NEW NUMERICAL CALCULATING METHODS FOR SOUND PROPAGATION MODELING IN JAPAN

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ABSTRACT: Several new numerical methods for sound propagation modeling developed by authors are described. The Finite Difference Time Domain method and Wide Angel Parabolic Equation Method are developed for an accurate sound propagation modeling. The FDTD method shows the usability in shallow water, though it requires a huge computer resource. The wide angle PE method based on the high order Pade series expanded by exponential operator shows that it has a potential for higher accuracy and shorter computing time.

INTRODUCTION

The solution of inversion problem in Ocean acoustic tomography (OAT) needs more accurate a priori information about sound propagation characteristics in the ocean.[1] In this paper, several new numerical methods for sound propagation developed by authors are described. The Finite Difference Time Domain (FDTD) method and wide angel elastic Parabolic Equation method are developed for an accurate sound propagation modeling.

The recent development of computer system enables the FDTD which is a famous calculation method in electro-magnetic fields[2] to be applied in the acoustics.[3] To confirm the validity of the FDTD that takes account of attenuation in a lossy seabed, the test problem of the PE II Workshop[4] was solved. The contour map of sound pressure and propagated pulse waveform has been calculated.

An accurate computational method based on elastic parabolic equation (PE) with complex Pade approximation series in the Arctic Ocean is also described. The comparison to the conventional fluid PE model shows the difference of sound propagation loss caused by the shear wave in an ice layer on the sea surface.

FINITE DIFFERENCE TIME DOMAIN METHOD

Formulation of FDTD

The basic equations of the FDTD method, that is taking account of attenuation, are given as follows [5]:

$$-\frac{1}{\mathrm{K}}\frac{\partial p}{\partial t} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \tag{1}$$

$$-r\frac{\partial v_x}{\partial t} = \frac{\partial p}{\partial x} + h v_x \tag{2}$$

$$-r\frac{\partial v_y}{\partial t} = \frac{\partial p}{\partial y} + h v_y \tag{3}$$

where p is sound pressure, v is the particle velocity, K is the bulk modulus, r is the density and t is time. The second part of the right hand side in eqs. (2) and (3) show an attenuation of the medium cased by absorption. The sound velocity c and the resistance coefficient h that is proportional to the particle velocity are given as follows,

$$c = w / \sqrt{g_1^2 - g_2^2} \tag{4}$$

$$h = \frac{2g_1g_2}{\sqrt{g_1^2 - g_2^2}}\, rc \tag{5}$$

where w is angular frequency. g_1 and g_2 are the wave number and attenuation constant, respectively.

Sound pressure field of test problem in shallow water

The FDTD method calculated the sound field of the test problem discussed in the PE II Workshop in shallow water. The convex seabed inclined at 2.86 degrees has an absorption coefficient of 0.5dB/λ and its sound velocity is 1700m/s, as shown in Fig.1. A sound source placed at the 100m depths projects a continuous sound at 25 Hz. Increments in space $\Delta x = \Delta y = 1.0$ m and in time $\Delta t = 0.2$ ms are used for calculation. The spreading loss is compensated in the calculated value by FDTD. Mur`s first order absorbing boundary conditions are provided because reflection waves from the outer boundary must be eliminated. The calculated contour map of the transmission loss is shown in Fig.2. The transmission loss pattern at 150 m depths by FDTD coincides with results from the coupled-mode method.[4]

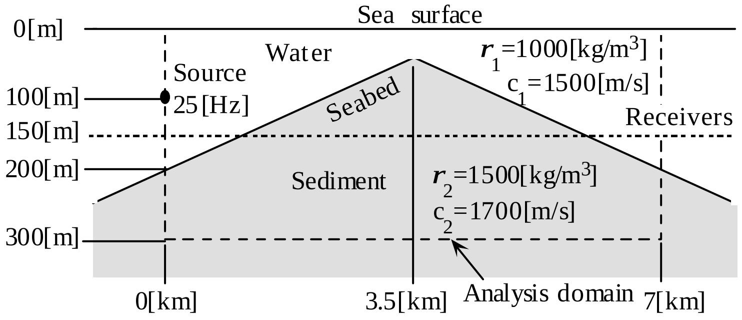

Fig.1 Geometry of the test problem discussed in PE II Workshop.

The FDTD method is capable of calculating the instantaneous sound. The propagation of pulse sounds is visualized by FDTD in the same test model. To reduce the grid dispersion error in FDTD, a Gaussian pulse signal is used to modulate the carrier sound of 25Hz, because It is necessary to suppress the side-lobes in the frequency domain of the pulse sound. The refracted pulse into the sediment is much more decayed than the pulse in water as shown in Fig.3. Because of the interference between sea surface and seabed, the pulse duration in the ocean is longer than that of projected pulse. The returning pulse from the top of the convex seabed is also clearly shown.

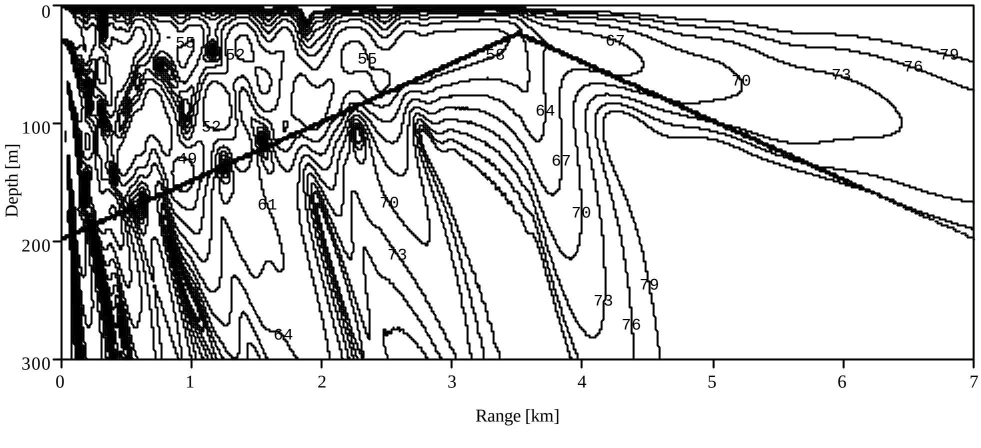

Fig.2 Contour map of the transmission loss by the FDTD method.

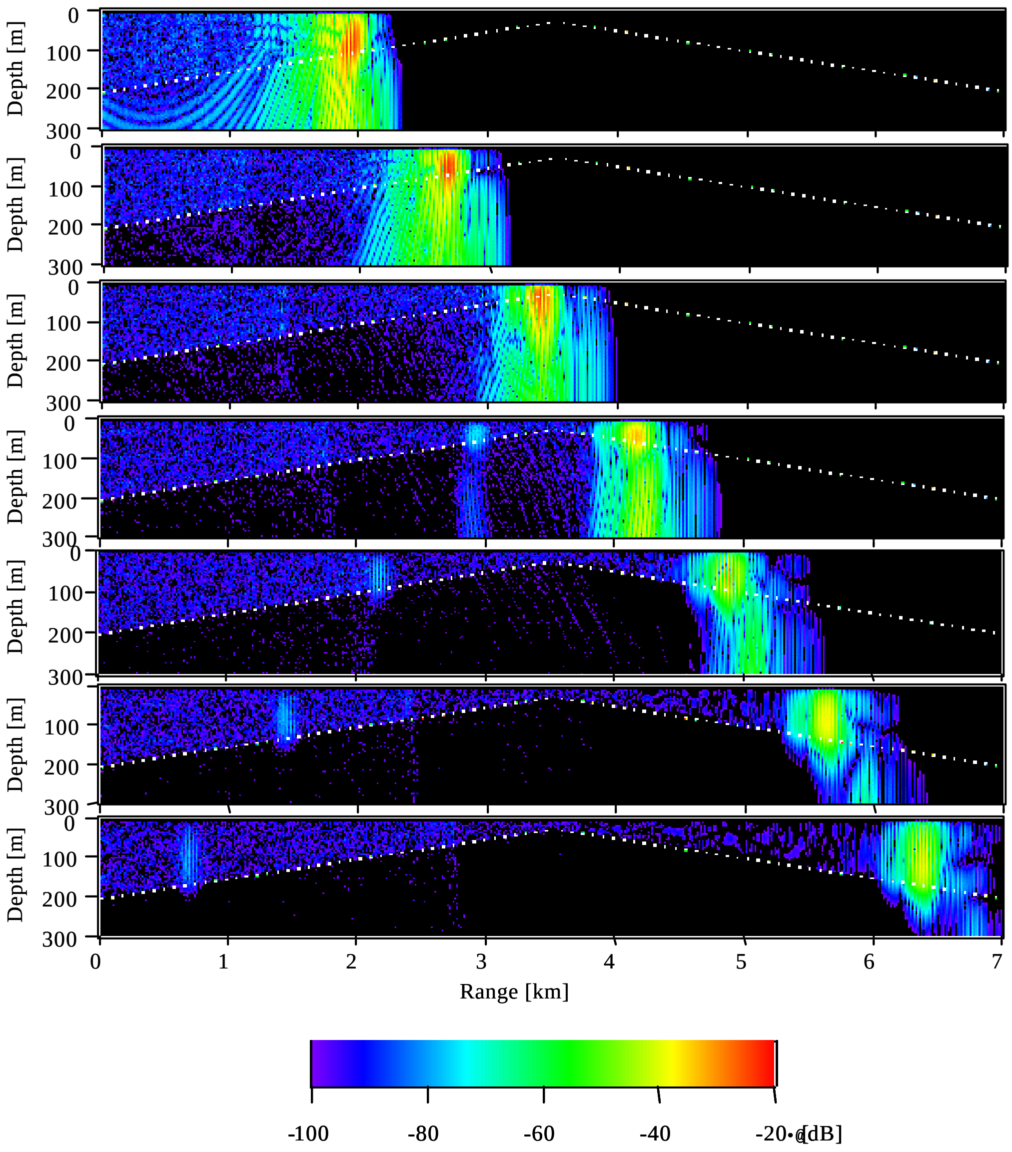

Fig. 3 Propagation of the pulse sound in shallow water with a convex seabed.

WIDE ANGLE ELASTIC PARABOLIC EQUATION

Formulation of wide angle elastic PE

The wide angle PE method based on the high order Pade series expanded by exponential operator shows that it has a potential for higher accuracy and shorter computing time.

Two dimensional dilatation equation is given by eq.(6) as follows,

$$(\lambda+2\mu)\left[\frac{\partial^2\Delta}{\partial x^2}+\frac{\partial^2\Delta}{\partial z^2}\right]+\rho\,\omega^2\Delta+2\frac{\partial\mu}{\partial z}\frac{\partial^2 w}{\partial x^2}+\omega^2\frac{\partial\rho}{\partial z}w+\left(\frac{\partial\lambda}{\partial z}+2\frac{\partial\mu}{\partial z}\right)\frac{\partial\Delta}{\partial z}+\frac{\partial}{\partial z}\left(\frac{\partial\lambda}{\partial z}\Delta\right)+2\frac{\partial}{\partial z}\left(\frac{\partial\mu}{\partial z}\frac{\partial w}{\partial z}\right)=0$$

$$\Delta=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}. \qquad (6)$$

where λ and μ are Lame constants. They are given by density ρ, longitudinal sound speed c_p, and shear sound speed c_s. Equation (6) gives the elastic wave equation of the outgoing sound in homogeneous condition [11], if $k_0=\omega/c_0$, c_0 is representative sound speed.

$$\frac{\partial}{\partial x}\begin{pmatrix}\Delta \\ w\end{pmatrix}=ik_0\left(-1+\sqrt{1+\frac{\mathbf{L}^{-1}\left(\mathbf{M}-k_0^{\ 2}\mathbf{L}\right)}{k_0^{\ 2}}}\right)\begin{pmatrix}\Delta \\ w\end{pmatrix} \tag{7}$$

An approximated Pade series is applied to the square root in the above wave equation (7). B.R.Wetton reported that calculation error based on real Pade series rapidly increased in the solution of outgoing wave equation.[14] In this paper, it is proposed that a complex Pade series are adopted to eq.(7) as follows,

$$f\left(X\right)\approx e^{i\alpha/2}\left[1+\sum_{j=1}^{n}\frac{a_n\left(\left(1+X\right)e^{-i\alpha}-1\right)}{1+b_n\left(\left(1+X\right)e^{-i\alpha}-1\right)}\right] \tag{8}$$

where α is rotating angle and is an arbitrary constant.

Propagation loss in the Arctic Ocean

To confirm the accuracy of the proposed PE method, sound propagation pulses are calculated in the Arctic Ocean with a common sound speed profile. It is supposed that the ice layer of 5 m covers the sea surface and the sound speed increases monotonously to the bottom. Figure 4 shows the receiving sound waveforms calculated by elastic PE using complex Pade. Because of the shear wave in ice layer on the sea surface, the difference of propagation loss around 4s is clearly shown in the figure. Figure 5 shows the receiving pulses as a function of depth. The comparison to the conventional fluid PE model shows the difference of sound propagation loss caused by the ice, though it is not shown here.

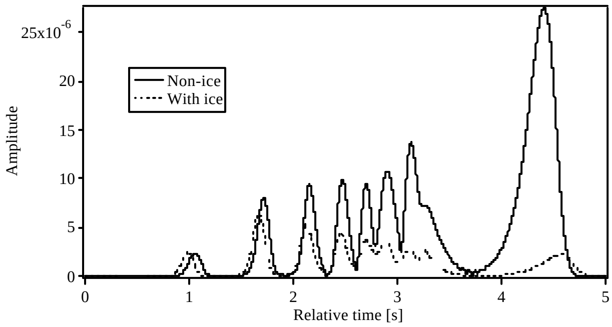

Fig. 4 Estimated receiving pulses calculated by elastic PE in the Arctic Ocean.

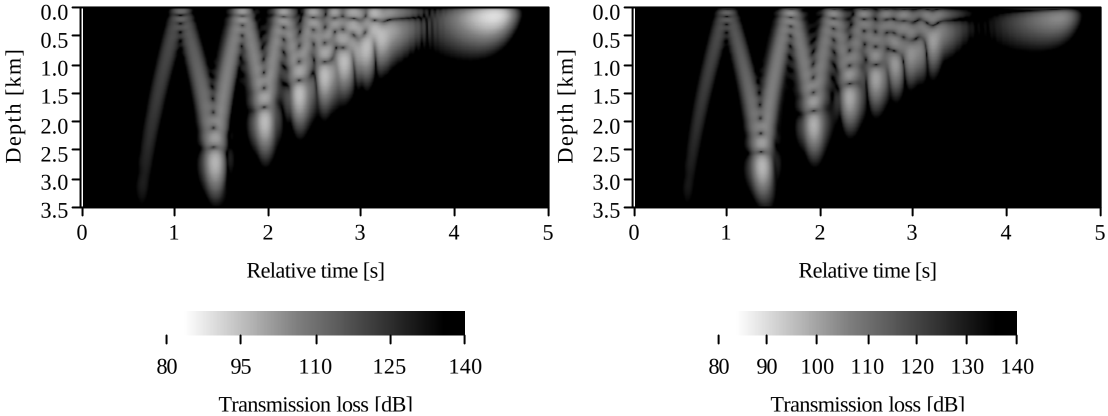

Fig.5 Pulse sound in the Arctic Ocean (Left:without ice layer, Right:coverd with ice layer).

CONCLUSIONS

The FDTD method is proposed to calculate the continuous and pulse sound field in shallow water with a convex lossy seabed. In the test problem of the PE II Workshop, the transmission loss pattern agrees well with the coupled-mode method when the continuous wave is projected. The propagation of pulse sound is visualized by FDTD in the same test model. We can clearly see the incoming pulse has returned from the top of the convex seabed. The FDTD method also shows the pulse waveforms with interference between direct wave and reflected waves. It is useful for us to recognize the sound propagation not only in water but also in sediment. These results show the validity of the FDTD method.

The applicability of the elastic PE method with complex Pade approximation in the Arctic Ocean has been described. It is demonstrated that the acoustic propagation properties can be predicted more accurately than by the conventional fluid PE method. It is expected that the influence of shear wave in an ice layer is appeared in an acoustical propagation characteristics in the Arctic Ocean. In the future, we intend to calculate range dependent models having more complex sound profiles with an ice layer of irregular bottom.

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