SOUND PROPAGATION CHARACTERISTICS IN ARCTIC OCEAN CALCULATED BY ELASTIC PE METHOD USING ROTATED PADE APPROXIMATION

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ABSTRACT: The purpose of this paper is to describe an accurate computational method for underwater acoustic propagation problems based on elastic parabolic equation (PE) with Rotated Pade approximation series in the Arctic Ocean. To confirm the validity of the proposed method, typical examples of sound propagation in benchmark model are calculated as comparing with the conventional fluid PE model. Propagation loss is estimated under the condition that the propagation path is 300 km in ocean covered with ice. The result shows the difference of propagation loss between ice and no-ice in ocean surface.

INTRODUCTION

It is important to accurately obtain the underwater sound propagation characteristics for the development of ocean acoustic tomography. The numerical calculation program of sound propagation for various oceans requires computation time and computer resources in general. The parabolic equation (PE) introduced by Tappert el. al.[1)] has a powerful simulation technique for various underwater sound propagation problems. For the last two decades, many reports on the PE method have been published.[2,3,4)] In recent years, an improved "wide angles" wave equation with operator splitting or the use of a recursive Pade formula has been introduced.[5,6,7)] An elastic PE method has the ability to take account of the influence of the shear wave in the ocean bottom.[8,9,10)] Recently, various programs of the Acoustic Thermometry of the Ocean are being planned in the Arctic Ocean. However, it is difficult to analyze precisely the underwater sound propagation in the Arctic Ocean, because the abrupt variations of the density and sound speed occur at interfaces in the ice-water and water-sediment.

In this paper, by comparing waveforms of acoustic propagation sounds in Arctic Ocean between ice and no-ice on ocean surface, we clarify the influence of shear wave in surface ice. In order to obtain more accurate solutions in Arctic Ocean, an elastic PE method is combined with the complex rotated Pade series. A computer simulation of acoustic propagation is performed under the condition that the propagation path is 300 km in ocean of 3.5 km depth.

FORMULATION OF THE ELASTIC PARABOLIC EQUATION MODEL

Two-dimensional equations in coordinates (*x*,*z*) are given by Newton's motion equation, where *x* and *z* are the horizontal propagation direction and depth direction below the ocean surface and *u* and *w* are displacements in the *x* and *z* directions. Azimuthal symmetry in the cylindrical coordinates is assumed.

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left(\lambda\Delta+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right),$$
$$\rho\frac{\partial^2 w}{\partial t^2}=\frac{\partial}{\partial z}\left(\lambda\Delta+2\mu\frac{\partial w}{\partial z}\right)+\frac{\partial}{\partial x}\left(\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right), \quad (1)$$

$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}, \quad (2)$$

where λ and μ are Lame constants that are given by density ρ, longitudinal sound speed, and shear sound speed.

The two-dimensional dilatation equation is given by differentiating eq. (1) with respect to *x* and *z* and summing them. For the case of constant wave propagation in a stratified medium, we must solve the following equations.

$$(\lambda+2\mu)\frac{\partial^2\Delta}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2\Delta}{\partial z^2}+\rho\,\omega^2\Delta+2\frac{\partial\mu}{\partial z}\frac{\partial^2 w}{\partial x^2}$$
$$+\omega^2\frac{\partial\rho}{\partial z}w+\left(\frac{\partial\lambda}{\partial z}+2\frac{\partial\mu}{\partial z}\right)\frac{\partial\Delta}{\partial z}+\frac{\partial}{\partial z}\left(\frac{\partial\lambda}{\partial z}\Delta\right)+2\frac{\partial}{\partial z}\left(\frac{\partial\mu}{\partial z}\frac{\partial w}{\partial z}\right)=0, \quad (3)$$
$$\mu\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial z^2}+\rho\omega^2 w+(\lambda+\mu)\frac{\partial\Delta}{\partial z}+2\frac{\partial\mu}{\partial z}\frac{\partial w}{\partial z}+\frac{\partial\lambda}{\partial z}\Delta=0.$$

Equation (3) gives the elastic wave equation of the outgoing sound as follows,

$$\frac{\partial}{\partial x}\begin{pmatrix}\Delta\\ w\end{pmatrix}=ik_0\left(-1+\sqrt{1+\frac{\mathbf{L}^{-1}\left(\mathbf{M}-k_0{}^2\mathbf{L}\right)}{k_0{}^2}}\right)\begin{pmatrix}\Delta\\ w\end{pmatrix}, \quad (4)$$

where $k_0=\omega/c_0$, c_0 is representative sound speed and matrices **L** and **M** contain differential operators of the z direction in eq. (4). The square root function $f(X)$ given by eq. (4) as

$$f(X)=-1+\sqrt{1+X}, \quad (5)$$

$$X=\frac{\mathbf{L}^{-1}\left(\mathbf{M}-k_0{}^2\mathbf{L}\right)}{k_0{}^2}, \quad (6)$$

where X contains operator matrices **L** and **M** in eq. (4). To solve eq. (4) numerically, the square root function $f(X)$ is expanded to Pade series $g(X)$.

$$f(X)\approx g(X)=\sum_{j=1}^{n}\frac{a_{j,n}X}{1+b_{j,n}X}, \quad (7)$$

where $a_{j,n}$ and $b_{j,n}$ are Pade coefficients and a real number. In this paper, it is proposed to adopt the rotated Pade series by Milinazzo for the elastic PE in order to avoid calculation error. The square root function $f(X)$ is approximated by the following rotated Pade series expansion.[11)]

$$f(X)\approx g(X)=e^{i\alpha/2}\sum_{j=1}^{n}\frac{a_{j,n}\left[(1+X)e^{-i\alpha}-1\right]}{1+b_{j,n}\left[(1+X)e^{-i\alpha}-1\right]}, \quad (8)$$

where α is rotation angle and is an arbitrary constant. Substituting eq. (8) into eq. (4), we obtain

$$\frac{\partial}{\partial x}\begin{pmatrix}\Delta\\ w\end{pmatrix}=ik_0\sum_{j=1}^{n}\frac{A_{j,n}X}{1+B_{j,n}X}\begin{pmatrix}\Delta\\ w\end{pmatrix}, \quad (9)$$

where n is the number of terms and $A_{j,n}$ and $B_{j,n}$ are rotated Pade coefficients which are calculated from $a_{j,n}$, $b_{j,n}$, and α. Rotated Pade coefficients are complex numbers and maintain good accuracy in case of X<-1.

NUMERICAL RESULTS AND DISCUSSION

ASA Benchmark problem

To confirm the accuracy of the proposed method, the ASA benchmark problem is performed. Figure 1 shows the sound fields calculated by elastic PE using the rotated Pade series. The solution agrees closely with a solution by the hybrid coupled wave-number integration approach.[12] It is clear that the elastic PE method can analyze sound propagation in shallow water correctly.

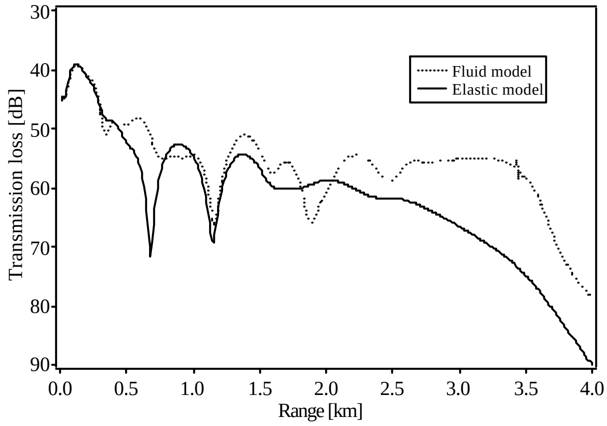
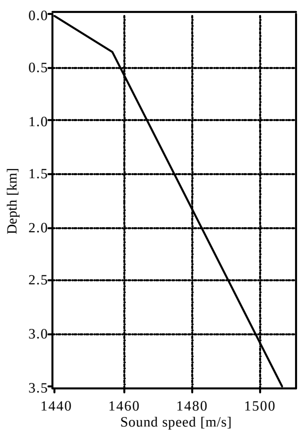


Fig. 1 Results of ASA Benchmark Problem.

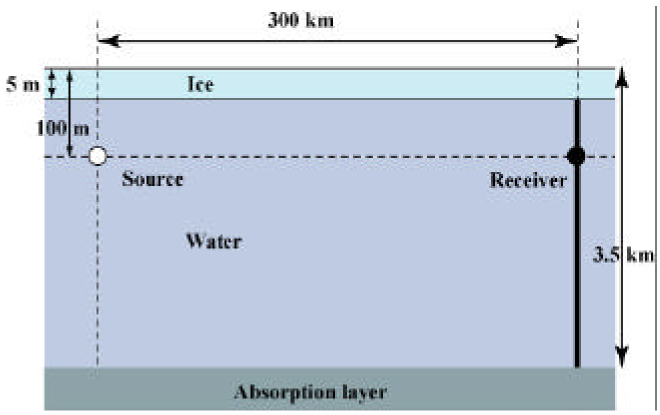
Calculation of pulse propagation in Arctic Ocean

We have simulated on the basis of the data by tomography experiment in the Greenland Sea area. In the winter's Arctic Ocean, the sound speed profile is dependent on only water pressure, since the water temperature is almost fixed. Therefore, the sound speed profile increases linearly with the depth, and has a very large jump of sound speed at boundary interface of the ice/water as shown in Fig. 2. The bottom medium is absorption layer. Acoustical parameters of density, sound speed and attenuation constants for ice layer are shown in Table I.

The propagation waveform was calculated by elastic PE and FFT methods in order to predict the arrival pulse train. The propagated sound in frequency domain is given by the product of the impulse response of the propagation pulse and the frequency characteristic of response of



(a) Sound speed profile



(b) Calculation model of Arctic Ocean in winter.

Fig.2 The elastic model of the Arctic Ocean and Sound speed profile in winter season.

Table I Acoustical parameters of ice

ρ [g/cm^3]	c_p [m/s]	α_p[dB/λ]	c_s [m/s]	α_s [dB/λ]
1	3000	0.18	1400	1.08

the projected pulse. It is assumed that a Green's wide angle source [13)] was placed 100 m below the ocean surface and projected the sound pulse given by the following equation.

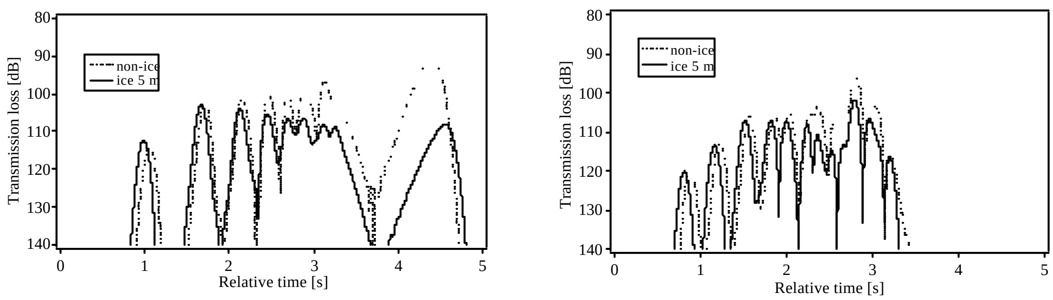
$$s(t) = \sin\left(\omega(t-t_0)\right)\exp\left(-150.0(t-t_0)^2\right). \qquad (10)$$

To calculate the frequency characteristics of the impulse at the receiving point, the sound pressure of continuous waves was calculated 1024 times by the elastic PE while changing the center frequency at every 10Hz. The predicted waveform propagated in Arctic Ocean was finally obtained from de-convolution using the inverse FFT method.

Figure 3 shows the predicted received waveforms calculated by the elastic PE with rotated Pade approximation. Solid and dashed lines are shown by covered with ice layer model and without ice model, respectively. The last pulse near 4.2 second is very different between two models, because shear wave exists in ice layer as shown in Fig. 3(a). Sound wave reflected under sea-ice is propagated to the downward and the sound wave refracted in water is propagated to the ice layer again. Therefore, it is predicted that the effect of the ice layer is greatly affected, when the sea surface is being covered with ice layer. Because the last pulse incidents at a small grazing angle in ice layer, shear wave generates greatly. Therefore, sound propagation loss becomes large compared with no-ice. As shown in Fig. 3(b), received waveform does not difference between two models when depth is 1000 m. Because sound pulse is little number of times that hits ice, reflection loss becomes small. These results confirm the usefulness of the elastic PE method.

CONCLUSIONS

The applicability of the elastic PE method with the Rotated Pade approximation to acoustic propagation in shallow water and has been presented. It is demonstrated that the acoustic propagation properties in shallow water can be predicted more accurately than by the



(a) Received depth is 100 m

(a) Received depth is 1000 m

Fig. 3 Estimated receiving pulse calculated by elastic PE in Arctic Ocean.

conventional fluid PE method. The elastic PE method applied in the numerical analysis of the Arctic Ocean. It is expected that the influence of shear wave in ice layer will appear in the acoustical propagation characteristics in Arctic Ocean with covered ice layer. In the future, we intend to calculate range dependent models having more complex sound profiles with irregular interfaces.

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REFERENCE

[1] F. D. Tappert, J. B. Keller and J. S. Papadakis: Lecture Notes in Physics **70** (1977) 224.

[2] L. Boden, J. B. Bowlin and J. L. Spiesberger: J. Acoust. Soc. Am. **90** (1991) 954.

[3] D. F. St. Mary and D. Lee: Comput. Acoust. (1988) 409.

[4] T. Anada, T. Fujii, K. Morita, T. Tsuchiya and N. Endoh: Jpn, J. Appl. Phys. **36** (1997) 3336.

[5] T. Tsuchiya, T. Okuyama, T. Fujii, N. Endoh and T. Anada: Jpn, J. Appl. Phys. **38** (1999) 3351.

[6] M. Hada, T. Anada, T. Tsuchiya and N. Endoh: Jpn, J. Appl. Phys. **38** (1999) 3361.

[7] T. Tsuchiya, T. Okuyama, N. Endoh, T. Anada, T. Nakamura and I. Nakano: J. Appl. Phys. **39** (2000) 3212.

[8] M. D. Collins: J. Acoust. Soc. Am. **86** (1989) 1459.

[9] R. R. Greene: J. Acoust. Soc. Am. **77** (1985) 1991.

[10] S. C. Wales and J. J. Mccoy: Wave Motion **5** (1983) 99.

[11] F. A. Milinazzo, C. A. Zala and G. H. Brooke: J. Acoust. Soc. Am. **101** (1997) 760.

[12] J. T. Goh and H. Schmidt: J. Acoust. Soc. Am. **100** (1996) 1409.

[13] F. B. Jensen, W. A. Kuperman, M. B. Porter and H. Schmidt: *Computational Ocean Acoustics*, ed. R. T. Beyer (AIP Press, Woodbury, New York, 1990) Chap. 4, p.348

[14] A. Bamberger, B. Engquist, L. Halpern, and P. Joly: SIAM J. Appl. Math. **48** (1988) 129.