ZARAGOZA '89
ENVIRONMENTAL ACOUSTICS

8º SYMPOSIUM FASE'89
«ACUSTICA AMBIENTAL»
Zaragoza, Abril 1989

A NUMERICAL METHOD FOR THE DESIGN OF HELMHOLTZ RESONATORS

A. Adobes, A. Bouizi and E. Luzzato

Direction des Etudes et Recherches - EDF - Département Acoustique et Mécanique Vibratoire -
1, Av du Général-de-Gaulle, 92141 Clamart FRANCE.

INTRODUCTION

High sound pressure levels at low frequencies in a narrow band of the frequency spectrum are often present in machinery rooms. In this case, acoustic resonators can be used to reduce the sound levels in the room. A Helmholtz resonator is composed of a small cavity related to the room by a neck. This paper presents a numerical method to determine the acoustical performances of such a resonator. This method takes into account all the relevant design parameters. The comparison of numerical results to measurements on real resonators shows good agreement.

THEORETICAL PROBLEM

The case of a single Helmholtz resonator baffled in an infinite rigid perfectly reflecting plane is considered.

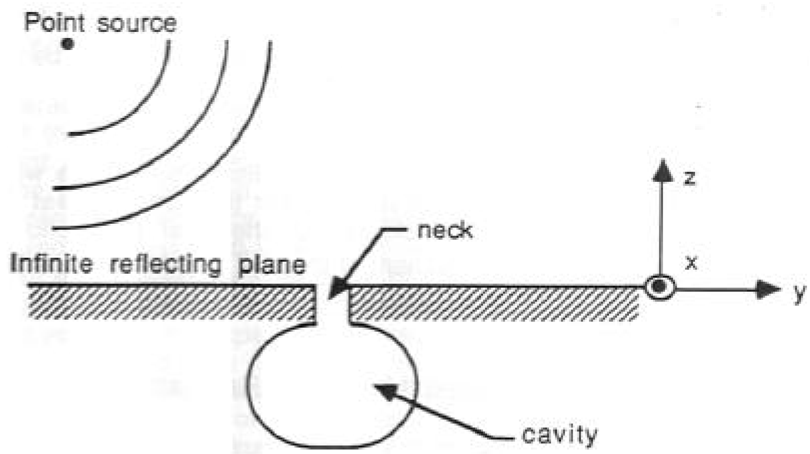

Figure 1 : Model of a Helmholtz resonator baffled in an infinite reflecting plane.

The neck and the cavity can be of any shape. The resonator is submitted to pure tone acoustic waves generated by a point source. Therefore it is assumed that all the variables fields such as the acoustic pressure and velocity fields, p and V are multiplied by $e^{-j\omega t}$, where ω denotes the excitation pulsation. The model must lead to the determination of both the pressure and velocity fields at the outer end of the neck so that an estimate of the resulting absorption coefficient can be found. The Helmholtz problems in the upper semi-infinite medium and inside the resonator are respectively solved using a boundary integral method and a finite element method.

The equation to be solved in the semi-infinite upper region is the Helmholtz-Huygens equation :

$$p(r) = -\iiint_{\Omega} s(r_o)\ G(r,r_o)\ dv_o + \iint_{\partial\Omega} [\ G(r,r_0)\frac{\partial p(r_0)}{\partial n} - p(r_0)\frac{\partial G(r,r_0)}{\partial n}\ ]\ d\sigma_0 \quad (1)$$

where s is a sound source density, $G(r,r_o)$ is an appropriate Green's function and n denotes the normal to the plane pointing downwards.

Let $g(r,r_o)$ be the generalized three-dimensionnal Green's function. This generalized Green's function is a solution of the free field Helmholtz equation with a Dirac distribution source at r_o. r denotes the vector position of the observation point in the propagation medium Ω, or on its boundary $\partial\Omega$. In the case of a semi-infinite medium bounded by an infinite reflecting plane , the appropriate Green's function is:

$$G(r,r_o) = g(r,r_o) + g(r,r'_o)$$

r'_o is the position vector of the image point source of r_o with respect to the infinite reflecting plane. The derivative of $G(r,r_o)$ with respect to the normal direction to the reflecting plane cancels. The boundary conditions are $\frac{\partial p}{\partial n} = j\omega\rho V_n$ at the outer end of the neck and $\frac{\partial p}{\partial n} = 0$ on the infinite perfectly reflecting and rigid plane baffle, where $V_n = V\ .\ n$ and ρ denotes the fluid density.

Equation (1) can then be rewritten at the outer end of the neck as:

$$p(r_p) = -\iiint_{\Omega} s(r_0)\ G(r_p, r_0)\ dv_0 + 2j\omega\rho \iint_{\partial\Omega_p} g(r_p, r_0)\ V_n(r_0)\ d\sigma_0 \quad (2)$$

r_p denotes an observation point at the outer end of the neck and $\partial\Omega_p$ is the surface of the outer end of the neck.

The equation to solve inside the resonator is the differential Helmholtz equation :

$$\Delta p + k^2 p = 0 \quad (3)$$

where k denotes the wave number. The equation must be solved according to the boundary conditions $\frac{\partial p}{\partial n} = j\omega\rho V_n$ at the outer end of the neck and $\frac{\partial p}{\partial n} = j\omega\rho\beta p$ on the neck walls of the resonator. β denotes an acoustic admittance, which will be taken equal to :

$$\beta = (\ j - 1\ )\ ((\ \gamma - 1\ )\ k\ d_h + (\ \frac{k_T}{k}\ )^2\ k\ d_v\ ) \quad (4)$$

to account for the phenomena of viscous and thermic dissipation at the walls [1]. γ is the ratio of the specific heat at constant pressure over the specific heat at constant volume, d_h the viscous boundary layer thickness, d_v the heat conduction boundary layer thickness and k_T the tangential component at the wall of the wave number k.

DISCRETIZATION AND SOLUTION

Boundary Element Outside And Finite Element Inside The Resonator:

The discretization of equation (2) leads to the matrix equation:

$$P_{(1)} = S + \mathbb{R}\ U \quad (5)$$

The components of vector $P_{(1)}$ are defined by the values of $p(r_p)$ on the discretization points of the outer end of the neck mesh. $\mathbb{R}$ is a matrix whose general term is :

$$R_{mn} = 2\,j\,\omega\,\rho \iint_{\partial\Omega_p} g(r_m, r_o)\ e_n(r_o)\ d\sigma_o \quad (6)$$

Functions e_n are the basis functions for the fluid velocity which are chosen equal to the basis functions of the pressure inside the resonator. Vector U represents the nodal components of the normal velocity field at the outer end of the neck. Vector S represents the contribution of the sound source s and its components are determined by the volume integral (2).
We use the classical finite element method to solve equation (3). The acoustic pressure field can be defined in terms of finite element basis functions N_m (x, y, z). The resulting matrix equation is:

$$\begin{bmatrix} \mathbb{B}^{11} & \mathbb{B}^{12} \\ \mathbb{B}^{21} & \mathbb{B}^{22} \end{bmatrix} \begin{bmatrix} P_{(1)} \\ P_{(2)} \end{bmatrix} = \begin{bmatrix} H\,U \\ 0 \end{bmatrix} \tag{7}$$

Matrix $\mathbb{B}$ represents the discretized Helmholtz operator. Its general term is :

$$B_{mn} = \int_{\Omega_r} [\nabla N_m . \nabla N_n - k^2 N_m N_n]\, dv - j\,\omega\,\rho \int_{\partial\Omega_\beta} \beta\, N_m N_n\, d\sigma \tag{8}$$

Ω_r denotes the domain inside the resonator and $\partial\Omega_\beta$ the part of the walls inside the resonator where an acoustic admittance is imposed.The representation of matrix $\mathbb{B}$ in four blocks allows one to separate the terms related to the outer end of the neck $P_{(1)}$ from the terms related to the cavity $P_{(2)}$. The general term of matrix $\mathbb{H}$ is :

$$H_{mn} = j\,\omega\,\rho \iint_{\partial\Omega_p} N_m(x,y)\; e_n(x,y)\; dx\; dy \tag{9}$$

Solving Of The Coupled Problem :

The coupled problem can be solved by gathering equations (5) and (7) into a general system of equations. The inversion of this system provides the solution containing pressure vector, $P_{(1)}$, the nodal components of the fluid velocity, U, at the outer end of the neck and pressure vector $P_{(2)}$ in the cavity.

$$\begin{bmatrix} \mathbb{I} & 0 & -\mathbb{R} \\ \mathbb{B}^{11} & \mathbb{B}^{12} & -\mathbb{H} \\ \mathbb{B}^{21} & \mathbb{B}^{22} & 0 \end{bmatrix} \begin{bmatrix} P_{(1)} \\ P_{(2)} \\ U \end{bmatrix} = \begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

$\mathbb{I}$ denotes the identity matrix.

COMPARISON OF MEASUREMENT AND NUMERICAL RESULTS - CONCLUDING REMARKS

Several resonators of various shapes were built in order to evaluate the accuracy of the method. Each resonator was installed as shown in figure 2. The measurement of the transfer function between the output of the amplifier and the output of the microphone set at the bottom of the resonators displayed a maximum whenever a resonant frequency was reached. This peak could then be located with a good accuracy. Vectors $P_{(1)}$ and U were used to compute the power absorbed by the resonator for each frequency. The curve of the power absorbed versus frequency could then be established and also displayed a maximum for the numerical resonant frequency. It matched the experimental resonant frequency by less than 3% for all the prototypes under study. In the case of the construction depth saving, quarter-wavelength resonator discribed on figure 3, the numerical resonant frequency was 130 Hz which was also the result of the measurement performed in the anechoïc room.

The advantage of the method we have described is twofold.
Unlike usual formulae working accurately only for particular shapes - cylindrical, parallelepipedic... -, it provides with a high accuracy the resonant frequency for Helmholtz resonators with a complex shape such as industrial resonators. Moreover, this model can be interfaced with numerical models used for indoor acoustic prediction. For instance, the absorbing behavior of an array of resonators can be evaluated in a given room.

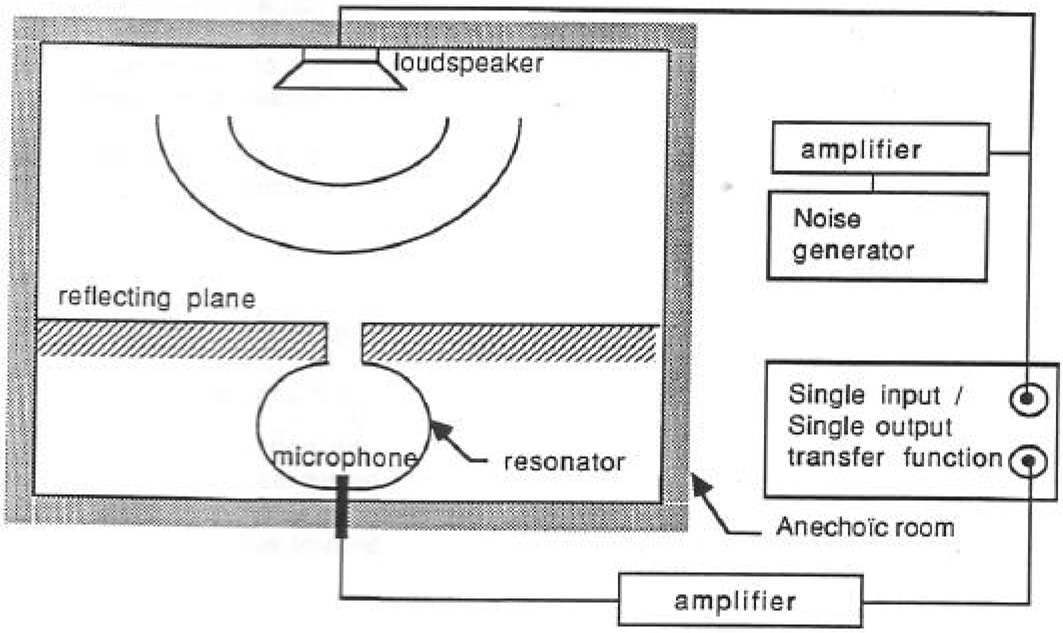

Figure 2 : Experimental configuration

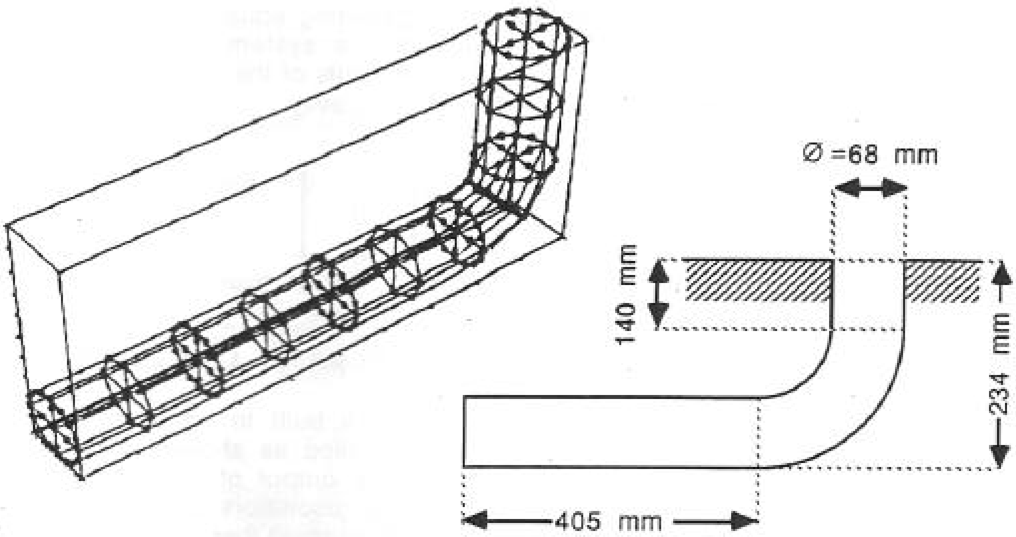

Figure 3 : Modelling of a quarter-wavelength resonator discretized with 20 20-node elements

REFERENCES

[1] P.M. MORSE and K.U. INGARD, 1968, Theoretical acoustics. New York : McGraw-Hill Book Company, Inc.
[2] A. ADOBES, 1987, Méthode d'analyse numérique des performances acoustiques d'un résonateur à col. HE-24 / 88.08 Département Acoustique EDF , Clamart FRANCE.
[3] A. BOUIZI, 1986, Mise en oeuvre d'un code de calcul d'éléments finis en vue du traitement de l'équation d'Helmholtz en espace clos. Département Acoustique EDF , Clamart FRANCE.
[4] A. ADOBES, A. BOUIZI, E. LUZZATO, 1988, Modelling of a stiff panel backed by a closed cavity : a study of a plate resonator, Proceedings of the Spring Conference, Acoustics '88 - University of Cambridge -