ABSTRACT
Different methods are discussed to describe the diffraction of light by monofrequent, pulsed and adjacent superposed ultrasound. The modulation of light (amplitude and phase) is very sensitive to variations in the ultrasonic parameters, whence the acousto-optic interaction is extremely useful in non-destructive testing. In the optical nearfield of a diffracted laser beam, it is even possible to reconstruct all of the acoustic wave parameters. The ability of measuring the amplitude and phase of reflected and transmitted waves is a powerful tool to discover information about the surface on which scattering of sound occurs.

INTRODUCTION
It was not until laser light was ready to be used in the 1960’s that the scattering of light by sound, first predicted by the French Léon Brillouin in 1921 [1] and first experimentally observed by the French Lucas and Biquard and the Americans Debye and Sears in 1932 [2,3], became a hot topic of investigation by many scientists. The first theoretical explanations of the Acousto-Optic effect came from the French Léon Brillouin (1931) [4] and the Indians Raman and Nagendra Nath (1935) [5-10]. The often called ‘Flemish School of Acousto-Optics’ [11] finds its roots at the Ghent University around 1950 [12] and has continued to exist in Belgium at the Ghent University (R. Mertens et al) and the KUL University Campus Kortrijk (O. Leroy et al.) from 1972. This paper does not cover the complete field of acousto-optics. It is our aim to describe some basic ideas of the problem in acousto optics and to present just some of the discoveries of Leroy and co-workers, from his early years at the Ghent University until his accorded emeritus status at the KUL University Campus Kortrijk.

SOME TOPICS OF LIGHT DIFFRACTION BY ULTRASOUND

Diffraction of light by monofrequent ultrasound (general Raman – Nagendra Nath Theory [8])
The configuration is shown in figure 1. The sound travels in the xdirection while the light is incident in the xz-plane and is diffracted by the sound. The different parameters used are shown in table 1. If the sound frequency is of the order of MHz, then the electric field of the laser light in the sound column [0,L] can be written as
\[ E(x, z, t) = e^{i\omega \Phi(x, z, t)} \]  

**Figure 1:** The geometrical configuration

| \( \omega = 2\pi v \) | = angular light frequency |
| \( \omega^* = 2\pi v^* \) | = angular sound frequency |
| \( \mu_0 \) | = refractive index in undisturbed media |
| \( \mu \) | = maximum variation of the refractive index from \( \mu_0 \) |
| \( k = 2\pi/\lambda \) | = wave number of the light |
| \( k^* = 2\pi/\lambda^* \) | = wave number of the sound |
| \( \vartheta = \phi = \varphi \) | = different notations for the angle between a light ray and the z-axis |

**Table 1:** Description of the applied symbols

With \( \Phi(x, z, t) \) a solution of the Helmholtz wave equation [8,12,13]

\[ \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{\mu^2(x, t)}{c^2} \frac{\partial^2 \Phi}{\partial t^2} \]  

With boundary conditions

\[ \Phi(x, 0, t) = \Phi_0(x, 0) = e^{-i\mu_k x \sin \Theta} \]  
\[ \left( \frac{\partial \Phi}{\partial z} \right)_{z=0} = -i\mu_0 k e^{-i\mu_k x \sin \Theta} \]

With

\[ \Phi_0(x, z) = e^{-i\mu_k (x \sin \Theta + z \cos \Theta)} \]

valid in the region \( z<0 \) and a solution of (2)

Hence

\[ E_0(x, z, t) = e^{i[\omega - \mu_k (x \sin \Theta + z \cos \Theta)]} \]  

Then, we write

\[ \Phi(x, z, t) = \Phi_0(x, z) \Phi_1(x, z, t) \]

whence \( \Phi_1(x, z, t) \) adopts the periodicity of the ultrasound [9] and, due to symmetry considerations, it can be expanded in a discrete Fourier transform [8]

\[ \Phi_1 = \sum_n \Phi_n(z, t) e^{inkx} \]

The single progressive sound wave will result in

\[ \mu(x, t) = \mu_0 + \mu \sin 2\pi \left( \frac{v^* t - x}{\lambda^*} \right) \]
which, if we neglect terms containing $\mu^2$, ultimately gives the famous differential-difference set of equations [9]

$$2 \frac{d\Phi_n}{d\xi} - \left( \Phi_{n-1} e^{ia\xi \sin \phi} - \Phi_{n+1} e^{-ia\xi \sin \phi} \right) = \frac{in^2 \lambda^2 \cos^2 \phi}{\mu_0 \mu_0^2} \Phi_n$$

(9)

in which

$$\xi = \frac{2\pi \xi}{\lambda} (x \sin \phi + z \cos \phi)$$

(10)

Relation (9) is often written in the form

$$2 \frac{d\Psi_n}{d\xi} - \Psi_{n-1} + \Psi_{n+1} = in(\rho^2 - 2a \sin \phi)\Psi_n$$

(11)

In which

$$\rho^2 = \rho \cos^2 \phi$$

(12)

$$\rho = \frac{\lambda^2}{\mu_0 \mu_0^2}$$

(13)

$$\Phi_n = \Psi_n e^{ia\xi \sin \phi}$$

(14)

Solving equations (11) or extending them to more complicated situations has been one of the main research topics in Acousto-Optics since 1936.

$\rho$ is called the ‘diffraction regime parameter’ and

$$Q = \frac{2\pi L}{\lambda^2 \cos \phi}$$

(15)

is the Klein-Cook parameter. Two extremity regimes exist. The Raman-Nath regime is defined as the situation in which $\rho = 0$, involving low frequencies (< 5 MHz) and leading to the generation of many diffraction orders, cfr. figure 2.

![Figure 2: Diffraction in the Raman-Nath regime](image)

The Bragg regime is defined as the situation in which $\rho \geq 5$, involving high frequencies (>15MHz) and generating a zero order and a first order diffracted light beam, cfr. figure 3.

![Figure 3: Diffraction in the Bragg regime](image)
**Diffraction of light by superposed ultrasound (a pulse)**

If the ultrasonic wave is built up by a fundamental tone $v^*$ and N-1 harmonics, then (8) becomes

$$\mu(x, t) = \mu_0 + \sum_{n=1}^{N} \mu_n \sin \left[ 2\pi \left( v^* t - \frac{y}{\lambda} \right) + \delta_n \right]$$

$\mu_n$ being the maximum variation of the refractive index caused by the nth harmonic, having a phase shift $\delta_n$. Substitution of (16) in the wave equation (2) and expanding the function in a Fourier series gives:

$$2 \frac{d\Psi_n}{d\xi} - \sum_{r=1}^{N} \frac{\mu_r}{\mu_1} \left[ \Psi_{n-r} e^{-i[r\alpha t + \delta]} - \Psi_{n+r} e^{i[r\alpha t + \delta]} \right] = i n \rho^{2} - 2a \sin \phi) \Psi_n$$

in which

$$\rho = \rho \cos^2 \phi ; \quad \rho = \frac{\lambda^2}{\mu_0 \mu_1 \lambda^2} ; \quad \xi = \frac{2\pi \mu_1}{\lambda} (x \sin \phi + z \cos \phi) ; \quad a = \frac{\lambda}{\mu_1 \lambda^2}$$

The set of equations (17) is called the generalized Raman-Nath system.

Now, we consider a generating function $G(\xi, \eta) = \sum_n \Psi_n(\xi, \eta) \eta^n$

Multiplication by $\eta^n$ and summation in (17) then produces

$$2 \frac{\partial G}{\partial \xi} - F(\eta) G + 2i a \eta \sin \theta \frac{\partial G}{\partial \eta} = 0$$

with

$$F(\eta) = \sum_{r=1}^{N} \frac{\mu_r}{\mu_1} \left[ \eta^r e^{-i[r\alpha t + \delta]} - \eta^{-r} e^{i[r\alpha t + \delta]} \right]$$

and boundary condition

$$G(0, \eta) = 1$$

Equation (20) was solved, from which all amplitudes $\Psi_n(\xi, \eta)$ could be analytically expressed.

**Diffraction of light by adjacent sound**

(8) is now of the form

$$\mu(x, t) = \mu_0 + \mu_1 [H(z) - H(z - L)] \sin 2\pi \left( v^* t - \frac{y}{\lambda} + \delta \right) + \mu_2 [H(z - L)] \sin 2\pi \left( v^* t - \frac{y}{\lambda} + \delta \right)$$

and for $\Theta = 0$, the system of differential equations is found and an analytical solution is obtained for $\rho = 0$. The result is amenable to study the influence of the phase difference between the two adjacent sound waves.

**THE RECONSTRUCTION OF THE ACOUSTICAL PARAMETERS**

Multiple angle incidence method

If a convergent light beam penetrates the sound column, then after diffraction, in some discrete directions, it is found that the light is an exact image of the sound that causes the diffraction. This phenomena has been used to get an image of an object that disturbs the sound in the sound column in the vicinity of the penetrating converging laser beam [17]. This effect has been studied theoretically by considering light beams incident simultaneously in 2N angles that are a multiple of the smallest Bragg angle $\Theta_B$. It has been found that in the direction $N\Theta_B$ the light is a perfect analytical representation of the sound through which the light beams are diffracted [18].
Optical near-field method

The region \( z \geq L \) but very close to the sound column is called the optical near field or Fresnel region. In that zone, the diffracted orders are not (yet) separated from each other. If the sound column is a pulse described by

\[
\mu(x, t) = \mu_0 + \sum_{n=1}^{N} \mu_n \sin \left[ \frac{2\pi n (v^* t - \frac{x}{\lambda^*}) + \delta_n}{\lambda} \right]
\]

(24)

then it is found [19] that in the optical nearfield \( z = L \) the sound intensity is given by

\[
I = 1 + C_0 \sum_{n=1}^{N} v_n n^2 \sin \left[ \frac{2\pi n (v^* t - \frac{x}{\lambda^*}) + \delta_n}{\lambda} \right]
\]

(25)

Whence in the optical nearfield, the interference of all the diffracted orders create an exact image of the ultrasound.

SOME APPLICATIONS IN NON DESTRUCTIVE TESTING

Roughness measurement of a corrugated surface

On the left side of figure 5, the effect of the roughness on the phase of the reflected sound for Rayleigh angle incidence is measured using the Acousto Optic technique, while the right side of figure 5 shows a picture obtained from an electron microscope [21].

Investigation of a diamond coating on a glass plate

On the left side of figure 6, the effect of the absence or presence of a diamond coating (on a glass plate) on the phase of the reflected sound was measured acousto-optically for Lamb angle incidence. A comparison can be made with the result of a roughness measurement using a needle (right side of figure 6) [20].
ACKNOWLEDGEMENTS
Nico F. Declercq is sponsored by ‘The Flemish Institute for the Encouragement of the Scientific and Technological Research in Industry (I.W.T.)’

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