations spreading from it. If the box truly were acoustically transparent, i.e., not affecting the radiation impedance of the fan, then the measured sound power would be directly transferable into dipole force using equation 1. Unfortunately it’s not. The rubber-cloth acts as a finite baffle.

As the source mechanism is of dipole type and the direction of the main dipole is normal to the rubber-cloth, the sound power levels at low frequencies will increase because the baffling effect. The measured sound power will therefore not correspond to a dipole in free space, which is our model of the fan.

The difference in measured sound power levels due to the box is given by

\[ \Delta L_W = L_{W,\text{box}} - L_{W,\text{free}} \]

where \( L_{W,\text{box}} \) is the sound power measured when the fan is mounted to the box and operated at a negligible pressure drop. In that way no load is added which is similar to the measurement of \( L_{W,\text{free}} \) where the same fan is mounted on a stand. A thin plastic film large enough to prevent flow from going directly from the outlet to the inlet is attached in order to achieve the same flow conditions as in the measurement of \( L_{W,\text{box}} \). The plastic film is assumed to be acoustically transparent.

Figure 4 shows \( \Delta L_W \). The baffling effect of the box is clear since the broadband levels at low frequencies are amplified. By fitting a curve to the broadband part, previously shown to be of dipole character, a correction curve is achieved. At certain frequencies \( \Delta L_W = 0 \). These frequencies coincide with the monopole-dominated frequencies in figure 1. But the source mechanism is not of monopole type. Instead there is this rotating dipole changing its direction continuously meaning that it sometimes is almost in parallel with the baffle and therefore not affected by it.

Based on the above results the sound power of a fan, measured at a certain running speed and load, can be divided into a monopole and a dipole part. The monopole part contains the frequencies observed to be monopole-dominated, e.g., the bladepassing frequency. The baffle doesn’t affect these frequencies and therefore no correction shall be applied. The dipole part contains the dipole-dominated frequencies to which the correction shall be applied. Finally, the monopole part and corrected dipole part are added and the result converted into the force of a free dipole using equation 1.

In And Outlet Flow Conditions

If the right source strength shall be measured in the ISO-box then the in/outlet flow conditions must be similar to those in the real structure. By manufacturing exact copies of the in and outlet ducts of the structure, flow conditions close to these can be expected. In order not to change the radiation impedance of the fan, the ducts were made acoustically transparent by covering a steel-bar frame with a thin plastic film.
The effect of the transparent ducts is principally higher broadband levels. The tones are not affected which means that the flow profile still is quite good and also that the ducts can be considered as acoustically transparent.

COMPARISON BETWEEN MEASURED AND PREDICTED SOUND PRESSURE

A prediction of the sound pressure level at a receiver point, according to the above-described procedure, is now compared with the measured one in figure 5. The structure is a cabin used for cooling electronic equipment.

The slight shift in frequency at the tones is due to differences in the running conditions. Maybe the load on the fan or the voltage connected to it wasn’t exactly the same in the ISO-box as in the real structure. The modal behaviour of the prediction shows the same behaviour as the measurement meaning the source character truly is of dipole type. The more resonant behaviour of the prediction is probably due to the absence of flow when measuring the ACF. The discrepancy at 1.6 kHz is most likely created by flow induced noise. The overall slightly lower broadband levels are expected since the flow profile in the real structure most likely is worse than the one created by the transparent ducts. The discrepancy at 125 Hz is due to the fact that this tone, which is the second rotation frequency, is slightly more of monopole-type and therefore excluded from the baffling correction. A refinement of the baffling correction when a tone is about equally much of dipole-type as of monopole-type would probably improve the prediction in this frequency-band.

CONCLUSIONS

An axial fan can be modelled by an equivalent dipole oriented in the axial direction of the fan. When the sound power of a fan is measured according to ISO 10302 the right flow condition can be simulated using transparent ducts. In order to convert the measured sound power into the force of the equivalent dipole, the baffling effect of the ISO-box must be corrected for. Multiplying the resulting force with a transfer function describing the airborne transmission path, predicts the sound pressure very satisfactorily.

REFERENCES