STATISTICAL EVALUATION OF THE UNDERWATER DETECTION

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ABSTRACT

Statistical evaluation of the passive structure for the vessel detection is considered. The passive detection structure is based on the statistical likelihood ratio test and on the Neyman-Pearson statistical criterion. The assumption is that a vessel is approximately so-called noisy vessel. It means that the underwater acoustic vessel noise is approximately a stationary ergodic stochastic zero mean Gaussian process with finite variance. Then the source of the vessel noise is approximately only phenomenon of the vessel propeller. The interference (ambient deep-sea noise) is a stationary ergodic stochastic zero mean Gaussian process with finite variance too.

INTRODUCTION

Each vessel is a platform with many acoustic sources. These sources are mutually statistically independent. Their sound emissions through the vessel structure form the acoustic field in the sea mass around the vessel. The sound emission into the sea mass around the vessel is especially interesting because the result of this emission is the vessel underwater acoustic noise or the vessel underwater acoustic signal. The vessel underwater acoustic signal is the fundamental phenomenon to detect the vessel by means of the passive sonar. Therefore the research of the vessel underwater acoustic signal is very important. The research results have to enable the vessel constructors to reduce the power of vessel acoustic sources and so to reduce the intensity of the underwater acoustic field around the vessel. In this way, the range of the vessel detectability decreases too and the vessel becomes acoustically invisible for the longer distances. On the other hand, the passive sonar constructors tend to construct the sonar with maximal sensitivity to detect so weak underwater vessel acoustic noise.

The sound sources that are on a vessel we can group in two different groups. In the first group we have the sources of cavitation due to the vessel propellers. In the second group we have the sources of sinusoidal oscillations due to operating machinery, mechanisms and propellers. The cavitation is a Gaussian stochastic process with wide-band spectrum. The sinusoidal oscillations are stable and/or unstable sinusoidal oscillations with the narrow-band spectra [1]. So, the vessel underwater acoustic signal is assumed to be a sum of a stochastic process and a finite number of sinusoidal oscillations. The stochastic process is Gaussian and is due to the cavitation phenomenon of the vessel propellers [1]. The sinusoidal oscillations have random phases, approximately stable amplitudes and are due to vessel operating machinery, mechanisms and propellers. The vessel underwater acoustic noise, far away of the vessel, propagates trough the sea as an acoustic medium. The sea has its own underwater noise (ambient noise) that is of stochastic nature. So, the passive vessel detection is statistical detection. The ambient sea noise, as a stochastic process, in the deep-sea locations has a Gaussian distribution too [2].

We assumed that the underwater acoustic noise, of a noisy vessel, is a sum of a zero mean Gaussian stochastic process with a finite variance due to propellers cavitation and of a finite number of sinusoidal waves with extremely small amplitudes. The deep-sea underwater ambient acoustic noise is supposed to be a zero mean Gaussian stochastic process with a finite variance as well. We considered the probability density function of the instantaneous values of the underwater acoustic noise of the noisy vessel and of the deep-sea underwater ambient acoustic noise. To detect the vessel underwater acoustic noise we chose the algorithm of the optimal statistical detection with the likelihood ratio statistical test and the Neyman-Pearson statistical criterion.

To evaluate the algorithm of the optimal statistical detection we can choose two statistical methods: the exact or direct method and the approximate or indirect method. The direct evaluation measure is the value of detection probability versus the value of the false alarm probability with signal-to-noise ratio as a parameter. The indirect evaluation measure is the so-called deflection coefficient that is the function of the difference between statistical expectations of the two possible probability density functions of the receiving signal.

UNDERWATER ACOUSTIC SIGNAL

Vessel Underwater Acoustic Noise

As stated before, the vessel underwater acoustic noise is a sum of the Gaussian stochastic process and a finite number of sinusoidal waves. We assume that the Gaussian stochastic process is a stationary ergodic and zero mean with finite variance, and that the sinusoidal waves have constant amplitudes and random phases uniformly distributed in the interval $(0,2\pi)$. The mathematical model of such a waveform has the following form

$$x_Q(t) = \sum_{q=1}^{Q} a_q \sin(2\pi f_q t + \varphi_q) + s(t) \tag{1}$$

where: a_q - is constant amplitude of q-th sinusoidal wave,
f_q - is constant frequency of q-th sinusoidal wave,
Q - is a finite number of sinusoidal waves,
φ_q - is random phase of q-th sinusoidal wave,
$s(t)$ - is zero mean Gaussian stochastic process.

The probability density function of the instantaneous values of (1) for the $Q=1$ is of the following form [3, 4]

$$p(x_1, a_1) = \frac{1}{\sigma_s\sqrt{2\pi}} \sum_{m=0}^{\infty} \frac{1}{m!}\left(\frac{-x_1^2}{2\sigma_s^2}\right)^m {}_1F_1\left(m+\frac{1}{2}, 1, \frac{-a_1^2}{2\sigma_s^2}\right) \tag{2}$$

where: σ_s^2 - is finite variance of the Gaussian stochastic process $s(t)$,
${}_1F_1(.,.,.)$ - is confluent hypergeometric function.

The probability density function of the instantaneous values of the vessel underwater acoustic noise (2) becomes approximately Gaussian for small values of the amplitude a_1. This situation is present for so-called noisy vessel. The probability density function of the instantaneous values of the noisy vessel underwater acoustic noise (2) becomes now approximately of the following form

$$p(x_Q) = \frac{1}{s_s\sqrt{2p}} exp\left(-\frac{x_Q^2}{2s_s^2}\right). \qquad (3)$$

The power spectral density of (1) consists of Q spectral lines with finite and constant amplitudes and of wide-band continuous nonwhite spectrum with a slope of about -6±1 dB/octave [1]. The power spectral density of (1) for a noisy vessel is only the wide-band continuous nonwhite spectrum with a slope of about -6±1 dB/octave.

Vessel Underwater Acoustic Receiving Signal

On the receiving location, far away of the vessel, the vessel underwater acoustic noise has an addendum more – ambient deep-sea noise $n(t)$. Therefore the waveform of the receiving signal has the following form

$$y_{Qr}(t) = x_{Qr}(t) + n(t) \qquad (4)$$

and by means of Eq. (1) the Eq. (4) becomes

$$y_{Qr}(t) = \sum_{q=1}^{Q} a_{qr}\, sin(2pf_q t + f_q) + s_r(t) + n(t) \qquad (5)$$

where $n(t)$ is zero mean Gaussian stochastic process with the finite variance $(s_n)^2$, and with the probability density function of the instantaneous values

$$p(n) = \frac{1}{s_n\sqrt{2p}} exp\left(-\frac{n^2}{2s_n^2}\right). \qquad (6)$$

The power spectral density of the ambient deep sea noise $n(t)$ is also the wide-band continuous with nonwhite spectrum and a slope of about -6±1 dB/octave.

On the receiving location the waveform of the noisy vessel underwater acoustic signal has the following form

$$y_{Qr}(t) = s_r(t) + n(t) \qquad (7)$$

where y_{Qr}(t) is zero mean ergodic stationary Gaussian stochastic process with the finite variance

$$s_{yr}^2 = s_{sr}^2 + s_n^2 . \qquad (8)$$

Now, we can write the probability density function of the instantaneous values of the noisy vessel underwater acoustic receiving signal $y_{Qr}(t)$, or

$$p(y_{Qr}) = \frac{1}{s_{yr}\sqrt{2p}} exp\left(-\frac{y_{Qr}^2}{2s_{yr}^2}\right) \qquad (9)$$

or, by means of Eq. (8), Eq. (9) becomes

$$p(y_{Qr}) = \frac{1}{\sqrt{2p(s_{sr}^2 + s_n^2)}} exp\left[-\frac{y_{Qr}^2}{2(s_{sr}^2 + s_n^2)} \right]. \qquad (10)$$

But, if the noisy vessel underwater acoustic noise is not present, the Eqs. (7) and (8) become respectively

$$Y_{Qr}(t) = n(t) \quad \text{and} \quad s_{yr}^2 = s_n^2 \qquad (11)$$

and Eq. (10) becomes

$$p(y_{Qr}) = \frac{1}{s_n \sqrt{2p}} exp\left(-\frac{y_{Qr}^2}{2s_n^2} \right). \qquad (12)$$

DETECTION ALGORITHM

Now we can write the general likelihood ratio for the case when the noisy vessel underwater acoustic receiving signal is defined with Eqs. (7) and (11). The likelihood ratio of the instantaneous values is of the following form

$$L = \frac{p_1(y_{Qr})}{p_0(y_{Qr})} \qquad (13)$$

where: - $p_1(y_{Qr})$ is probability density function when the signal is present and is defined by Eq. (10),
- $p_0(y_{Qr})$ is probability density function when no signal is present and is defined by Eq. (12).

Now we can write an approximated form of Eq. (13) for the case when a vessel is noisy, or

$$L \approx exp\left(\frac{y_{Qr}^2}{2s_n^2} r \right), \quad \text{where} \quad r = \frac{s_{sr}^2}{s_n^2} << 1 \ . \qquad (14)$$

Eq. (13) is the general form of likelihood ratio for passive detection of the vessel underwater acoustic noise. Eq. (13) is really the detection algorithm for the alternative hypothesis H_1: $L \geq L_0$ and for the null hypothesis H_0: $L < L_0$, where L_0 is a threshold defined by statistical criterion.Eq. (14) is the special case when we detect so called "noisy vessel" or when the vessel underwater acoustic noise approximately consists only of the cavitation noise. For this case we can write the detection algorithm for the alternative hypothesis H_1: $(y_{Qr})^2 \geq L_{0nv}$ and for the null hypothesis H_0: $(y_{Qr})^2 < L_{0nv}$, for

$$L_{0nv} \approx 2s_n^2 r^{-1} ln L_0 \ , \quad \text{where} \quad r << 1 \ . \qquad (15)$$

DETECTION EVALUATION

To evaluate the underwater passive detection structure performance we have to consider the probability of detection, or

$$D = \int_c^{\infty} p_1[l(y)] dl(y) \qquad (16)$$

where $l(y)$ is the reduced likelihood ratio, and p_1 is the probability of the alternative hypothesis H_1. The threshold level c, for the Neyman-Pearson criterion, is defined by means of the so called false alarm probability α, or

$$\alpha = \int_{c}^{\infty} p_0[l(y)]dl(y), \tag{17}$$

where p_0 is the probability of the null hypothesis H_0.

<u>Direct Evaluation Method</u>

For our case the probability density functions p_1 and p_0 are of gamma distribution with degree of freedom equal K, which is a number of discrete samples, or

$$p_1[l(y)] = \frac{l^{\frac{K}{2}-1} exp\left[-\frac{l}{2\left(\sigma_n^2+\sigma_s^2\right)}\right]}{\left[2\left(\sigma_n^2+\sigma_s^2\right)\right]^{\frac{K}{2}} \Gamma\left(\frac{K}{2}\right)} \tag{18}$$

and

$$p_0[l(y)] = \frac{l^{\frac{K}{2}-1} exp\left[-\frac{l}{2\left(\sigma_n^2\right)}\right]}{\left[2\left(\sigma_n^2\right)\right]^{\frac{K}{2}} \Gamma\left(\frac{K}{2}\right)} \tag{19}$$

for

$$l(y) = \sum_{k=1}^{K} y_k^2 \tag{20}$$

where $\Gamma(.)$ is the complete gamma function. Substituting Eqs. (18) and (19) into Eqs. (16) and (17) respectively (with K>30) we reach the one relation that approximate the detection probability, or [5]

$$D(\alpha,\rho,K) = 1 - \Phi\left[\frac{\Phi^{-1}(1-\alpha)+\sqrt{2K}}{\sqrt{1+\rho}} - \sqrt{2K}\right] \tag{21}$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} exp\left(-\frac{t^2}{2}\right)dt \tag{22}$$

is the Laplace integral. The function $D=f(\alpha,\rho)$ is the family of characteristics that is called ROC (Receiver Operating Characteristics) diagram with the α and ρ as the parameters. If we wish to detect a vessel in the real circumstances, the detection probability value D, has to be greater than 0.5. So, for example, if we chose the value of the false alarm probability α=0.001 and if the number of discrete samples is K=20000, with the value of parameter $\rho_{min}\approx 0.03114$, we have the detection probability value D_{min}=0.5 that is a minimum value to detect an unknown vessel. For the maximum detection probability value D_{max}=1 the value of parameter is $\rho_{max}\approx 0.08470$.

<u>Indirect Evaluation Method</u>

Very often we define the so-called deflection coefficient d as the indirect evaluation measure, which is an approximation of the exact evaluation measure. The square of the deflection coefficient is defined as

$$d^2 = \frac{\left[E\{l_1(y)\} - E\{l_0(y)\}\right]^2}{V\{l_0(y)\}} \tag{23}$$

where $E\{.\}$ is statistical expectation, and $V\{.\}$ is variance. The $l_1(y)$ and $l_0(y)$ are reduced likelihood ratios for alternative and null hypotheses respectively.

For the so-called noisy vessel the square of the deflection coefficient becomes of the following form

$$d^2 = \frac{K\sigma_x^4}{2\sigma_n^4} = \frac{K}{2}\left(\frac{\sigma_x^2}{\sigma_n^2}\right)^2 \tag{24}$$

or, by means of the parameter ρ, becomes of the form

$$d^2 = \frac{K}{2}\rho^2 \quad \text{or finally} \quad d(\rho, K) = \rho\sqrt{\frac{K}{2}}\,. \tag{25}$$

For the same values of α and K as before, the deflection coefficients for D_{min} and D_{max} become of the following values d_{min}=3.114 and d_{max}=8.470 respectively.

CONCLUSIONS

The paper shows the final relations of direct and indirect evaluation performance of the underwater passive optimal detection of the so-called noisy vessel. We chose the statistical likelihood ratio test and the Neyman-Pearson criterion for the optimal detection. Eq. (21) shows that the direct or exact evaluation for passive detection of the so-called noisy vessel is a function of the parameters α, ρ and K. On the other hand, Eq. (25) shows that the indirect or approximate evaluation is a function of the parameters ρ and K only. So, the deflection coefficient is an indirect measure of the detection probability *D.*

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