ON MODELLING OF ROOM ACOUSTICS BY A SOUND ENERGY TRANSITION APPROACH

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Alarcão, Diogo [1]; Bento Coelho, J. L. [1]; Tenenbaum, Roberto A.[2]
1 CAPS-Instituto Superior Técnico;
P-1049-001Lisboa, Portugal
Tel. 351 218 419 393
Fax 351 218 465 303
E-Mail: bcoelho@ist.utl.pt
2 COPPE- Federal University of Rio de Janeiro
Rio de Janeiro, Brasil

ABSTRACT

Fast, reliable computation techniques are needed for “real-time” design applications. A method was derived from first statistical principles, by assuming random walks of energy packets in an enclosure. The transition probabilities of the energy transfer matrix are based on the solid angle magnitudes of the enclosure walls, subtended at some particular wall centre. The room impulse response and the steady-state sound pressure level are computed. From the room impulse response, the decay curve and reverberation times are computed and some room sound quality indexes are calculated.

INTRODUCTION

The assessment of room acoustical parameters is of major importance in room acoustics especially at the design stage. Different theoretical models have been applied for the prediction of sound fields in enclosures, but in most of the cases they are derived from three basic methods: the acoustic wave equation, the diffuse-field theory and the geometrical room acoustics.

In this paper, an alternative geometrical-statistical method is presented, describing a simple technique with low computation times, for a very wide range of room geometry configurations [1]. The sound field inside the enclosures is considered to be contributed by discrete energy packets, that are radiated from one, or several, sources. The motion of these energy packets, or sound particles, or phonons, is completely determined by an equation of motion that considers the transition amplitudes, when the sound particle changes its location inside the enclosure. Physical phenomena such as sound absorption in the air and in the surface walls are accounted for.

When considering that successive transitions occur at discrete time intervals, given by the mean reflection time, then the equations represent a homogeneous Markov chain of first order for the energy packets. The basis of this technique was earlier considered by Gerlach and Mellert [2] and by Kruzins and Fricke [3].

THEORY

The sound field inside an enclosure will be seen as resulting from energy packets (sound particles), that are emitted from sound sources. These energy packets are then reflected diffusely from the enclosure surface several times until some receiver detects them.

Let *P(**s**,* 0*)* denote the value of the probability density of the sound particle at point *R*, located over the surface of the enclosure, and given by the position vector ***s*** at time t = 0. Therefore, *P(**s**,* 0*) dS* yields the probability of the particle being located over an infinitesimal surface element *dS* at initial time t = 0. The form of the probability density *P* must obey the normalisation condition for any value of time *t*:

$$\iint_S P(\mathbf{s},\,0)dS = \iint_S P(\mathbf{s},\,t)dS = 1 \qquad (1)$$

Assume that the initial phonon distribution *P(**s**, 0)* is known. To determine the "motion" of the sound particle inside the enclosure with volume *V* and with total surface *S,* it is necessary to determine the value of *P*, as from the statistical physics' master equation:

$$\frac{dP(\boldsymbol{s},\,0)}{dt} = \iint_S W_{dt}^0(\boldsymbol{r}\to\boldsymbol{s})P(\boldsymbol{r},\,0)dS - \iint_S W_{dt}^0(\boldsymbol{s}\to\boldsymbol{r})P(\boldsymbol{s},\,0)dS \qquad (2)$$

which can be integrated over the entire surface *S* of the enclosure. $W_{dt}^0(\boldsymbol{s}\to\boldsymbol{r})$ is the transition amplitude density, or transition probability density per unit time when the sound particle changes from position ***s*** to position ***r*** in the time interval *dt*. The superscript 0 means that the transition amplitude densities are to be calculated at time t = 0.

The probability density *P(**s**, t)* can be calculated as follows:

$$\begin{aligned}
&P(\boldsymbol{s},\,dt) = P(\boldsymbol{s},\,0) + \frac{dP(\boldsymbol{s},\,0)}{dt} = \\
&P(\boldsymbol{s},\,0) + \iint_S W_{dt}^0(\boldsymbol{r}\to\boldsymbol{s})P(\boldsymbol{r},\,0)dS - \iint_S W_{dt}^0(\boldsymbol{s}\to\boldsymbol{r})P(\boldsymbol{s},\,0)dS = \\
&P(\boldsymbol{s},\,0)\left[1 - \iint_S W_{dt}^0(\boldsymbol{s}\to\boldsymbol{r})dS\right] + \iint_S W_{dt}^0(\boldsymbol{r}\to\boldsymbol{s})P(\boldsymbol{r},\,0)dS
\end{aligned} \qquad (3)$$

The quantity given by the integral inside the rectangular brackets equals the transition amplitude density when the phonon passes from the state ***s*** to any other state ***r*** in the time interval *dt*. Equation (3) can thus be written as:

$$\begin{aligned}
&P(\boldsymbol{s},\,dt) = P(\boldsymbol{s},\,0)W_{dt}^0(\boldsymbol{s}\to\boldsymbol{s})dS + \iint_S W_{dt}^0(\boldsymbol{r}\to\boldsymbol{s})P(\boldsymbol{r},\,0)dS \\
&P(\boldsymbol{s},\,dt) = \iint_S W_{dt}^0(\boldsymbol{r}\to\boldsymbol{s})P(\boldsymbol{r},\,0)dS
\end{aligned} \qquad (4)$$

Equation (4) allows the determination of the probability density distribution for the sound particle at all subsequent time *t* by writing:

$$\begin{aligned}
&t = dt_1 + dt_2 + dt_3 + ... + dt_n \\
&P(\boldsymbol{s},\,t) = P(\boldsymbol{s},\,\sum_{k=1}^{n} dt_k) \\
&P(\boldsymbol{s},\,t) = \iint_S ... \iint_S P(\boldsymbol{s_1},\,0)W_{dt}^0(\boldsymbol{s_1}\to\boldsymbol{s_2})W_{dt_2}^{dt}(\boldsymbol{s_2}\to\boldsymbol{s_3})...W_{dt_n}^{dt_{n-1}}(\boldsymbol{s_n}\to\boldsymbol{s})d\boldsymbol{s_1}d\boldsymbol{s_2}...d\boldsymbol{s_n}
\end{aligned} \qquad (5)$$

In equation (5), $W_{dt}^{0}(\boldsymbol{s_1} \rightarrow \boldsymbol{s_2})$ is the transition amplitude density per unit time when the phonon changes its position from an infinitesimal surface element dS_1, located at position vector $\boldsymbol{s_1}$, to an infinitesimal surface element dS_2, located at position vector $\boldsymbol{s_2}$.

The characterisation of the phonon will consider its location inside the enclosure and also the acoustic energy that it carries. Therefore, the probability densities, as given by (5), can be interpreted as the "equation of motion" for the acoustic energy density. If the initial energy density per unit time over the enclosure surface S at time t = 0 is defined as $\boldsymbol{P}(\boldsymbol{s}, 0)$, then equation (5) can be re-written as:

$$\boldsymbol{P}(\boldsymbol{s}, t) = \iint_S \ldots \iint_S \boldsymbol{P}(\boldsymbol{s_1}, 0) W_{dt}^{0}(\boldsymbol{s_1} \rightarrow \boldsymbol{s_2}) W_{dt_2}^{dt}(\boldsymbol{s_2} \rightarrow \boldsymbol{s_3}) \ldots W_{dt_n}^{dt_{n-1}}(\boldsymbol{s_n} \rightarrow \boldsymbol{s}) d\boldsymbol{s_1} d\boldsymbol{s_2} \ldots d\boldsymbol{s_n} \quad (6)$$

with the condition that:

$$\iint_S \boldsymbol{P}(\boldsymbol{s}, 0) d\boldsymbol{s} = \boldsymbol{P} \quad (7)$$

where $\boldsymbol{P}$ is the total acoustic energy per unit time inside the enclosure at t = 0. Equation (6) must be altered in order to include surface sound absorption and air attenuation:

$$\boldsymbol{P}(\boldsymbol{s}, t) = \iint_S \ldots \iint_S \boldsymbol{P}(\boldsymbol{s_1}, 0) W_{dt}^{0}(\boldsymbol{s_1} \rightarrow \boldsymbol{s_2})(1 - \boldsymbol{a}(\boldsymbol{s_1})) \ldots W_{dt_n}^{dt_{n-1}}(\boldsymbol{s_n} \rightarrow \boldsymbol{s})(1 - \boldsymbol{a}(\boldsymbol{s_n})) d\boldsymbol{s_1} d\boldsymbol{s_2} \ldots d\boldsymbol{s_n} \quad (8)$$

The sound attenuation in the air inside V can be taken into account by considering an exponential factor, thus obtaining:

$$\boldsymbol{P}(\boldsymbol{s}, t) = \iint_S \ldots \iint_S \boldsymbol{P}(\boldsymbol{s_1}, 0) W_{dt}^{0}(\boldsymbol{s_1} \rightarrow \boldsymbol{s_2})(1 - \boldsymbol{a}(\boldsymbol{s_1})) e^{-m|\boldsymbol{s_1} - \boldsymbol{s_2}|} W_{dt}^{0}(\boldsymbol{s_2} \rightarrow \boldsymbol{s_3})(1 - \boldsymbol{a}(\boldsymbol{s_2})) e^{-m|\boldsymbol{s_2} - \boldsymbol{s_3}|} \ldots$$
$$W_{dt_n}^{dt_{n-1}}(\boldsymbol{s_n} \rightarrow \boldsymbol{s})(1 - \boldsymbol{a}(\boldsymbol{s_n})) e^{-m|\boldsymbol{s_n} - \boldsymbol{s}|} d\boldsymbol{s_1} d\boldsymbol{s_2} \ldots d\boldsymbol{s_n} \quad (9)$$

where the factor m stands for the air absorption coefficient [4]:

$$m = 5.5 \times 10^{-4} \frac{50}{h} \left(\frac{f}{1000} \right)^{1.7} \quad (10)$$

for the sound frequency f and a relative humidity h.

Equation (9) establishes the correct determination of the phonon motion inside the enclosure with volume V. The product of transition amplitudes in equation (9) means that successive transitions are considered as independent and only depending on the immediate previous transition. The phonon motion can be regarded as a Markov process.

If the entire enclosure surface S is divided into M homogeneous and finite surfaces S_j, and it is further assumed that the probability of finding a sound particle is constant over S_j, the above multiple surface integrals in equation (9) are converted into M sums:

$$\boldsymbol{P}(S_j, t) = \sum_{a1=1}^{M} \ldots \sum_{an=1}^{M} \boldsymbol{P}(S_{a1}, 0) W_{dt}^{0}(S_{a1} \rightarrow S_{a2})(1 - \boldsymbol{a}(S_{a1})) e^{-mD(S_{a1}, S_{a2})} \cdot$$
$$W_{dt_2}^{dt}(S_{a2} \rightarrow S_{a3})(1 - \boldsymbol{a}(S_{a2})) e^{-mD(S_{a2}, S_{a3})} \ldots W_{dt_n}^{dt_{n-1}}(S_{an} \rightarrow S_j)(1 - \boldsymbol{a}(S_{an})) e^{-mD(S_{an}, S_j)} \quad (11)$$

where $D(S_i, S_j)$ is the mean distance between surface S_i and surface S_j. If the transition time intervals are assumed to be equal, i.e. $dt = dt_1 = dt_n = \boldsymbol{t}$, where $\boldsymbol{t}$ is a reference time interval, then:

$$t = k \times dt = k\boldsymbol{t} \tag{12}$$

If the transition amplitudes are assumed to remain invariant with time, then

$$\boldsymbol{P}(S_j, k\boldsymbol{t}) = \sum_{a1=1}^{M} \ldots \sum_{ak=1}^{M} \boldsymbol{P}(S_{a1},\ 0\ ) T_{a1a2} T_{a2a3} \ldots T_{ak\ j} \tag{13}$$

which represents a homogeneous Markov chain of first order and where:

$$T_{ij} = Q_{ij}(1 - \boldsymbol{a}(S_i))e^{-mD(S_i,S_j)} = W(S_i \to S_j)(1 - \boldsymbol{a}(S_i))e^{-mD(S_i,S_j)} \tag{14}$$

with $D(S_i, S_j)$ being the mean distance between surfaces S_i and S_j. Equation (13), together with definition (12), can be re-written in a matrix form [1] (as in [2-3]):

$$[\boldsymbol{P}]_{(k\boldsymbol{t})} = [\boldsymbol{P}]_{(0)} [T]^{\mathbf{k}} \tag{15}$$

where $[P]_{(k\boldsymbol{t})}$ is an M-dimensional row vector with entries $\boldsymbol{P}_{j,k} = \boldsymbol{P}(S_j, k\boldsymbol{t})$, which define the energy density over S_j at time $k\boldsymbol{t}$. $[P]_{(0)}$ is an M-dimensional row vector, called the starting vector:

$$[\boldsymbol{P}]_{(0)} = [\boldsymbol{P}_{1,0}, \boldsymbol{P}_{2,0}, \boldsymbol{P}_{3,0}, \ldots, \boldsymbol{P}_{M,0}] \tag{16}$$

whose entries are determined so as to represent the initial acoustical energy density (or sound particle density distribution) over the various surfaces of the enclosure at t=0. The initial acoustical energy densities $\boldsymbol{P}_{j,0}$ are assumed to be constant over each surface S_j. $[T]^{\mathbf{k}}$ represents the k^{th} matrix power of the $M \times M$ transition matrix T, with entries defined by (14). The entries of the starting vector $[P]_{(0)}$ are defined by considering N omni-directional sound sources radiating spherical waves

$$\boldsymbol{P}_{j,0} = \boldsymbol{P}(S_j, 0) = \frac{1}{4\boldsymbol{p}c} \sum_{l=1}^{N} \frac{\Omega_j^l \Gamma_l(\boldsymbol{J},\boldsymbol{q}) \Pi^l}{S_j} \tag{17}$$

where Ω_j^l is the solid angle of surface S_j subtended at source l, Π^l is the total acoustical energy of source l and $G_l(\boldsymbol{J},\boldsymbol{q})$ is its directivity function.

The reference transition time, $\boldsymbol{t}$, can be defined [5] as $\boldsymbol{t} = 4V / cS$, the average time between successive reflections in a room with diffuse reflecting walls for a large phonon ensemble, where c is the speed of the sound in the air. Equation (15) determines the acoustic energy density inside the enclosure by taking into account the exchange of energy due to successive reflections, or radiations, occurring at discrete time intervals $k\boldsymbol{t}$. If the starting vector $[P]_{(0)}$ is multiplied by matrix T at discrete intervals, defined by the transition time $\boldsymbol{t}$, then the sound energy can be computed in “real time”.

The transition probabilities $Q_{ij} = P(S_i \to S_j)$ must be defined by taking into account the area of each surface, as well as the “viewing angle” of the surfaces, as being viewed, for example, from the centre of S_i ([6], [7]). The probability of a sound particle being radiated from S_i to S_j can be estimated by the solid angle through which S_j is seen from the centre of the surface S_i.

The intensity of the sound, which is scattered into Ω_j, the solid angle of a finite surface S_j subtended at the receiving point {*x, y, z*}, is given by [5]:

$$I_j^k(x,y,z) = \frac{\Pi_{j,k}\Omega_j \times (1-\alpha_j)}{\pi} \tag{18}$$

where $\Pi_{j,k}$ is the incident energy density per second hitting the surface S_j. The total radiated steady-state mean value, I_r, can be given by:

$$I_r(x,y,z) = \sum_{j=1}^{n}\sum_{i=0}^{k} I_j^i(x,y,z) \tag{19}$$

where k is the last transition where the energy densities over the surfaces have become negligible, by comparison with the initial value. The sound pressure level can be obtained from this quantity by using ([1,6]) $I_r = p_r^2/3\rho c$, with ρ being the air density and where the sound field incident on any surface of the enclosure is assumed to be neither direct ($I = p^2/\rho c$) nor diffuse ($I = p^2/4\rho c$). The factor 3 provided a better agreement with predictions for a highly reverberant enclosure.

The contribution of the direct sound, emitted from the source, should be added to obtain the total sound pressure field L_p ([1,6]):

$$Lp = 10\log\left[\frac{c\rho}{(2\times 10^{-5})^2}\left(\frac{1}{4\pi}\sum_{l=1}^{N}\frac{\Pi^l}{r_l^2} + 3I_r(x,y,z)\right)\right] \tag{20}$$

where r_l is the distance between the receiving point {*x, y, z*} and the source *l*.
The impulse response can be determined ([1,6]) by considering the various k transitions over a time interval $k\tau$

$$I_r(k\tau) = \sum_{j=1}^{n}\frac{e_j^k\Omega_j(1-\alpha_j)}{\pi} \tag{21}$$

ROOM SOUND QUALITY INDEXES

Sound quality in rooms is, nowadays, increasing in importance as a scientific task, instead of being left, as it was in the past, to the acoustician's or even the architect's sensitivity. More specifically, one of the main objectives of any simulation technique in rooms is the prediction of its sound quality. Once the room impulse response is calculated, several attributes for the evaluation of the room's sound quality can be determined.

The first of these indexes is the clarity factor, C_{80}, which can be derived from the impulse energy decay, $p^2(t)$, as

$$C_{80} = \frac{\int_0^{80} p^2(t)\,dt}{\int_{80}^{\infty} p^2(t)\,dt} \tag{22}$$

where the numerator means the sound energy of the impulse response in the first 80 ms after the arriving of the direct sound, known as first reflections, and the denominator is the energy of the reverberant sound, that is, the one that arrives after t=80 ms. The clarity factor, expressed in decibels, establishes the degree to which discrete sounds stand apart from one another and is one of the indexes that can be roughly related with the intelligibility of the room. The greater the clarity factor, the more suitable the room is for speech purposes. For music performance, a clarity index of 0 dB is considered as a minimum acceptable for good acoustics.

Another index easily computed from the impulse energy decay is the support factor, *ST1*, which is defined by the following equation

$$ST1 = 10\log \frac{\int_{20}^{100} p^2(t)\,dt}{\int_{0}^{10} p^2(t)\,dt}. \qquad (23)$$

This quality index, like the clarity factor, is a rate, expressed in dB, between two segments of the squared impulse response of the room. The first segment is the impulse energy decay in the time interval from 20 to 100 ms. The second one measures the impulse decay energy in the first 10 ms. The main difference to the clarity factor is that both sound pressure signals must be evaluated at a receiving point, located at one meter from the sound source, which must be omnidirectional. The *STI* is a measure of the degree to which, for example, the sound emitted by a musician's instrument is reflected both to himself and to the neighbourhood.

Since the time average squared sound decay after the sound energy interruption can be easily computed by a reverse integration of the squared impulse response [8, 9], all sound quality indexes depending on the decay curve at any point in the room can be calculated from the corresponding impulse response. The first of them is, of course the reverberation time itself. However, it is well known that the subjective perception by listeners of the sound quality in rooms depends mainly on the initial part of the impulse response [5, 7]. That means that the early decay time, *EDT*, defined as the time spent for the first 10 dB of decay of the sound is also an important index to be considered in a room acoustics assessment.

Another interesting room sound quality index is the bass rate, *BR*, which is defined as the ratio between the low and the midrange reverberation times, as given by

$$BR = \frac{RT_{125} + RT_{250}}{RT_{500} + RT_{1000}}, \qquad (24)$$

where RT_n stands for the reverberation time in the octave band with n as central frequency.

COMPUTER IMPLEMENTATION AND RESULTS

A computer program, COLISEO, was written for the calculation of the steady-state sound pressure distribution and the room impulse response. This allowed the calculation of reverberation times and room sound quality parameters, such as early decay time, clarity, support factor and bass ratio. Comparisons with other methods and with experimental results showed the method to be reliable, very flexible and with low computation time.

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