MULTI-DIMENSIONAL SOUND FIELD REPRODUCTION BASED ON THE FINITE DIFFERENCE TIME DOMAIN METHOD

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ABSTRACT

Numerical analysis methods based on the wave theory such as FDM, FEM, BEM and other new techniques are being applicable to architectural acoustic problems. Among the methods, the Finite Difference Time Domain (FDTD) method is useful for analyzing transient acoustic phenomena in rooms. As an application of the method, a sound field reproduction system based on the FDTD method is developed. In the system, directional impulse responses are calculated by the method and these are reproduced from a multi-channel loudspeaker system installed in an anechoic room. In this paper, basic reproducibility of the system was experimentally examined from physical and psycho-acoustical viewpoints.

INTRODUCTION

Various kinds of numerical analysis techniques such as the Finite Difference Method, Finite Element method and Boundary Element Method have become popular in acoustics recently. Among these methods, the Finite Difference Method in time domain, which is usually called the Finite Difference Time Domain (FDTD) method, is suitable for analysis of acoustical transient phenomena in rooms. The authors have investigated several applications of the FDTD method to room acoustic problems [1-4]. In this study, for the aim of *auralization* of impulse responses in room sound fields, a multi-channel sound field reproduction system based on the FDTD method is developed and the reproducibility of the system is examined from physical and psycho-acoustical viewpoints. In this paper, the outline of the system including the calculation method of directional impulse responses necessary for the reproduction system is introduced and experimental results are shown.

SOUND FIELD REPRODUCTION SYSTEM BASED ON THE FDTD METHOD

Outline of the Reproduction System

Figure 1 shows outline of the numerical simulation/reproduction system introduced in this study. Although the system is for 2-dimension, a configuration for 3-dimensional sound field is possible in principle. At first, directional impulse responses in orthogonal directions of +*y*(front), -*y*(rear), +*x*(right) and -*x*(left) at a receiving point are calculated using the FDTD method. Next, the responses are directly reproduced from four loudspeakers installed in an anechoic room.

The loudspeakers are arranged to be equidistant and orthogonal directions in a plane. By the system, 2-dimensional sound field assumed in a numerical simulation is realized in an anechoic room. The idea of this system is based on the 6-channel recording/reproduction system developed in our laboratory [5].

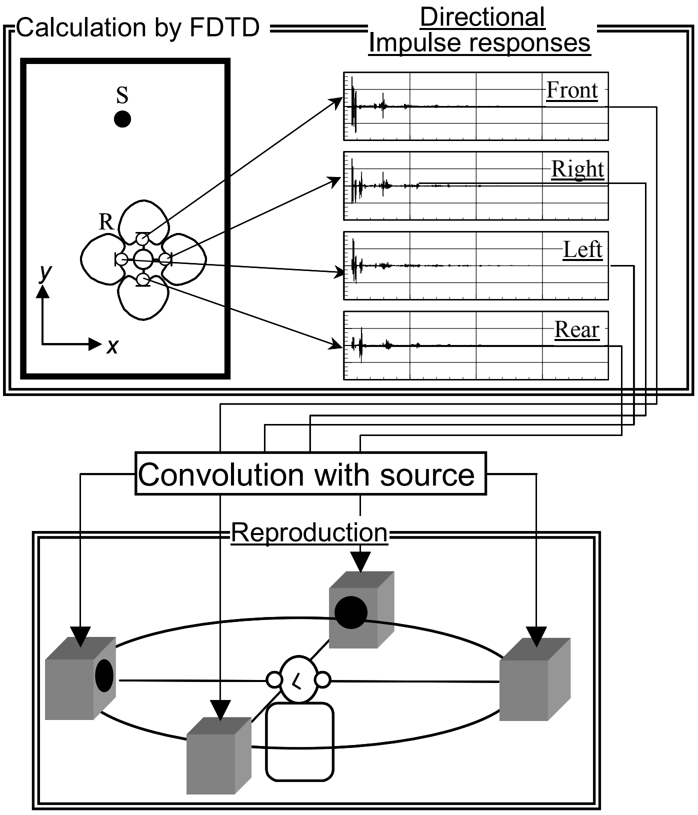


Fig.1 Numerical sound field simulation system

Calculation of Directional Impulse Responses by the FDTD Method

The sound wave in a 2-dimensional sound field is described by the following differential equations. Equations (1) and (2) are momentum equations in *x*- and *y*- directions, respectively, and Eq. (3) is the continuity equation.

$$\frac{\partial p(x,y,t)}{\partial x} + \rho \frac{\partial u_x(x,y,t)}{\partial t} = 0 \tag{1}$$

$$\frac{\partial p(x,y,t)}{\partial y} + \rho \frac{\partial u_y(x,y,t)}{\partial t} = 0 \tag{2}$$

$$\frac{\partial p(x,y,t)}{\partial t} + \kappa \left(\frac{\partial u_x(x,y,t)}{\partial x} + \frac{\partial u_y(x,y,t)}{\partial y} \right) = 0 \tag{3}$$

where, p is the sound pressure, u_x and u_y are the particle velocities in *x*- and *y*- directions, respectively, ρ is the density of the air and κ is the volume elastic modulus of the air.

The spatial and time derivatives in Eqs.(1) to (3) are approximated by centered finite differences. By adopting the staggered grid system with square-grids ($\Delta x = \Delta y \equiv \Delta h$) as shown in Fig.2, the following equations are obtained for the discrete system.

$$u_x^{n+1}(i+1/2, j) = u_x^{n}(i+1/2, j) - \frac{\Delta t}{\rho \Delta h} \left\{ p^{n+1/2}(i+1, j) - p^{n+1/2}(i, j) \right\} \tag{4}$$

$$u_y^{n+1}(i, j+1/2) = u_y^{n}(i, j+1/2) - \frac{\Delta t}{\rho \Delta h} \left\{ p^{n+1/2}(i, j+1) - p^{n+1/2}(i, j) \right\} \tag{5}$$

$$p^{n+1/2}(i,j)=p^{n-1/2}(i,j)-\kappa\Delta t\left\{\frac{u_x^n(i+1/2,j)-u_x^n(i-1/2,j)}{\Delta h}+\frac{u_y^n(i,j+1/2)-u_y^n(i,j-1/2)}{\Delta h}\right\} \tag{6}$$

where, Δt is the discrete time step, and Δh is the spatial interval. Indices *n*, *n*+1/2, *n*-1/2 and *n*+1 denote time steps.

As the initial condition, a continuous distribution of sound pressure expressed by the following equation and shown in Fig.3 was set so as to assume an impulse source.

$$p(x,y)=\begin{cases}1+\cos\pi\left(\dfrac{r}{N\Delta h}\right) & (r<N\Delta h)\\ 0 & (r>N\Delta h)\end{cases}$$

(7)

where, *r* is the distance from the source point (x_0, y_0) given by $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$. *N* is the parameter indicating the spatial extent of the impulsive source and is set to be 10 in this study. Under such an initial condition, the sound pressure and particle velocities at each grid point were successively calculated according to Eqs. (4), (5) and (6).

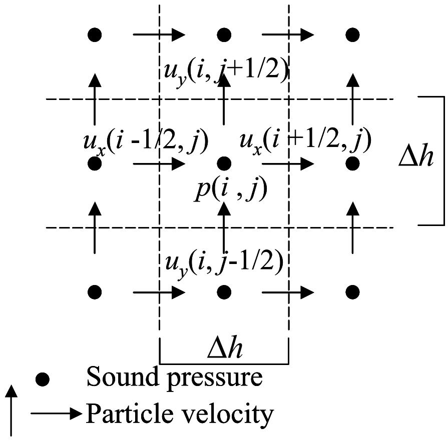


Fig.2 Staggered grid system for the FDTD method

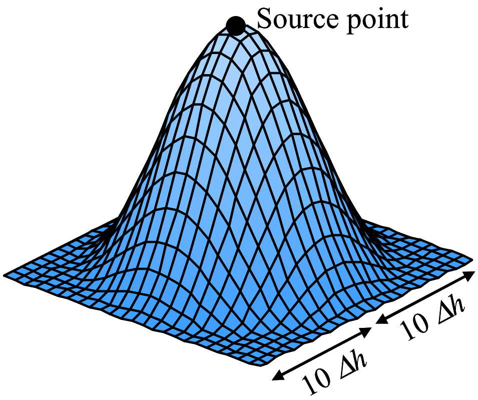


Fig.3 Initial condition of the source assuming the impulse

In order to obtain a directional impulse response, direction of sound incidence to the receiving point must be identified. For the purpose, sound intensity in x– and y– directions is calculated. In the grid system shown in Fig. 2, the sound intensity $I_x^n(i,j)$ and $I_y^n(i,j)$ at the grid point (*i,j*) are approximately calculated using the sound pressure and particle velocities in *x*– and *y*– directions as follows.

$$I_x^{n+1/2}(i,j)=p^{n+1/2}(i,j)\frac{u_x^n(i+1/2,j)+u_x^n(i-1/2,j)}{2}$$

(8)

$$I_y^{n+1/2}(i,j)=p^{n+1/2}(i,j)\frac{u_y^n(i,j+1/2)+u_y^n(i,j-1/2)}{2}$$

(9)

Thus, the incidence angle θ is calculated as follows.

$$\theta=\tan^{-1}\left(\frac{I_y^n(i,j)}{I_x^n(i,j)}\right)$$

(10)

The directional sound pressure response is obtained using the sound pressure $p^{n+1/2}(i,j)$ and a directivity factor $f(\theta)$ as follows. In this study, a cardioid shown in Eq. (12) was chosen as the directivity function. (The authors are examining the applicability of other functions.)

$$p_{dir}^{n+1/2}(i,j) = p^{n+1/2}(i,j) \cdot f(\theta)$$

(11)

$$f(\theta) = \frac{1+\cos\theta}{2}$$

(12)

EXPERIMENTAL INVESTIGATION ON REPRODUCIBILITY OF THE SYSTEM

In order to see reproducibility of the system, experimental studies from physical and psycho-acoustical viewpoints were performed. Figure 4 shows the sound field under investigation. Sound sources (S_0 to S_{90}) were set 10 m distant from a receiving point R every 15 degrees of incidence angle in a free field. For each source, directional impulse responses at R were calculated. In the calculation, spatial grid size Δh and time interval Δt were set to be 0.02 m and 0.04 ms, respectively. As a free field boundary condition, normal acoustic impedance of ρc was assumed on the hypothetical boundaries. Calculated directional impulse responses were convolved with a white noise with frequency components lower than 1.4 kHz (cut off frequency of 1k Hz band in 1/1 octave).

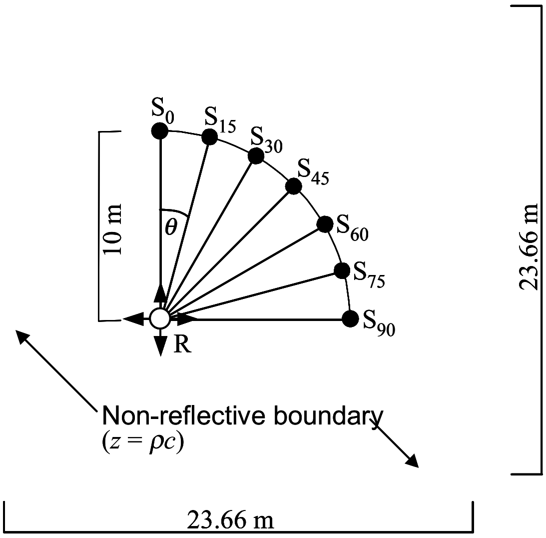
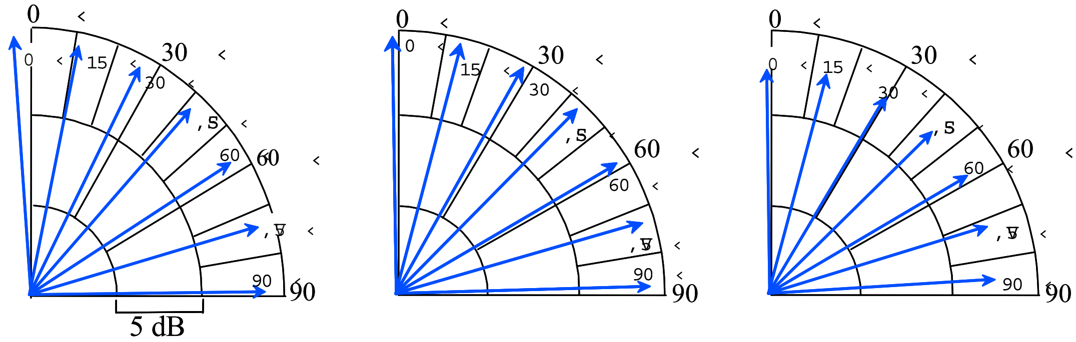


Fig.4 Geometrical setting of the sources and receiving point

Sound Intensity Measurement

In order to see physical reproducibility of the system, the test signal was reproduced from the system installed in an anechoic room and sound intensity at the center point was measured. Figure 5 (a), (b) and (c) show the experimental results. Length and direction of the arrows indicate magnitude and incidence angle of the measured sound intensity. For all frequencies, directions of measured sound intensity agree well with sound incidence directions assumed in the simulation model.



(a)250 Hz in 1 Oct. band (b)500 Hz in 1 Oct.band (c)1k Hz in 1 Oct.band

Fig.5 Measurement results of the sound intensity

Sound Source Localization Test

Next, in order to see the reproducibility from a psycho-acoustical viewpoint, sound localization test was performed. As shown in Fig.6, the interrupted test signal was reproduced in the system from 0 to 330 degrees in every 30 degrees randomly, and a subject sitting at the center point of the system answered the incidence angles. Figure 7 shows the average of the results for nine subjects. Simulated directions and perceived directions by the subjects are in good agreement.

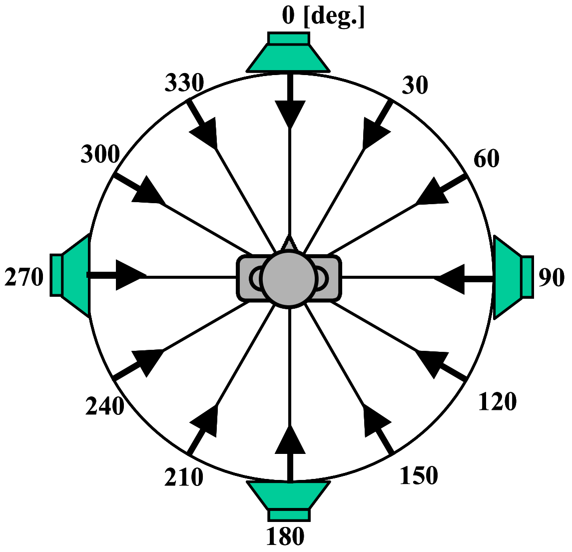


Fig 6. Experimental setting for the localization test

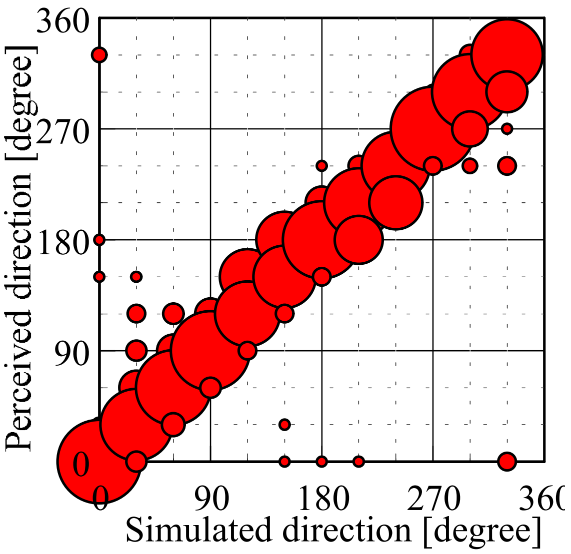


Fig.7 Experimental result

EXAMPLE OF NUMERICAL SIMULATION

Figure 8 shows a 2-D rectangular room under investigation. The room has triangular diffusers along the sidewalls. In the room, directional impulse responses at receiving point *R* were calculated. In addition, the transient sound pressure distribution in the room was calculated.

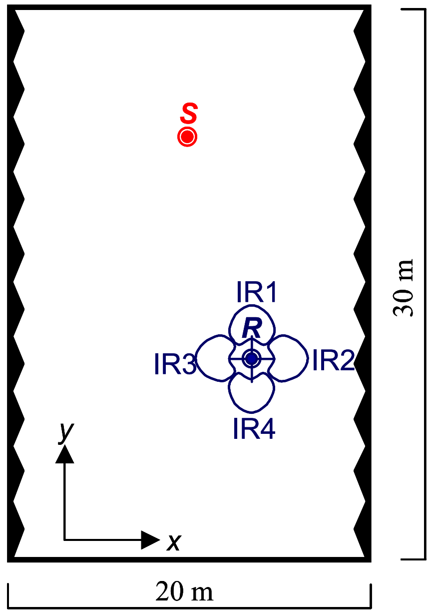


Fig.8 2-D room under investigation

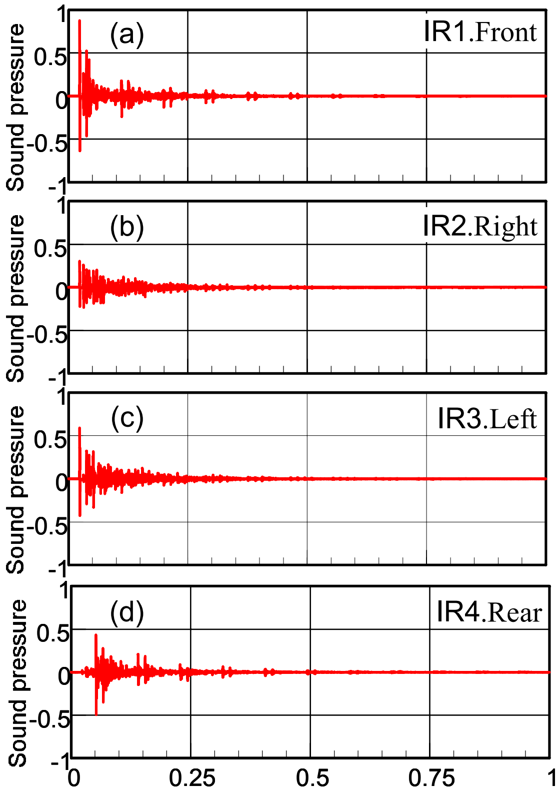


Fig.9 Directional impulse responses at the receiving point

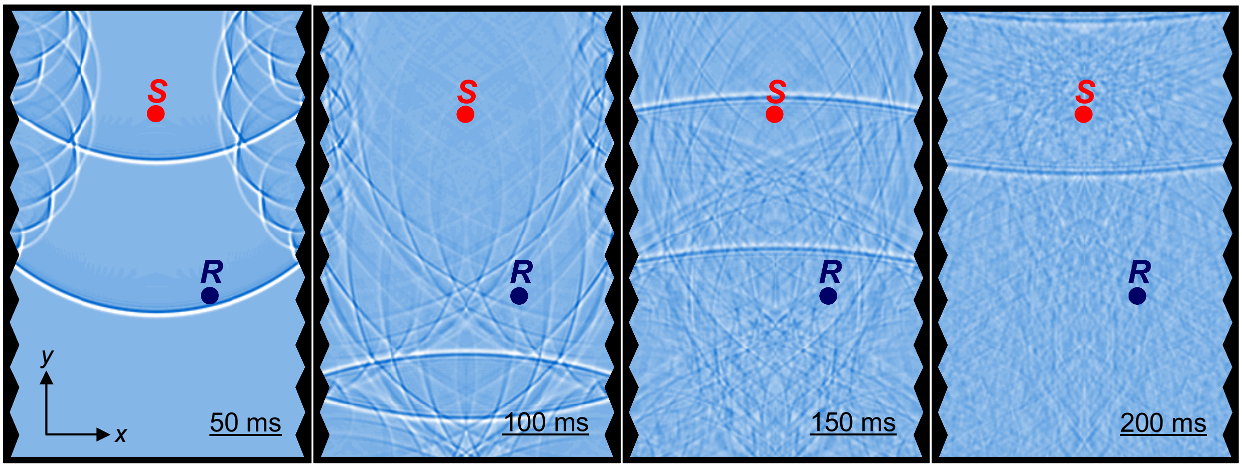


Fig.10 Sound pressure distribution

Fig. 9 and 10 show the directional impulse responses at *R* and sound pressure distribution in the room, respectively. In the sound pressure distribution, Fluttering echoes in *y*-direction caused by the parallel flat walls are obviously seen. On the other hand, the sound waves in x-direction are scattered by the diffusive sidewalls. In the directional impulse responses in front and rear directions shown in Fig. 9 (a) and 9 (d), the fluttering echoes are clearly seen. As shown in the figures, directions of sound incidence are precisely reflected in the directional impulse responses. Using this system, the authors have been making subjective investigation on the effect of diffusive walls for preventing fluttering echoes [6].

CONCLUSIONS

As an application of the FDTD method to *auralization* of room sound field, a multi-dimensional sound field reproduction system based on the numerical simulation was developed. Through the experimental study described in this paper, basic reproducibility of this system has been confirmed by both of physical and psycho-acoustical investigations. This system can be efficiently applied to investigation of acoustical effects by various parameters in room acoustic design, such as room shapes, arrangements and shapes of wall diffusers and suspended panel arrays.

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