# PRECISION ENHANCEMENT OF DOPPLER ULTRASOUND SPECTRAL ESTIMATION BY FINDING TFD OPTIMAL PARAMETERS

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**ABSTRACT**
A fundamental problem in Doppler ultrasound blood flow measurements is the computation of the signal's instantaneous frequency, which is generated when the signal is acquired. In fact, it is proportional to the flow velocity. The Time-Frequency Distributions Cohen Class (TFD) have efficiently estimated the instantaneous frequency for quasi-stationary signals such as arterial blood-flow. However, the precision of the spectral estimations depends on diverse factors. In this work a systematic study is conducted which indicates the relationship between the precision of the spectral estimations and the particular parameters of each kernel of the TFD distributions (e.g. Choi Williams, Bessel and Born Jordan). It considers the SNR and the sample length to obtain the optimum parameters that minimize the RMS error in the estimation.

## INTRODUCTION

Previous works have suggested that some time frequency distribution parameters affect the accuracy of the spectral estimations such as the pseudo instantaneous mean frequency and the RMS mean bandwidth (Cardoso, *et al.*, 1996). A preliminary study has already been conducted obtaining encouraging results (García, *et al*., 2000), this work complement those studies.

## TIME FREQUENCY DISTRIBUTION

The time frequency distributions (TFD) of the Cohen class considered in this work are the Bessel, the Born Jordan and the Choi Williams distributions (Cohen, 1989), their definitions follow.

<u>Bessel Distribution</u>

The discrete Bessel TFD when it is evaluated at discrete time zero and optimized is:

$$DBD(0,k) = 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{m=max\{-2\alpha|\tau|,-N+1+|\tau|\}}^{min\{2\alpha|\tau|,N-1-|\tau|\}} \frac{1}{\pi\alpha|\tau|}\sqrt{1-\left(\frac{m}{2\alpha\tau}\right)^2}\,x(m+\tau)x^*(m-\tau)\right] - 2|x(0)|^2 \qquad (1)$$

where $k$ is the discrete frequency taking integer values from *0* to *N-1*, $\alpha$ is a scaling factor taking the half of any natural value, and *W(n)* is a Hanning window of length *2N-1* (Boashash, and Black, 1987; Guo, and Durand, 1994).

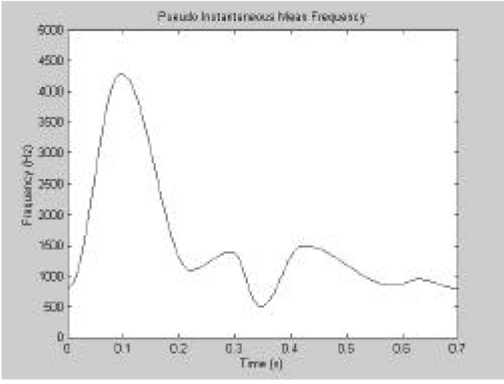



Fig 1: Signal's pseudo instantaneous mean frequency (PIMF) wave form of the simulated Doppler ultrasonic quasi-stationary signal that represents a typical blood flow in the Carotid artery.

Born Jordan Distribution

The discrete Born Jordan TFD when it is evaluated at discrete time zero and optimized is:

$$DBJD(0,k) = 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{m=max\{-2\alpha|\tau|,-N+1+|\tau|\}}^{min\{2\alpha|\tau|,N-1-|\tau|\}} \frac{1}{4\alpha|\tau|} x(m+\tau)x^*(m-\tau)\right] - 2|x(0)|^2 \quad (2)$$

where $k$ is the discrete frequency taking integer values from *0* to *N-1*, $\alpha$ is a scaling factor taking the half of any natural value, and *W(n)* is a Hanning window of length *2N-1* (Boashash, and Black, 1987; Cohen, 1989).

Choi-Williams Distribution

The discrete Choi-Williams TFD when it is evaluated at discrete time zero and optimized is:

$$DCWD(0,k) = 4\,\mathrm{Re}\left[\sum_{\tau=0}^{N-1} W(\tau)W^*(-\tau)e^{-j\frac{2\pi k\tau}{N}} \sum_{m=-N+1+|\tau|}^{N-1-|\tau|} \sqrt{\frac{1}{4\pi\tau^2/\sigma}}\, e^{-\frac{m^2}{4\tau^2/\sigma}} x(m+\tau)x^*(m-\tau)\right] - 2|x(0)|^2 \quad (3)$$

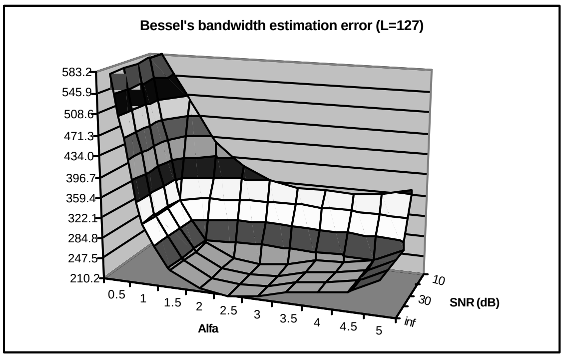
where $k$ is the discrete frequency taking integer values from *0* to *N-1*, $\sigma$ is a scaling factor taking any positive real value, and *W(n)* is a Hanning window of length *2N-1* (0 (Boashash, and Black, 1987; Choi, and Williams, 1989).

**DOPPLER ULTRASOUND SIGNAL SIMULATION**

In order to characterize the pseudo instantaneous mean frequency (PIMF) and the RMS mean bandwidth (RMSMB) error estimation, it is used a simulated Doppler ultrasonic quasi-stationary signal that represents a typical blood flow in the Carotid artery. The signal has the following characteristics: signal's duration *0.7s* and constant RMSMB *100Hz.* Its PIMF wave form is shown in figure 1. The simulation procedure is accurate described in (Cardoso, *et al.*, 1996). In this work, a sampling rate $f_o=19200Hz$ is considered, i.e. *T=13440* samples. Note that the sampling rate must be four times the signal's maximum frequency when TFDs are used.

Noise generation

White noise is added to the signal before starting the analysis procedure. In this work, signal noise ratios (SNR) of -10 dB, -20 dB, -30 dB and –40 dB, and a noiseless case are considered in this study.



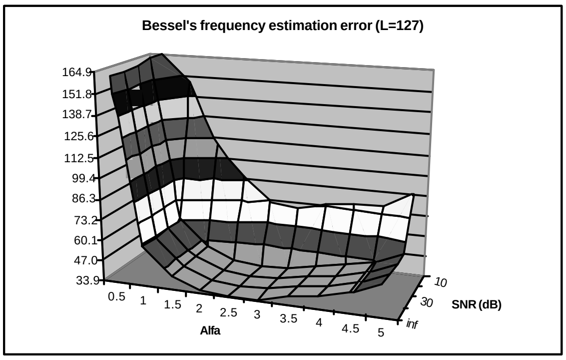




Fig 2. RMSBW (bandwidth) and PIMF (frequency) estimation error functions of the Bessel TDF versus its parameter ($\alpha$) and the SNR for the case of the sampling window length of $L=127$.

## SPECTRAL ESTIMATION

The spectral estimation of both the RMSB and the PIMF are treated as in (Cardoso, *et al.*, 1996; Fan, and Evans, 1994). Their procedures have a common part. First, a signal portion of length $L$ is taken from the $n^{th}$ to the $(n+L-1)^{th}$ elements of the whole signal, it will be called the $n^{th}$ signal window. $L$ can be *63*, *127* and *255*, and $L=2N-1$. The elements of the window are numbered in the discrete time domain from $1-N$ to $N-1$. Second, the analytic signal of this window is calculated. The elements of the analytic signal are also numbered in the discrete time domain from $1-N$ to $N-1$. Third, the TFD of this analytic signal is calculated using equation (1), (2) or (3), depending on the case. The elements of the TFD are numbered in the discrete frequency domain from *0* to $N-1$. Note that the components corresponding to negative frequencies, which are numbered from $N/2$ to $N-1$, are all equal to zero. Finally, the pseudo instantaneous power distribution (PIPD) of this TFD is calculated. Its elements are also numbered in the discrete frequency domain from *0* to $N-1$. Observe that the components corresponding to negative frequencies, which are numbered from $N/2$ to $N-1$, are also equal to zero. The PIPD is defined as:

$$PIPD(0,k)=\begin{cases} TFD(0,k) & TFD(0,k)\geq 0 \\ 0 & TFD(0,k)<0 \end{cases} \tag{4}$$

In case of the PIMF calculation, the pseudo instantaneous mean frequency associated to the $n^{th}$ window signal is stated by:

$$PIMF(n)=\frac{\sum_{k=0}^{N/2-1} f_k \bullet PIPD(0,k)}{\sum_{k=0}^{N/2-1} PIPD(0,k)} \tag{5}$$

where $f_k$ is the real frequency associated to the discrete frequency $k$. Observe that $n$ can be considered as the whole signal's discrete time variable, running from *0* to $T$-$L$. Indeed, it represents the total amount of fully overlapped signal windows of length $L$ in the complete signal (an overlapping of L-1 elements). That is, the *PIMF(1)* correspond to the 1st signal window; the *PIMF(2),* to the 2nd signal window; and so on. On the other hand, in case of the RMSMB calculation, the RMS mean bandwidth associated to the $n^{th}$ window signal is stated by:

$$RMSMB(n)=\sqrt{\frac{\sum_{k=0}^{N/2-1} \left(PIMF(n)-f_k\right)^2 \bullet PIPD(0,k)}{\sum_{k=0}^{N/2-1} PIPD(0,k)}} \tag{6}$$

with the same considerations as in equation (5).

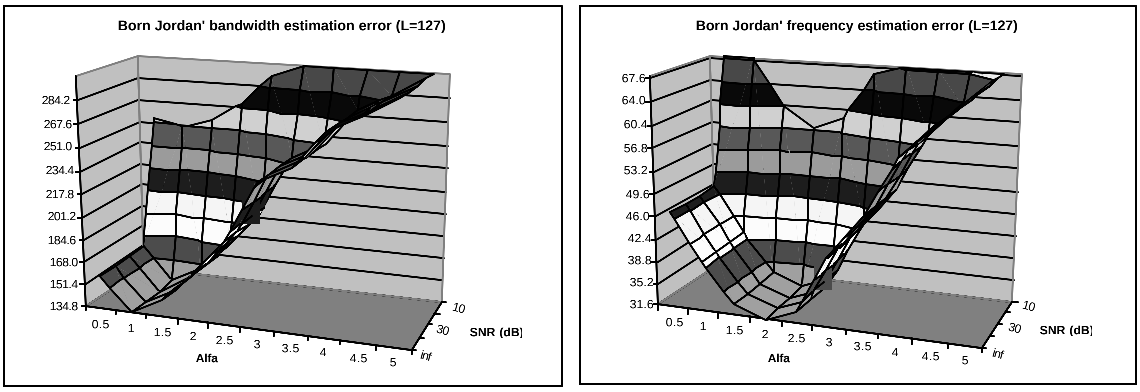



Fig 3. RMSBW (bandwidth) and PIMF (frequency) estimation error functions of the Born Jordan TDF versus its parameter ($\alpha$) and the SNR for the case of the sampling window length of *L=127*.

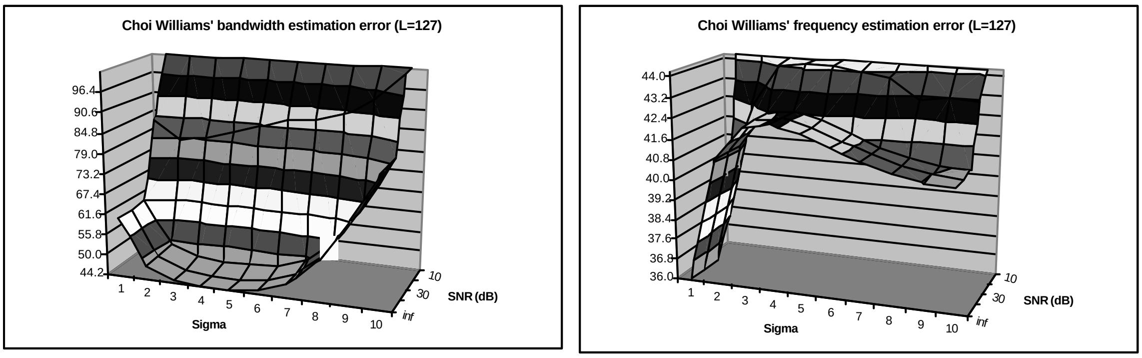



Fig 4. RMSBW (bandwidth) and PIMF (frequency) estimation error functions of the Choi Williams TDF versus its parameter ($\sigma$) and the SNR for the case of the sampling window length of *L=127*.

**ERROR ESTIMATION**

Typically, in any spectral estimation, the error has two independent components (Cardoso, *et al.*, 1996). The first component represents the mean of the errors of the estimated values respect to the theoretic values. That error will be called the bias. The second component represents the standard deviation (std). Then, the root mean square (RMS) error is estimated according to:

$$error_{RMS} = \sqrt{bias^2 + std^2} \tag{7}$$

In case of calculating the error estimation of the PIMF, it can be done with:

$$bias = \frac{1}{m}\sum_{n=0}^{m-1}\left(PIMF_{estimated}(n) - PIMF_{theoretic}(n)\right) \tag{8}$$

$$std^2 = \frac{1}{m}\sum_{n=0}^{m-1}\left(PIMF_{estimated}(n) - PIMF_{theoretic}(n)\right)^2 \tag{9}$$

where $m$ is the total amount of fully overlapped signal windows of length $L$ in the whole signal of length $T$, in consequence, $m = T - L + 1$.

Whereas, in case of calculating the error estimation of the RMSBW, it can be done with:

$$bias = \frac{1}{m}\sum_{n=0}^{m-1}\left(RMSBW_{estimated}(n) - RMSBW_{theoretic}(n)\right) \tag{10}$$

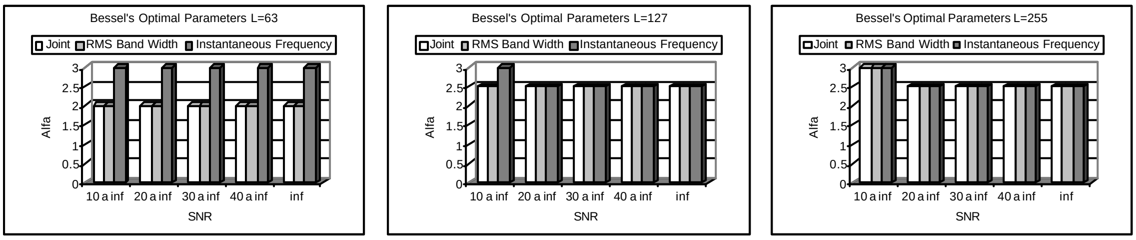



Fig 5: Bessel's optimal parameters as a function of the SNR dynamical range (10-inf dB, 20-inf dB, 30-inf dB, 40-inf dB, inf dB), and the sampling window length (L=63, L=127, L=255). Three cases are considered: RMSBW estimation error; PIMP estimation error; and, simultaneous RMSBW and PIMF estimation error.

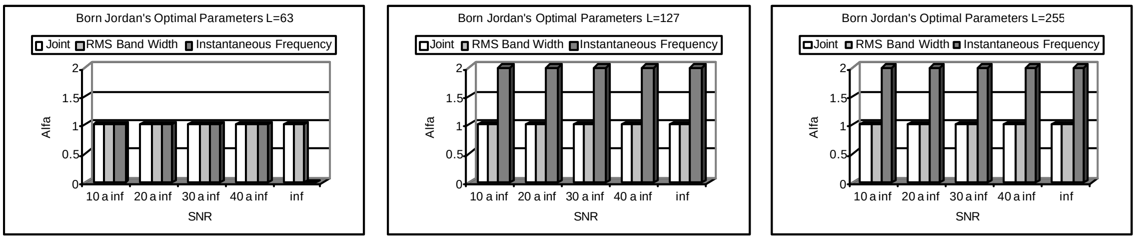



Fig 6: Born Jordan's optimal parameters as a function of the SNR dynamical range (10-inf dB, 20-inf dB, 30-inf dB, 40-inf dB, inf dB), and the sampling window length (L=63, L=127, L=255). Three cases are considered: RMSBW estimation error; PIMP estimation error; and, simultaneous RMSBW and PIMF estimation error.

$$std^2 = \frac{1}{m}\sum_{n=0}^{m-1}\left(RMSBW_{estimated}(n) - RMSBW_{theoretic}(n)\right)^2 \quad (11)$$

with the same considerations as in equations (8) and (9).

## RESULTS

It is considered that there are four relevant factors that determine the error in the spectral estimations: the signal itself, the value of the TFD's parameters (or the TFD's scaling factors), the sampling window length and the SNR. An error function is constructed for each TFD and for each sampling window length, varying the TFD's parameter and the SNR. It is observed that the error function has a minimum value considering traces at constant SNR. That minimum value occurs at the so called optimal value of the TFD's parameter. Detailed results for each TFD are shown in figures 2, 3 and 4, corresponding to the case of a sampling window length of *L=127*. Results corresponding to the cases of the sampling window length *L=63* and *L=255* are very similar.

In practice, a dynamical range of SNR should be considered. For this reason, the optimal parameters that minimize the RMS error estimation associated to a SNR dynamical range are calculated. The SNR dynamical ranges considered in this work are 10-inf dB, 20-inf dB, 30-inf dB, 40-inf dB and the noiseless case. Furthermore, sometimes it is required to calculate only the PIMF; but in other occasions, only the RMSBW or both of them are required. The three previous cases are considered and the results are shown in figures 5, 6 and 7.

## CONCLUSIONS

The optimal parameters of time frequency distributions (TFD) have been calculated in this work. The optimal parameters are those that minimize the error estimation of spectral parameters such that the pseudo instantaneous mean frequency (PIMF) and the RMS mean bandwidth

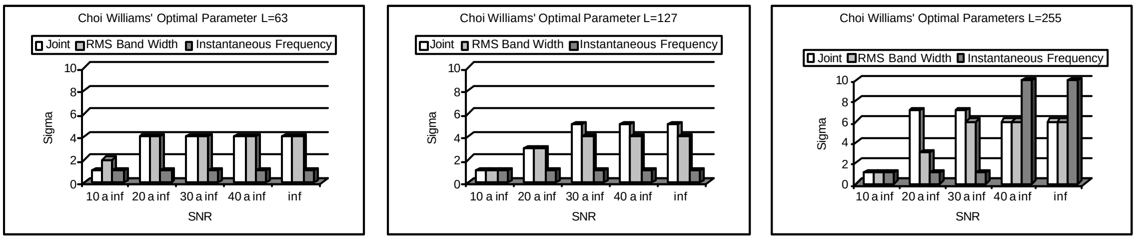



Fig 7: Choi Williams' optimal parameters as a function of the SNR dynamical range (10-inf dB, 20-inf dB, 30-inf dB, 40-inf dB, inf dB), and the sampling window length (L=63, L=127, L=255). Three cases are considered: RMSBW estimation error; PIMP estimation error; and, simultaneous RMSBW and PIMF estimation error.

(RMSBW). Three TFD have been considered: the Bessel, the Born Jordan and the Choi Williams distributions. Four relevant factor that affect the spectral estimations are modeled: the signal itself, the TFD's parameters or scaling factors, the sampling window length and the SNR. The signal considered is a simulated Doppler ultrasonic quasi-stationary one that represents a typical blood flow in the Carotid artery with sampling windows of L=63, L=127 and L=255, and SNR of 10 dB, 20 dB, 30 dB, 40 dB and a noiseless case. The optimal parameters have been calculated by constructing the error estimation functions that characterizes both, the PIMF and the RMSBW, versus the TFD's parameter and the SNR, considering the sampling window length constant. Then, the TFD's optimal parameters have been chosen to minimize the RMS error estimation associated to a dynamical range of SNR. The SNR dynamical ranges considered are 10-inf dB, 20-inf dB, 30-inf dB, 40-inf dB and inf dB. Figures 5, 6 and 7 show the detailed results obtained. In general, the RMSBW estimation and the simultaneous RMSBW and PIMF estimation have the same optimal parameters. For the Bessel TFD the parameters are $\alpha=2$ for L=63 and $\alpha=2.5$ for L=127 and L=255. For the Born Jordan TFD the parameters are $\alpha=1$ for L=63, L=127 and L=255. And for the Choi Williams TFD the parameter is $\sigma=4$ for L=63. On the other hand, the PIMF estimation has the following optimal parameters: Bessel's $\alpha=3$ for L=63, and $\alpha=2.5$ for L=127 and L=255; Born Jordan's $\alpha=1$ for L=63, and $\alpha=2$ for L=127 and L=255; and Choi Williams' $\sigma=1$ for L=63. In order to obtain more detailed information about the Choi Williams' parameters, refer to the figure 7.

## ACKNOWLEDMENTS

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