DEVELOPMENT AND ITS APPLICATION OF PARABOLIC EQUATION METHOD

PACS REFERENCE: 43.30.Bp

Anada Tetsuo; Tsuchiya Takenobu; Endoh Nobuyuki
Kanagawa University
3-27-1 Rokkakubashi, Kanagawa-ku,
YOKOHAMA 221-8686
JAPAN
Tel: +81-45-481-5661
Fax: +81-45-491-7915
E-mail: anada@cc.kanagawa-u.ac.jp

ABSTRACT: With the rapid progress of ocean acoustic tomography, parabolic equation are widely used for the study of sound propagation problems in ocean acoustics. In order to develop a new application of PE method, the following two problems will be discussed.

First, the rigorous and efficient method combined the Douglas operator scheme to PE based on the Pade approximation is given for the acoustic property of underwater acoustic lens in a real-time imaging sonar system. Secondly, a new time-domain PE method based on the slow-varying envelope approximation is considered for the forward wave propagation, diffraction and reflection in short pulse propagation problems.

1. INTRODUCTION

With the rapid progress of the ocean acoustic tomography[1], the Parabolic Equation methods (PEM) are very widely used for the study of sound propagation problems in ocean acoustics[2,6]. The advantages of PEM are its simple numerical implementation and its fast convergence within a reasonable cpu-time and memory. While, the experiment in ocean acoustics propagation is planned to investigate resources and the meteorological phenomenon of the Arctic Ocean among others. In the near future, it will be important to develop an underwater real-time image sonar system to promote investigation of the ocean.

The imaging sonar system installed on the autonomous underwater vehicle (AUV) is the forward-looking sonar using an acoustic lens system [7]. It is possible to reduce the enormous amount of calculation for two-dimensional beam forming. Furthermore, a high-resolution imaging system and size reduction of the equipment are achieved. Generally, the design of the acoustic lens as well as that of the optical lens is based on ray theory. However, it is difficult to know the sound field distributions, the effect of finite-size, loss, diffraction by the lens, and so forth. Though a sound field is obtained by the normal mode theory, the enormous mode number will be required in the operation of the high frequency. Therefore, other numerical methods are required. In this paper, numerical techniques based on the PE method are considered. First, the derivation of an efficient and accurate method combined the Douglas operator scheme to PE based on the Pade approximation is given for the acoustic property of an underwater acoustic 2D/3D lens in a real-time imaging sonar system. Secondly, a time-domain PEM based on the slow-varying envelope approximation is given for the forward wave propagation, diffraction and reflection in the short pulse propagation problems in underwater acoustics.

2. GENERAL FORMALISM

2.1 Numerical analysis of 2D/3D acoustic lens

The scalar Helmholtz equation for sound field is given by

$$\rho\nabla \bullet \left[\frac{1}{\rho}\nabla p(\mathbf{r},t)\right] - \frac{1}{c^2(\mathbf{r},t)}\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = 0 \qquad (1)$$

where **r** is the position vector, c(**r**,t) is the sound speed, ρ is the density function, p(**r**,t) is the pressure.

By assuming the acoustic sound wave propagates along principal direction, the sound field can be separated as a slowly varying envelope and a fast oscillating phase term, i.e.,

$$p(r,z) = \phi(r,z)\exp(jk_0 n_b r) / \sqrt{r}$$

where envelope function f(r,z) varies slowly with r.

Substituting a solution of the above form into Eq.(1), the reduced wave equation is transformed into the following equation for :

$$\frac{\partial^2\phi}{\partial r^2} + 2jk_0\frac{\partial\phi}{\partial r} + \rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial}{\partial z}\right)\phi + k_0^2(n(r,z)^2 - n_b^2)\phi = 0 \qquad (2)$$

where n_b=1 is reference refractive index.

Eq. (2) is split into two terms that governs the evolution of the forward and the backward sound wave of ϕ. By neglecting backward sound waves, the one-way equation can be obtained as follows:

$$\frac{\partial\phi}{\partial r} = jk_0\left(\sqrt{1+L}-1\right)\cdot\phi \qquad (3)$$

In the case of 2D model: $\mathbf{L} = k_0^{-2}\left(\rho(z)\frac{\partial}{\partial z}\frac{1}{\rho(z)}\frac{\partial}{\partial z} + k_0^2\left(n(r,z)^2-1\right)\right)$

The formal solution of Eq.(3) at the marching step r+Δr can be written as follows:

$$\phi(r+\Delta r,z) = \exp[jk_0\Delta r(\sqrt{1+L}-1)]\cdot\phi(r,z) \qquad (4)$$

In order to implement the exponential operator of Eq.(4), we apply the Pade-series/Pade-product approximation given first by Collins[3,4,5]:

$$\phi(r+\Delta r,z) = \left(1+\sum_{i=1}^{n}\frac{a_{i,n}L}{1+b_{i,n}L}\right)\phi(r,z) = \prod_{i=1}^{n}\frac{1+\lambda_{i,n}L}{1+\mu_{i,n}L}\phi(r,z) \qquad (5)$$

The nth-order Pade propagator in Eq.(5) may be decomposed into an n-step algorithm for which the ith partial step takes the following form

$$\phi^{q+i/n} = \frac{(1+\lambda_{i,n}L)}{(1+\mu_{i,n}L)}\phi^{q+(i-1)/n} \qquad (6)$$

In the case of 3D model: $\sqrt{1+L} \cong \frac{1+P_1X+P_2Y}{1+q_1X+q_2Y}$, $P_1 = P_2 = \frac{3}{4}$, $q_1 = q_2 = \frac{1}{4}$

$$X \equiv k_b^{-2}\left(\frac{\partial^2}{\partial x^2}+\frac{1}{2}k_0^2\left(n(x,y,z)^2-n_b^2\right)\right) \text{ and } Y \equiv k_b^{-2}\left(\frac{\partial^2}{\partial y^2}+\frac{1}{2}k_0^2\left(n(x,y,z)^2-n_b^2\right)\right)$$

When Eq.(6) is discretized with the difference scheme of Crank-Nicholson, the truncation error of the second-order differential is O(Δz^2). It is known that the instability phenomenon is generated in the numerical result. In order to avoid this instability phenomenon, the rotated branch-cut complex Pade coefficients is proposed. The Douglas operator scheme as the medium is homogeneous becomes O(Δz^4). Then, the central difference equation for the second derivative in the differential operator L is given as follows:

$$\rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial}{\partial z}\right) = \frac{1}{\Delta z^2}\frac{\delta_z^2}{1+\alpha\delta_z^2} \qquad (7)$$

where α=0 (Crank-Nicholson), 1/12 (Douglas scheme). The δz are a central difference, and Varga's treatment is applied to handle the density discontinuity on the boundary interface[7,8,9].

$$\delta_z^2 = \rho_-\phi_{i-1} - \rho_0\phi_i + \rho_+\phi_{i+1},\ \rho_\pm = \frac{2\rho(r,z)}{\rho(r,z)+\rho(r,z\pm\Delta z)},\ \rho_0 = \rho_+ + \rho_- \qquad (8)$$

When the Douglas operator is applied, we finally obtain the high accuracy scheme and lead to a tridiagonal system of complex linear equations. The present scheme allows us to use an efficient procedure such as the Thomas algorithm, so that the computational time is almost identical to that in the Crank-Nicholson scheme. Moreover, to avoid the effect of spurious reflections from the basement of computational window edge, we assume the transparent boundary condition at the ocean bottom proposed by Hadley [10]. Then, the real part of wavenumber k_z in the depth direction must be restricted compulsorily to be positive to ensure only radiation outflow. The sound waves pass through the boundary without reflection.

$$\phi^{q}_{m+1} = \phi^{q}_{m} \exp(jk_z \Delta z) \;, \quad \Delta z = z_m - z_{m-1} \tag{9}$$

where $k_z = \dfrac{1}{j\Delta z} \ln(\phi^{q}_{m} / \phi^{q}_{m-1})$ and $\mathrm{Re}(k_z) \geq 0$

By applying Alternating-Direction Implicit Methods in the case of the 3D model, two difference approximations in the x, y direction as a propagation axis z are used alternately on successive marching-steps of $1/2\Delta z$.

2.2 Time-Domain PE method based on the Slowly Varying Envelope Approximation

Based on SVEA, an acoustic short pulse propagation has been applied to the wave equation simultaneously leading to an algorithm suitable for studying forward wave propagation, wide angle of diffraction and reflection[11,12]. In comparison with a carrier frequency, the signal frequency will be regarded as slow wave propagation. By applying SVEA to the time term of the wave equation (1) and substituting a solution of the form $p(r, t)=\varphi(r, t)e^{-j\omega t}$ into wave equation, the following time-domain wave equation is obtained:

$$-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - 2j\frac{\omega}{c^2}\frac{\partial\varphi}{\partial t} = -\nabla \times \nabla \times \varphi + \frac{\omega^2}{c^2}\varphi \tag{10}$$

where ω is the angular frequency (carrier frequency).

The first term on time is much smaller than the second term. With first-order approximation, the second-order wave equation in time is reduced to a first-order equation as follows:

$$-2j\frac{\omega n^2}{c^2}\frac{\partial\varphi}{\partial t} = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} + \frac{\omega^2 n^2}{c^2}\varphi \tag{11}$$

Comparing with Eq.(11) in time domain and PE equation in frequency domain, these equations have the same form. It is quite obvious that PE codes can be extended to solve the time-domain parabolic equation in the slow-wave approximation. Therefore, we can be solved by using Alternating Direction Implicit Method for obtaining the solution of Eq.(11). Eq.(11) can deal with diffraction, reflection, and forward propagation.

3. Numerical results by parabolic equation method

3.1 Simulation of 2D/3D Acoustic Lens for imaging sonar system

As a new application field of the PEM, the underwater acoustic lens for the real time imaging sonar system is studied from the standpoint on wave theory. Transmitting frequencies are 400, 500 and 600 [Hz], and the direct image through the lens can be displayed on the receiving array, which consists of 128 x 128 elements as shown in Fig.1. The acoustic lens used in this sonar is a spherical single lens with the diameter of approximately 400[mm] and made of Acrylic resin having sound profile of 2,700[m/s] and damping factor of 0.5[dB] per wavelength. We simulated the 2D/3D sound field using the present lens geometry. The step-size in the transverse direction is $\Delta x=\Delta y=0.001$[m], the range-increment is $\Delta z=0.001$[m]. Figure 2 shows simulated results of sound field through the lens. From these results, it is seen that the focal position is about 0.4[m] from the center of the lens. We intend to compare simulated results with observed data and verify the accuracy of this PE method.

3.2 Simulation of acoustic pulse propagation by time-domain PE method

The longitudinal profile of the wave pocket along +z propagating direction is also Gaussian beam with full-width of 10[m]. The computational window is 200[m] by 1000[m], which is discretized in a 200 by 2000 grid as shown Fig.3. The time step used is 0.2 [ms]. The total duration simulated is 1[s]. The multimode channel waveguide is supported some guided modes and is reflected in the boundary of top and up sloping-bottom and the interference will be occurred. Figure 3 also shows the pulse propagation characteristics of the incident-pulse with Gaussian profile. It is observed that the pulse is confined in the underwater column, while the part of the pulse wave is radiated into the up sloping sediment layer. The validity of the present method is confirmed by the coincidence with our physical image.

4. Conclusion

Numerical techniques based on the PEM were given for ocean acoustic propagation problems. Our proposed FD-PE scheme can be significantly improved in allowing wider propagation angles and suppressed the vibration of Gibbs. Moreover, the underwater acoustic lens using ADI method was also studied for the acoustic imaging sonar system. By obtaining the acoustic characteristic of the lens, the working mechanism of lens was confirmed. Secondly, a newly developed time-domain PE method based on the slow-varying envelope approximation was derived for solving forward wave propagation, diffraction and reflection and given for short pulse propagation problems in underwater acoustics. In the near future, the algorithms proposed here will be applied and extended to sound field propagation in practical 3D ocean problems in frequency domain and time domain.

Reference

(1) W.Munk, P.Worcester, C. Wunsch, Cambridge Univ. Press, p.433, 1995.

(2) D. Lee, A. D. Pierce, J. of Computational Acoustics, Vol.3, No.2, pp.95-173, 1995.

(3) M.D.Collins, J.Acoust. Soc. Am.86,pp.1459-1464, Oct.1989.

(4) M.D.Collins, J.Acoust. Soc. Am.89, pp.1068-1075(1991).

(5) M.D.Collins, J.Acoust. Soc. Am.93, pp.1736-1742 (1993).

(6) Finn B. Jensen, William A. Kuperman, Micheal B. Porter and Henrik Schmidt, AIP Press, Woodbury, New York, Chap. 6, p.375,1990.

(7) Richtmer and Morton,,pp.216-217, John Wiley &Sons Inc., 1967.

(8) Lee,D. and M. H. Schltz, pp.79-117, World Scientific, Singapore, 1995.

(9) R.S. Varga, Prentice-Hall, Inc. Maruzen Co., LTD.

(10) G.R. Hadley, Opt. Lett., vol.16, no.9, pp.624-626, 1991.

(11) P.L. Liu, Q.Zhao, and F.S. Choa,"Slow-wave finite-difference beam propagation method", IEEE Photon., Technol. Lett. vol.7, pp.890-892, 1995

(12) G.H. Jin, et al, IEEE Photon. Technol. Lett. vol.9, pp.348-350,1997.

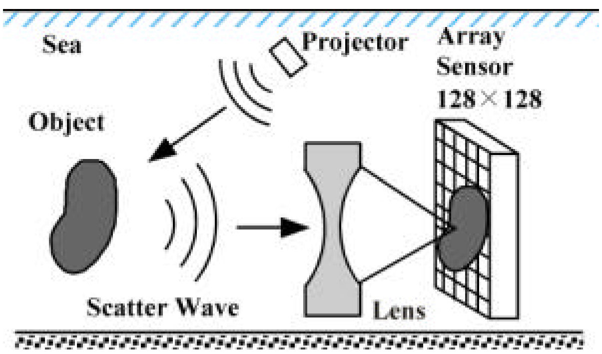

Figure 1. Real-time imaging sonar system used acoustic lens.

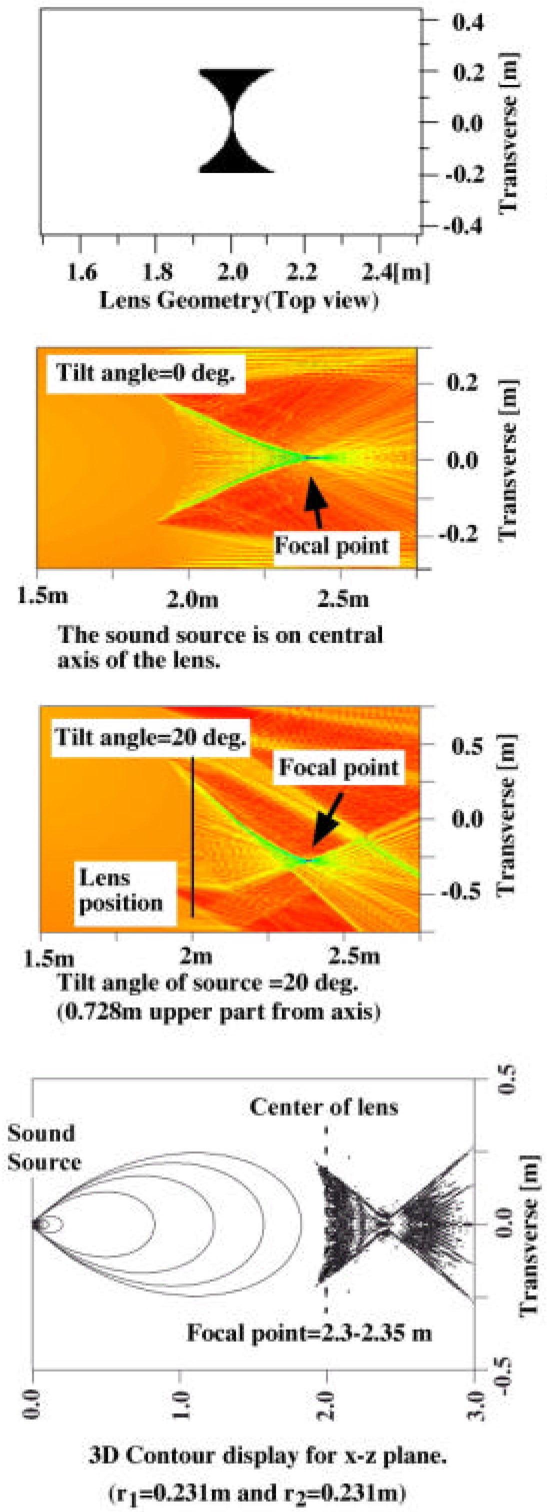

Fig.2 2D/3D model of acoustic lens and acoustical characteristics by tilt angle of incident sound source.

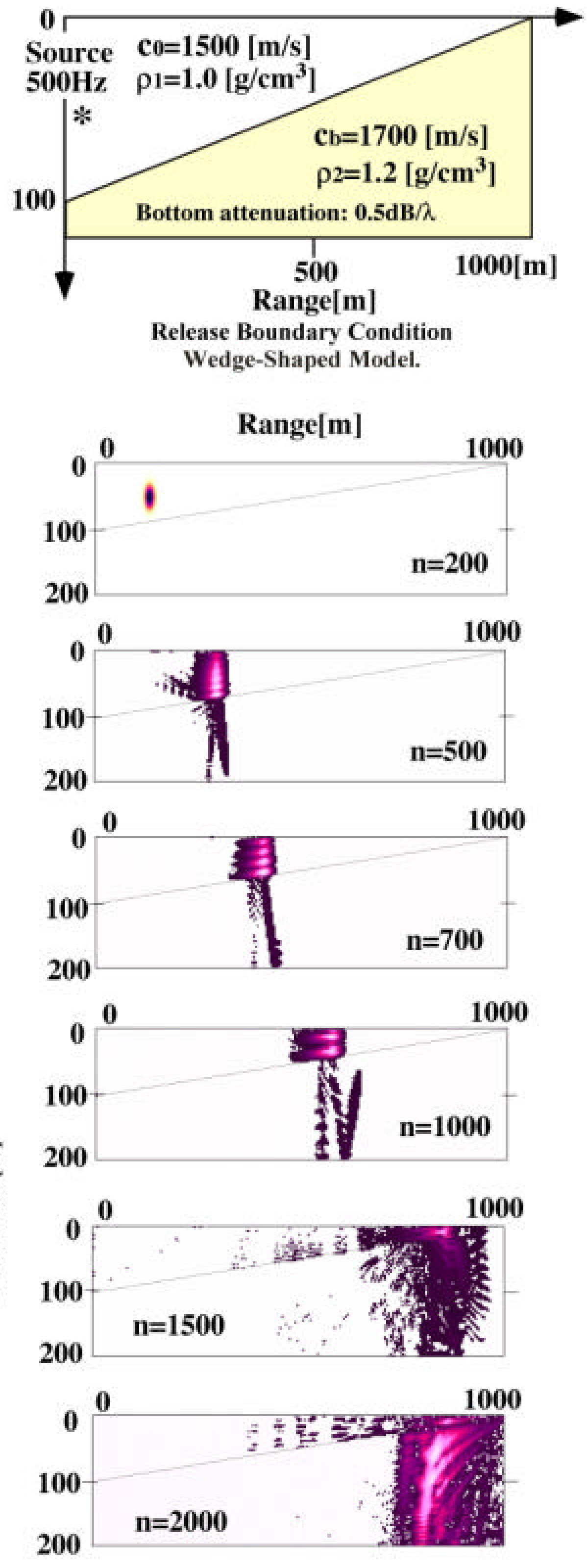

Fig.3 Pulse propagation in shallow water.