SURFACE AND INTERFACE WAVES IN PIEZOELECTRIC SANDWICH PLATES

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ABSTRACT
The goal of the present work deals with the analysis the propagation of shear horizontal (SH) and Lamb waves in piezoelectric sandwich structures. For both kinds of wave, the exact solutions are obtained using the formulation of state vector combined with the method of surface local impedance and the technique of the global matrix. The spectra of dispersion, the asymptotic behaviours as well as the profiles across the thickness of the composite structures are numerically obtained. The surface (Bleustein-Guylaev and piezoelectric Rayleigh) and interface (Maerfeld-Tournois and piezoelectric Stoneley) waves obtained at high frequency are examined and discussed.

1. INTRODUCTION

Constant research of new surface acoustic waves (SAW) devices, under the effect of the increasing demands for large-value data transmission in mobile communication and development of the components high frequency have contributed to the design and the optimization of filters, resonators and lines of delay to transverse surface waves and Rayleigh waves. The integration in adaptive or intelligent structures of piezoelectric sensors and actuators polymorphes becomes common. In spite of the technological interest of these new innovating components, making call in their design with piezoelectric crystals and ceramics, few theoretical results related to their high frequency behaviour are available due to the complexity of the analytical and numerical procedures (periodic stacks or not, material properties, interfaces and boundary conditions, …).

In this work, one presents a selection of numerical results concerning the propagation of harmonic guided waves, shear horizontal (SH) and Lamb (L) waves, in infinite sandwich plates made of perfect stacking of three layers of piezoelectric ceramics of the same thickness (010)-[100]-PZT5/(0$\bar{1}$0)-[100]-PZT5/(010)-[100]-PZT5 and (001)-[100]-PZT4/(00$\bar{1}$)-[100]-PZT4/(001)-[100]-PZT4 (Figs. 1-2). The major faces of these piezoelectric plates are free of mechanical stresses and covered with metallic thin films, of negligible mass, short-circuited. For each layer the orientations of the crystallographic cuts with respect to the sagittal plans are selected so that one has a decoupling between the orthogonal and parallel modes to the sagittal plans. The direct axes of polarization A_∞ are alternatively parallel and antiparallel to the axes x_3 , giving for these assemblies remarkable properties. The spectra of dispersion and the profiles of the fields along to the thickness of the plates are determined by numerical simulations.

The surface (Bleustein and piezoelectric Rayleigh) and interfaces (Maerfeld-Tournois and piezoelectric Stoneley) waves obtained at high frequency are examined and discussed.

2. GOUVERNING EQUATIONS

We consider symmetric sandwich structures that consist of three layers (*n=1, 2, 3*) of the same piezoelectric ceramics and of the same thickness $h/3$ but with opposite directions in polarization as illustrated in Figs. 1-2. The layers are assumed perfectly bonded. The top and bottom faces of the sandwich plates are free from tractions and completely coated with infinitesimally thin electrodes which are shorted. We are concerned with the transversely isotropic symmetry class, with the A_∞ axis oriented along the x_3 direction. Two types of stacks are considered. The first type of assembly (Fig. 1.) has the A_∞ axes parallel to the plane (x_1,x_3) of the sandwich plate, while the A_∞ axes are perpendicular to the plane (x_1,x_2) of the sandwich plate for the second type (Fig. 2.).

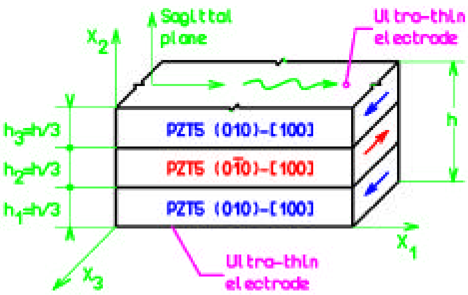

Fig. 1. Infinite piezoelectric sandwich plate in PZT5: geometry and notations.

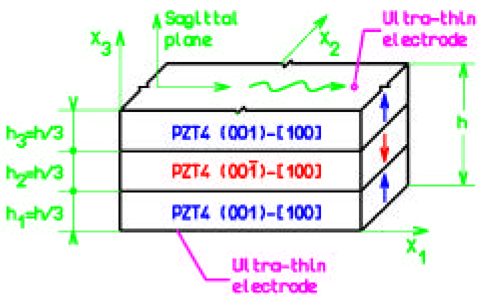

Fig. 2. Infinite piezoelectric sandwich plate in PZT4: geometry and notations.

The linear set of three-dimensional equations governing the behaviour of the (*n*)th piezoelectric layer consists of the equations of motion and Maxwell's equations (quasi-static approximation) [1]

$$T^{(n)}_{ij,j} = r^{(n)}\ddot{u}^{(n)}_i, \quad D^{(n)}_{i,i} = 0, \quad E^{(n)}_k = -f^{(n)}_{,k}, \tag{1}$$

as well as the following constitutive relations

$$T^{(n)}_{ij} = c^{(n)}_{ijkl}S^{(n)}_{kl} - e^{(n)}_{kij}E^{(n)}_k, \quad D^{(n)}_i = e^{(n)}_{ikl}S^{(n)}_{kl} + e^{(n)}_{ik}E^{(n)}_k, \tag{2}$$

where $S^{(n)}_{kl} = \frac{1}{2}\left(u^{(n)}_{k,l} + u^{(n)}_{l,k}\right)$. In Eqs (1-2) $u^{(n)}_i$ is the mechanical displacement vector, $r^{(n)}$ the mass density, $T^{(n)}_{ij}$ the stress tensor, $S^{(n)}_{ij}$ the strain tensor, $E^{(n)}_i$ the electric field, $f^{(n)}$ the electric potential and $D^{(n)}_i$ the electric displacement. The coefficients $c^{(n)}_{ijkl}$, $e^{(n)}_{kij}$ and $e^{(n)}_{ik}$ are the elastic stiffnesses, piezoelectric and dielectric permittivity constants. For these two particular assemblies, the piezoelectric coefficients have opposite signs while all the other material coefficients are the same across the thickness of the structures, i.e.,

$$e^{(1)}_{kij} = e^{(3)}_{kij} = e_{kij}, \quad e^{(2)}_{kij} = -e_{kij}. \quad c^{(n)}_{ijkl} = c_{ijkl}, \quad e^{(n)}_{ik} = e_{ik}, \quad r^{(n)} = r. \tag{3}$$

In the present analysis, we are concerned with the piezoelectric shear horizontal (SH) waves for the first structure and with the piezoelectric Lamb (L) waves for the second one respectively. Both plate modes SH and L propagate along the x_1 direction.

Thus the solutions are sought for in the complex harmonic plane wave form:

$$\begin{aligned} &i)\ u_1^{(n)} = u_2^{(n)} = 0, \quad u_3^{(n)} = \overline{u}_3^{(n)}(x_2)e^{j(k_1x_1 - wt)}, \quad f^{(n)} = \overline{f}^{(n)}(x_2)e^{j(k_1x_1 - wt)} \quad j^2 = -1, \\ &ii)\ u_1^{(n)} = \overline{u}_1^{(n)}(x_1, x_3)e^{j(k_1x_1 - wt)}, u_2^{(n)} = 0, u_3^{(n)} = \overline{u}_3^{(n)}(x_1, x_3)e^{j(k_1x_1 - wt)}, f^{(n)} = \overline{f}^{(n)}(x_1, x_3)e^{j(k_1x_1 - wt)}, \end{aligned} \tag{4}$$

where k_1 and w represent the wavenumber and circular frequency. In substituting Eqs ($4_{1,2}$) into the governing fields Eqs. (1-2) and keeping the quantities satisfying the interfaces continuity conditions, we obtain two sets of four and six coupled differential equations, respectively.

Now if we define, using the Voigt's notations, the Stroh's vectors [2-4] stated as

$$\overline{S}_{SH}^{(n)}(x_2) = \left(\overline{u}_3^{(n)}, \overline{f}^{(n)}, j\overline{T}_4^{(n)}, j\overline{D}_2^{(n)}\right)^T, \quad \overline{S}_L^{(n)}(x_3) = \left(j\overline{T}_3^{(n)}, \overline{u}_1^{(n)}, j\overline{D}_3^{(n)}, \overline{u}_3^{(n)}, j\overline{T}_5^{(n)}, \overline{f}^{(n)}\right)^T. \tag{5}$$

it can be shown that the coupled equations obtained from Eqs. (1-2) can be rewritten as first order matrix differential equations form [5-6]

$$\frac{d\overline{S}_{SH,L}^{(n)}}{dx_{2,3}} = jN_{SH,L}^{(n)}\overline{S}_{SH,L}^{(n)}, \quad n = 1,2,3, \tag{6}$$

where the block Stroh's matrices $N_{SH,L}^{(n)}$ are real and all the submatrices of $N_{SH,L}^{(n)}$ are symmetric. The solution of Eq. 6 can be expressed through the eigenvalues and eigenvectors of $N_{SH,L}^{(n)}$.

In the cases under investigation, the interfacial and boundary electromechanical conditions are given by:

$$\begin{aligned} &\overline{S}_{SH,L}^{(n+1)} = \overline{S}_{SH,L}^{(n)}, \quad at \quad x_2 = nh/3, \quad n = 1,2, \\ &\overline{S}_{SH}^{(n)} = \left(\overline{u}_3^{(n)}, 0, 0, j\overline{D}_2^{(n)}\right)^T, \quad at \quad x_2 = (n-1)h/2, \quad n = 1,3, \\ &\overline{S}_L^{(n)} = \left(0, \overline{u}_1^{(n)}, j\overline{D}_3^{(n)}, \overline{u}_3^{(n)}, 0, 0\right)^T, \quad at \quad x_3 = (n-1)h/2, \quad n = 1,3. \end{aligned} \tag{7}$$

3. NUMERICAL SIMULATIONS AND DISCUSSION

In this section, we present a series of numerical results concerning the spectra of dispersion, the generalized slowness curves and the profiles of the continuous fields in the thickness directions, which informs us about behaviours of the plate modes. The spectra of dispersion are obtained on a very broad range of frequency without the observation of numerical instabilities, this by associating the Stroh's formalism and a method of surface local impedance [4-6] for the treatment of the conditions (7). This original approach based on of the transfer the surface local impedance, via a recursive algorithm, preserves the advantage of the traditional matrix of transfer (dimension of the matrix of local impedance independent of the number of layer) while pushing away the phenomena of instabilities induced mainly by the quasi-electrostatic inhomogeneous partial waves. In order to overcome the loss of information about the spectra, the method is combined locally with the global matrix method (unconditionally stable) [6-8], but with increasing of time CPU [9].

The figure 3 represents the dispersion curves of a particular sandwich plate (010)-[100]-PZT5 /(0$\overline{1}$0)-[100]-PZT5 /(010)-[100]-PZT5 in short-circuited, from the elastic point of view, we have a simple plate but from piezoelectric point of view, it is a laminated plate symmetric with respect to the middle plane. In contrast with other cases [10], since all the layers of the sandwich plate are made of the same material the analysis of the generalized slowness curves (Fig. 4) of a layer of (010)-[100]-PZT5 makes cannot predict in which layer the signal will be predominant. In present situation the generalized slowness diagram informs us about the asymptotic values of the phase velocity at high frequency. At high frequency, the velocity of plate fundamental modes A_0, S_0

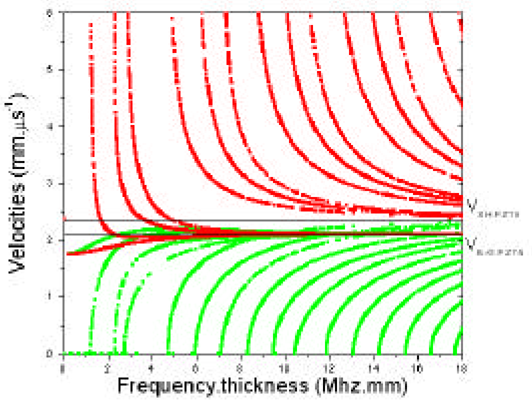

Fig. 3. Dispersion curves for SH waves in a (010)-[100] PZT5/(010)-[100] PZT5/(010)-[100] PZT5 short-circuited piezoelectric sandwich plate.

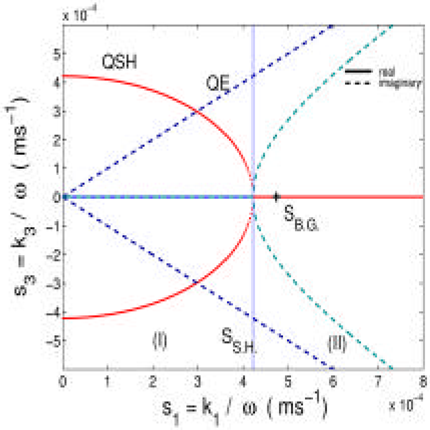

Fig. 4. Generalised slowness curves for (010)-[100] PZT5 layer.

towards that of Bleustein-Gulyaev waves [1], $v_{B.G.PZT5} = 2.112\ mm.ms^{-1}$. The signal takes on the form of two Bleustein-Gulyaev waves propagating in the two layers of the facing. The velocity of two plate modes A_1, S_1 tends towards that of Maerfeld-Tournois waves [1], $v_{M.T.PZT5} = v_{B.G.PZT5} = 2.112\ mm.ms^{-1}$. The signal is then composed of two waves of Maerfeld-Tournois located at the interfaces of the sandwich plate. The figure 5 (a) represents the superposition of these two types of behaviour. It is difficult to consider them separately because the curves merge into only one branch. The velocity of the higher order modes tends towards that of the bulk shear horizontal wave in the direction of the axis of wave guide $v_{SH.PZT5} = 2.37\ mm\,ms^{-1}$. The wave adopts the behaviour of a shear horizontal wave in each layer of the sandwich plate as depicted the figure 5 (b).

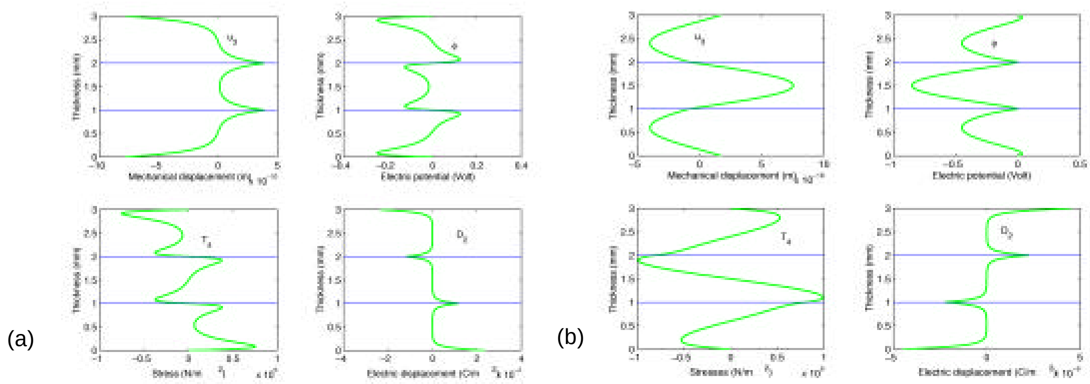

Fig. 5. Spatial distribution of fields in a (010)-[100] PZT5/(010)-[100] PZT5/(010)-[100] PZT5 short-circuited piezoelectric sandwich structure: (a) fxh=17.9 MHz and v_{ph} =2.11 mm.10^{-6} s^{-1}; (b) fxh=17.9 MHz and v_{ph} =2.47 mm.10^{-6} s^{-1}.

The figure 6 shows the spectra of dispersion of a sandwich plate (001)-[100]-PZT4/(00$\bar{1}$)-[100]-PZT4/(001)-[100]-PZT4 in short-circuited. As the previous case, the analysis of the generalized slowness curves of a layer of (001)-[100]-PZT4 (Fig. 7) cannot highlight in which layer of sandwich the high frequency signal will be predominant. The phenomenon of mode merging occurs again. At high frequency we distinguish three asymptotic behaviours. The velocity of the higher modes tends towards $v_{S.V.\ PZT4} = 2.59\ mm.ms^{-1}$. As observed in Fig. 8 (a) the acoustic signal propagates in each layer of the sandwich by taking on the behaviour of a bulk shear vertical wave. The velocity of modes A_1, S_1 tends toward the asymptotic limit $v_{S.P.\ \ PZT\,4} = 2.323\,mm.ms^{-1}$.

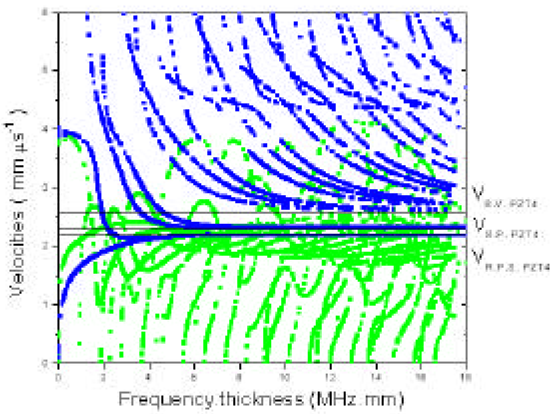

Fig. 6. Dispersion curves for Lamb waves in a (001)-[100] PZT4/(001)-[100] PZT4/(001)-[100] PZT4 short-circuited piezoelectric sandwich plate.

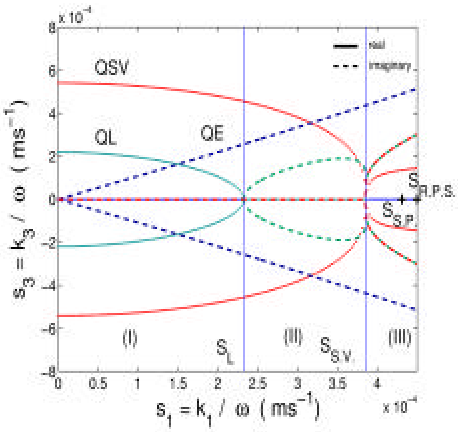

Fig. 7. Generalized slowness curves for (001)-[100] PZT4 layer.

The signal has the behaviour of two piezoelectric Stoneley waves propagating along both interfaces between the core and the facing layers of the sandwich plate as shown in figure 8 (b). The velocity of the fundamental modes A_0, S_0 tends asymptotically towards the velocity $v_{R.P.S\ \ PZT\,4} = 2.191\,mm.ms^{-1}$ the behaviour of the signal is that of two waves of Rayleigh propagating in the layers of the sandwich facing as show in figure 8 (c).

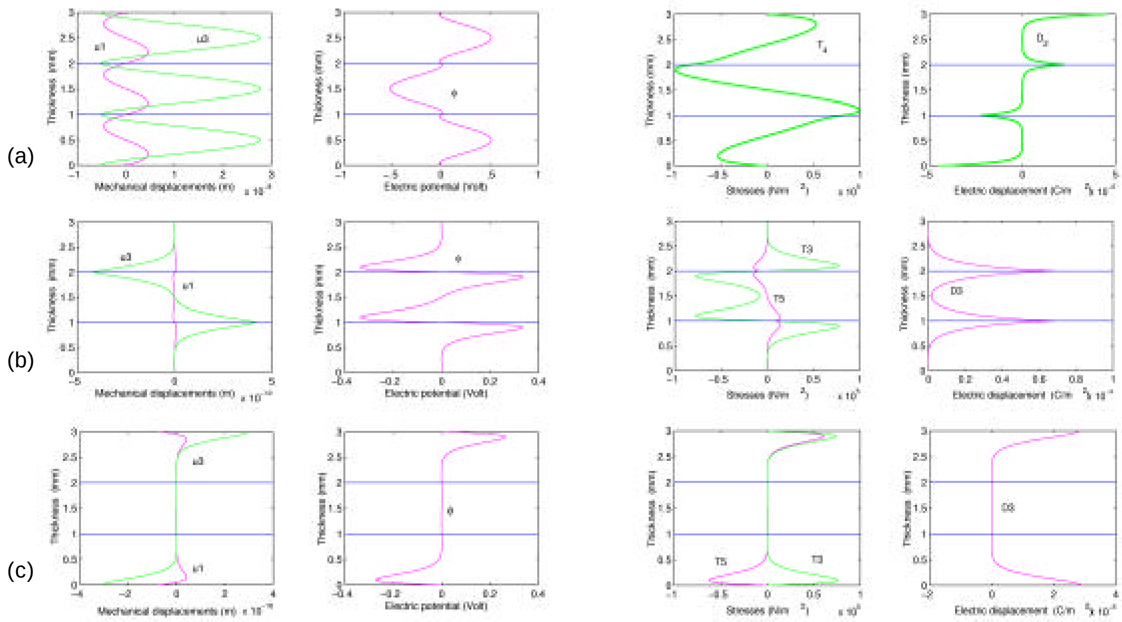

Fig. 8. Spatial distribution of fields in a (001)-[100] PZT4/(001)-[100] PZT4/(001)-[100] PZT4 short-circuited piezoelectric sandwich structure: (a) fxh=17.09 MHz and $v_{S.V.\ PZT4}$ =2.62 mm.10^{-6} s^{-1}; (b) fxh=15.5 MHz and $v_{S.P.PZT4}$=2.47 mm.10^{-6} s^{-1}; (c) fxh=14.25 MHz and $v_{R.P.S.\ PZT4}$ =2.47 mm.10^{-6} s^{-1}.

4. CONCLUDING REMARKS

In this study, the numerical simulations concerning two modes of propagation (SH) and (L) in short-circuited piezoelectric sandwich plates, made of materials at high symmetry (transverse isotropy), are presented. The proposed numerical approaches, robust at high frequency are limited to the method of the matrix of local impedance of surface combined locally with the technique of the global matrix. We will note from the two examined cases the large variety of the high frequency behaviours. The presence of fields located in the vicinity of the interfaces and the free surfaces suggests that these architectures can be exploited in new devices with interface acoustic waves (IAW) and surface acoustic waves (SAW) weakly dispersive. In addition these results could be used for benchmark for the validation of efficient and accuracy approximate models (pseudo-mixed formulation by finite elements, polynomial approach,...) and supplemented by taking into account losses, errors of parallelism of the poled axes and quality of the bond.

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