REGULARIZATION OF EXACT BOUNDARY INTEGRAL EQUATION FOR CALCULATION OF PULSED WAVE DIFFRACTION ON STRONGLY CURVED SURFACE

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ABSTRACT

The regularization of exact boundary integral Fredholm's equation is proposed in the report. This approach allows to calculate the scattered or diffracted pulsed wave field on strongly curvilinear surfaces for practically arbitrary geometry. Mathematically the essence of the method consists in a replacement of exact Fredholm's integral equations by their truncated analog, in which the contributions of geometrically shadowed areas are eliminated. This approach has a deep physical sense and allows to obtain the correct solutions when the direct numerical solution of exact integral equations leads to unstable results.

The most powerful method for solution of diffraction problems of acoustical, electromagnetic or seismic waves on curved surfaces is the method of integral equations. In the framework of this method the problem is reduced to solution of exact Fredholm's integral equations of first or second kind for wave field or its normal derivative taken on the scattered surface. For the pulsed wave fields, possessed the wide spectrum of wavelengths, this method is in essence the only one, which permits to obtain the solutions as in domains of short and long wavelengths as well as in the intermediate resonance domain

The two-dimensional problem of scattering a scalar wave field on the curvilinear boundary of half-space is considered in the report. This statement from the physical point of view corresponds to an acoustical approximation for signal scattering at the curvilinear boundary of elastic media.

In accordance with the Green's theorem the relation between wave field in internal points of medium and values of its normal derivative and itself on the curved surface is represented by integral

$$U(\vec{r}) = U_0(\vec{r}) + \int_S dS \left\{ U(\vec{r}_S) \frac{\partial}{\partial n_S} G(\vec{r}, \vec{r}_S) - G(\vec{r}, \vec{r}_S) \frac{\partial}{\partial n_S} U(\vec{r}_S) \right\} \qquad (1)$$

where $U_0(\vec{r})$ is the initial wave field from a source, and $G(\vec{r}, \vec{r}_S)$ - is the Green's function of free space.

From (1) it is possible to derive the Fredholm's equation of the second kind for the wave field or its normal derrivative on the curved surface. For example for the Dirichlet's boundary condition $\left(U(\vec{r}_S)=0\right)$ it will be equation for normal derivative $V(\vec{r}_S)=\frac{\partial}{\partial n_S}U(\vec{r}_S)$, which has the following form:

$$V(\vec{r})=2V_0(\vec{r})-2\int_S dS\,V(\vec{r}_S)\frac{\partial}{\partial n}G(\vec{r},\vec{r}_S) \tag{2.1}$$

For the case of Neuman's boundary condition $\left(\frac{\partial}{\partial n_S}U(\vec{r})\right|_{\vec{r}=\vec{r}_S}=0)$ the following Fredholm's equation of the second kind can be derived from (1) for wavefield on the boundary:

$$U(\vec{r})=2U_0(\vec{r})+2\int_S dS\,U(\vec{r}_S)\frac{\partial}{\partial n_S}G(\vec{r},\vec{r}_S) \tag{2.2}$$

The integrals by surface in (2) have to be considered in the sense of its principal values.

It is possible to obtain from (1) the diagram of radiation pattern $D(\varphi,t)$, which describes angular (φ) distribution of pulsed wavefield in the far wave zone. For the case of the Dirichlet's boundary condition it has a view:

$$D(\varphi,\omega)=1+\frac{i}{4}\int_S dS\;e^{-ik\left|\left(\vec{r}_s-\vec{r}_0\right)\right|\cos\left(\varphi-\varphi_s\right)}\,V(\vec{r}_S) \tag{3.1}$$

Similarly for the case of the Neuman's boundary condition one can be written.

$$D(\varphi,\omega)=1+\frac{i}{4}\int_S dS\;\cos\left(\varphi-\varphi_n\right)e^{-ik\left|\left(\vec{r}_s-\vec{r}_0\right)\right|\cos\left(\varphi-\varphi_s\right)}\,U(\vec{r}_S) \tag{3.2}$$

The numerical modeling of a wave field scattering by a curvilinear surface is usually based on integral equations mentioned above. Thus, the problem is reduced to solution of the equations (2). Then, substituting the obtained solution for the field on the surface or its normal derivative into relation (1), it is possible to calculate a wave field in any point of a half-space with the curvilinear boundary or to determine the radiation pattern (3).

It is clear that the efficient application of the given approach for arbitrary scattered surface is possible with use of numerical methods of solution. The numerical solution of the equations (2) can be obtained by the iteration method or by the method of finite-difference approximation of the Fredholm's integral equations of second kind (2), reducing them to a system of the linear algebraic equations for wave field in the certain boundary grid nodes.

For enough smoothed irregularities of the surface the solution of the integral equations does not bring any difficulties. For example, the solution of the problem on the pulsed reflected wavefield, generated by explosion under mountain of gaussian shape (fig.1) with height to width ratio equaled to 0.5, is shown in fig.2.

However, in case of the large slopes and curvatures of the surface irregularities in comparison with characteristic wavelength, the described methods became to be unstable and result to incorrect solutions. It occurs because for large slopes and curvatures of irregularities the multiply reflected on the surface waves begin to give essential contribution and there is also the contribution of mutually shadowed points of the surface. This contribution is cancelled in different orders of multiple scattering series corresponding to different orders of iterations [1]. Therefore the iterative process gives an incorrect result.

Thus, the necessity appears for regularization of initial integral equations (2)

To overcome this difficulty we suggest to eliminate certainly the contribution of mutually shadowed points of the surface that corresponds to the physics of the real wave field propagation.

If, following to [1], to estimate a solution of the problem of plane wave field scattering on an arbitrary curvilinear surface by the stationary phase method in the short-wave approximation, then after the first iteration it is possible to see, that the field is equal to the field of external source and the sum of contributions of all stationary phase points. Further there are two types of stationary phase points: the reflecting one and shadowed one.

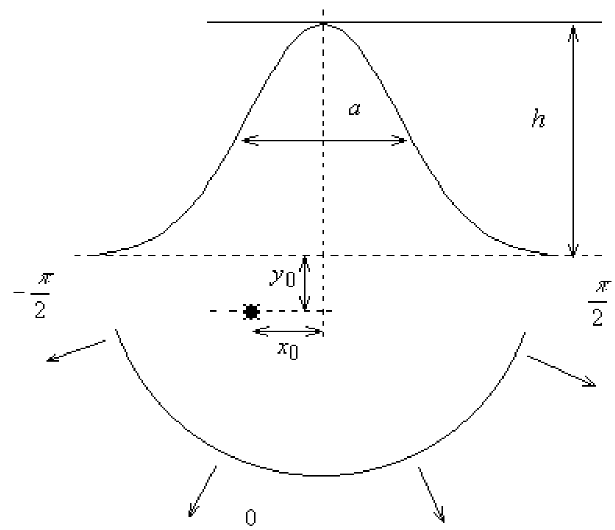

Fig. 1. The problem geometry.

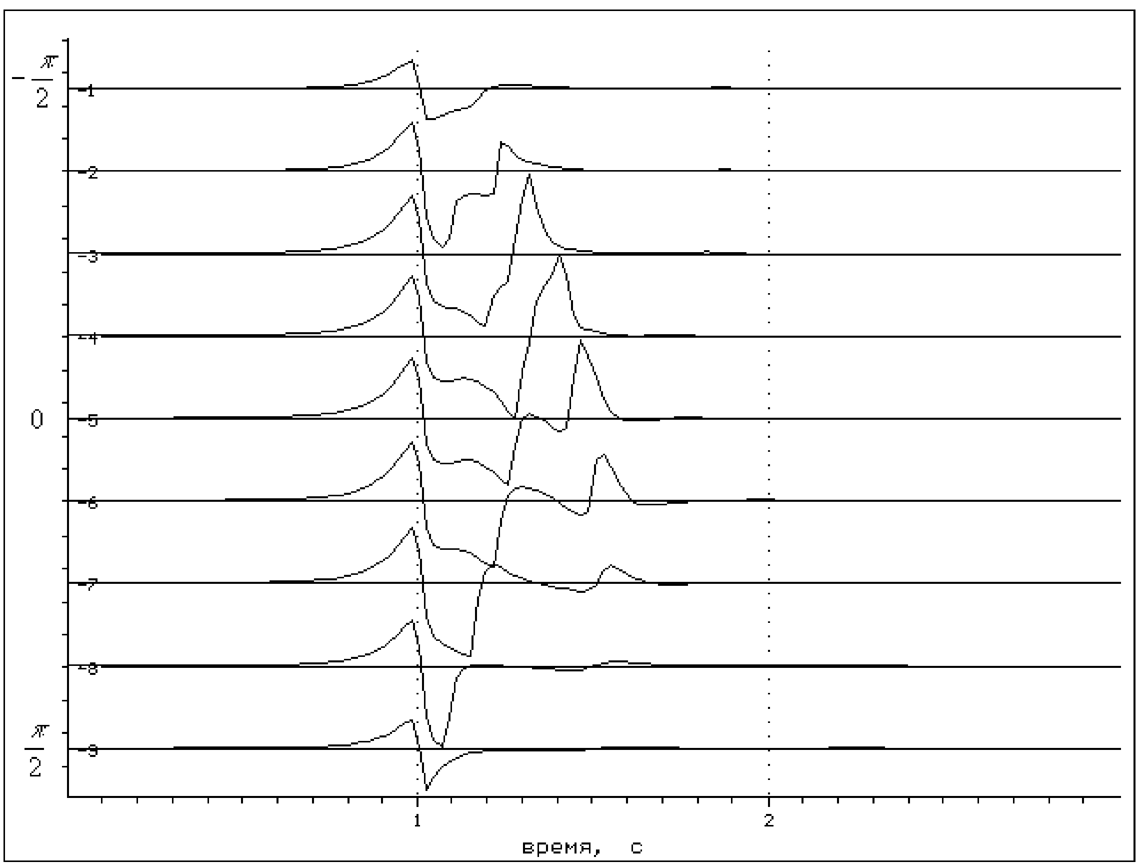

Fig. 2. The pulsed radiation diagram for the explosion under mountain in geometry is shown in fig.1 for $h/a = 0.5$.

The contribution of shadowed stationary phase points is cancelled at the next iteration. Therefore at any finite number of iterations it is not possible to achieve the exact solution. This contribution becomes to be important at account of contribution of multiply reflected waves.

The suggested regularization method is based on the approach when at solution of the integral equations the contribution of the source is taken into account only for the geometrically visible areas from the source, and the integration is fulfilled only on the mutually non-shadowed areas of the surface. To take into account the wavefield diffraction the visible areas were extended on a value proportional to wavelength. Thus, the contribution of mutually shadowed points of the surface is eliminated in the integral equation that corresponds to the physical picture of the real wavefield propagation. For more accurate description of the field currents in the light - shadow vicinity the approach suggested by Fock in [2] is used. Because the stationary phase points are closed one to another in the light - shadow vicinity, the wavefield in this area has a local character. It was shown in [2], that for arbitrary surface the field in the light - shadow vicinity is proportional to universal Fock's function, for which there are tables and asymptotic representations.

The developed approach has allowed us to obtain the correct solutions for surfaces with arbitrary large slopes and curvatures of irregularities. For example, the solution for the problem of explosion under mountain of gaussian shape with height to width ratio equaled to 3 is shown in fig. 3, which can not be calculated using the traditional approaches.

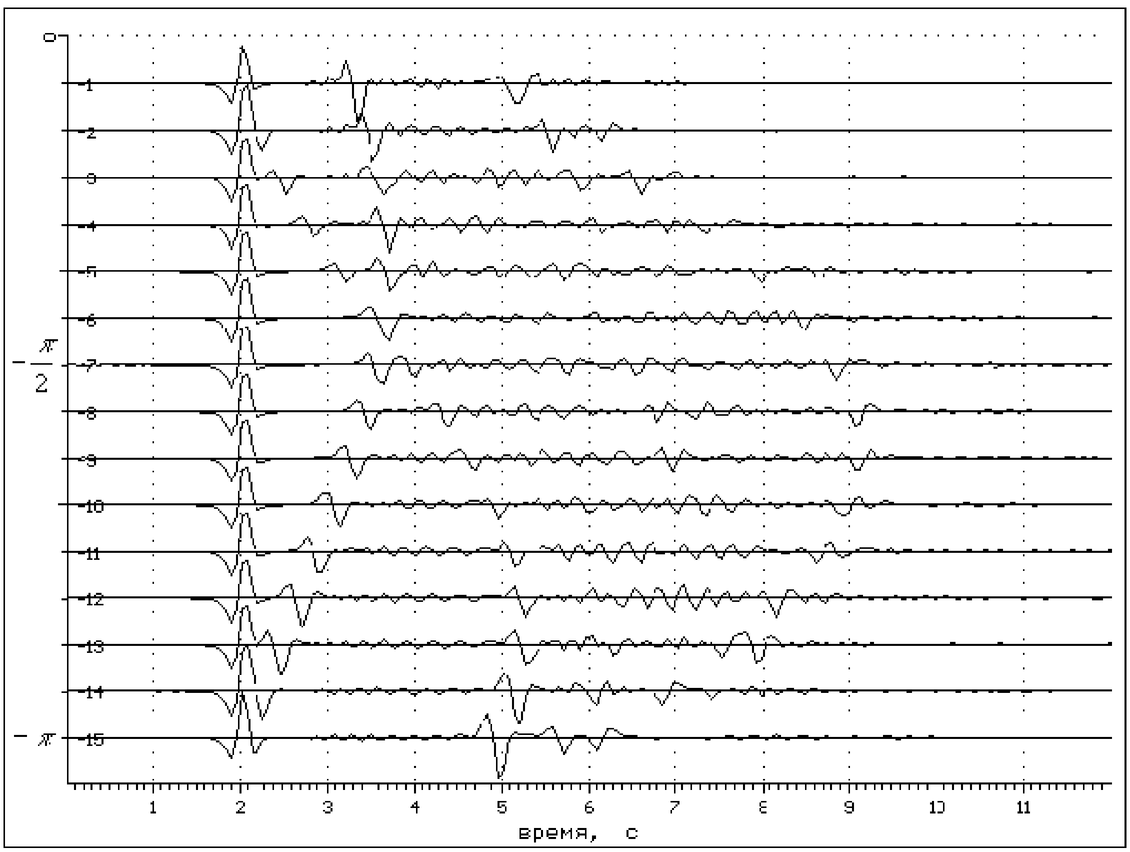

Fig. 3. The pulsed radiation for the explosion under mountain in geometry is shown in fig.1 for $h/a = 3$.

It is also shown in the report, that represented regularization method can be correctly derived from a mathematical point of view with use of the exact integral equations.

Let's consider the initial integral equation (2) relatively the field on the surface V with integral operator L

$$V = 2V_0 + LV \tag{4}$$

Let's divide the integral operator on two parts $L = L^{+} + L^{-}$, where L^{+} is operator dealt with geometrical optics contribution from visible area and L^{-} is operator dealt with the same contribution from shadowed area. Analogously we can divide the initial field of source $2V_0 = 2V_0{}^{+} + 2V_0{}^{-}$.

Then on the first iteration of equation (4) we have

$$V = 2V_0{}^{+} + 2V_0{}^{-} + L^{-}2V_0{}^{+} + L^{+}2V_0{}^{+} + L\,2V_0{}^{-} + L^2 V \tag{5}$$

Taking into account, further, that in the short wave approximation the contributions of shadowing points conceal in different terms of series [1], we can require the execution of condition

$$L^{-}2V^{+}{}_0 = -2V_0{}^{-}. \tag{6}$$

This condition can be considered as integral equation relatively unknown function V_0^{+}

$$V_0^{+} = V_0 + L^{-}2V_0^{+} \tag{7}$$

After simplification of equation (5) with condition (6) we can obtain:

$$V = 2V_0{}^{+} + L^{+}2V_0{}^{+} + L(2V_0{}^{-} + LV)$$

Iteration of this equation leads to the following representation of the solution:

$V = 2V_0{}^{+} + V^{1}$, where $V^{1} = 2L^{+}V_0{}^{+} + LV^{1}$

Analogously for V^n it is possible to obtain $V^{n} = L^{+^{n}}2V_0{}^{+} + \sum_{k=1}^{\infty} L^{k-1}L^{+}L^{+^{n}}2V_0{}^{+}$

Thus, we obtain the series which does not contain the contributions of mutually shadowed areas:

$$V = 2V_0{}^{+} + L^{+}2V_0{}^{+} + L^{+^{2}}2V_0{}^{+} + L^{+^{3}}2V_0{}^{+} + \cdots = 2V_0{}^{+} + \sum_{n=1}^{\infty} L^{+^{n}}2V_0{}^{+}$$

Hence the obtained series is the solution of the following integral equation:

$$V = 2V_0{}^{+} + L^{+}V$$

LITERATURE

1. Liszka E.G., McCoy J.J. Scattering at a rough boundary - Extensions of the Kirchhoff approximation. The Catholic University of America, Washington, 1981.
2. Fock A.A., Problems of diffraction and propagation of electromagnetic waves. “Soviet Radio ”, Moscow. 1970.