MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

The investigation of aeroelastic effects on the elastic wave propagation and radiated noise of periodic structures

Errico, Fabrizio[1]
Vibroacoustics & Complex Media Research Group, LTDS-CNRS UMR 5513
Ecole Centrale de Lyon, 34 Avenue Guy de Collongue, Ecully, 69134, France

Pasta-Lab, Dipartimento di Ingegneria Industriale, Università di Napoli Federico II, Via Claudio 21, Napoli, 80125, Italy

Franco, Francesco
Pasta-Lab, Dipartimento di Ingegneria Industriale, Università di Napoli Federico II, Via Claudio 21, Napoli, 80125, Italy

Petrone, Giuseppe
Pasta-Lab, Dipartimento di Ingegneria Industriale, Università di Napoli Federico II, Via Claudio 21, Napoli, 80125, Italy

De Rosa, Sergio
Pasta-Lab, Dipartimento di Ingegneria Industriale, Università di Napoli Federico II, Via Claudio 21, Napoli, 80125, Italy

Ichchou, Mohamed
Vibroacoustics & Complex Media Research Group, LTDS-CNRS UMR 5513
Ecole Centrale de Lyon, 34 Avenue Guy de Collongue, Ecully, 69134, France

ABSTRACT

In classic aeroelastic and flutter studies the natural modes of the structures are analysed with and without flow coupling. The study of travelling elastic waves in presence of flow, or travelling flutter waves, even though being suggested in many works in the literature, never had a deep and practical application. Here, within a finite element framework, a wave-based approach is coupled with supersonic and subsonic aerodynamic theories to analyse the effect of one-sided mean flow on the structural dispersion curves. Different polynomial eigenvalue problems arise, depending on the aeroelastic model used, as for the observed effects on the elastic wave propagation. The method is here applied both on homogeneous and complex-shaped periodic cells. The sound transmission is also computed and compared to the case in which aeroelastic effects are included.

Keywords: Aeroelasticity of Plates, Elastic Waves, Flow-Induced Vibrations and Noise

I-INCE Classification of Subject Number: 76;http://i-ince.org/files/data/classification.pdf

1. INTRODUCTION

[1] fabrizio.errico@ec-lyon.fr; fabrizio.errico@unina.it;

In the history of aviation, especially while conducting scientific researches and developments, structural failures caused by aeroelastic phenomena have affected many fighter aircraft, spacecrafts and jet engines. For this reason, the aeroelasticity of plates and shells, which differs from classic aeroelastic theories for lifting surfaces, has been widely studied in the last decades. The main issues arise form the difficulty in distinguishing between the vibrations induced by the external and self excitation, in addition to non-linearities that induce fatigue failures instead of catastrophic instantaneous failures [1].

In most of the studies on aeroelasticity of plates and shells, the modal approach is often preferred to a wave-based one, because it allows a clearer evaluation of the flutter conditions, analysing the effect of the aerodynamic auto-induced forces on each structural mode [1-8]. This approach has been successful in predicting, studying and designing even some of the most complex aerospace structures in the history of aviation.

However, very few works are present, generally dealing with infinite ideal structures, where the effect of the aerodynamic operator is analysed in terms of the elastic structural waves (dispersion curves). J.W. Miles presented a work discussing the flutter of an isotropic infinite panel in a two-dimensional incompressible flow, driving the wave speed relative to the panel, identifying the flutter conditions versus the circular frequency, [6,7]. This work has been furtherly developed by others, [3,6], and will here be used as the main reference for validation purposes.

At the same time, nowadays, much attention is placed on periodic/cellular structures, especially in the sectors where the main requirement for the structures is a high stiffness-to-mass ratio. Moreover, the periodic structures have peculiar filtering properties and can be modelled to realize frequency-selecting structures and metamaterials [9,10].

Here, within the framework of periodic structures, the effect of one-sided mean flow on the structural wave propagation is investigated. Supersonic flow is simulated on a periodic flat plate and the sound transmission loss is compared for different flow regimes, when aeroelastic effects are accounted in the model.

2. THE NUMERICAL METHOD

The method here presented is based on the Wave Finite Element Method (WFEM), [11,12]. This approach, instead of requiring the modelling of a whole periodic or homogeneous structure, uses the finite element of a single periodic cell, ideally extracted from the whole system. By imposing the periodic conditions to the cell model, the dynamics of the whole structural arrangement is analysed. First the mass and stiffness (**M** and **K**) matrices of the cell's finite element model (see Fig. 1) are extracted; this operation can be performed using any commercial or in-house FE package.

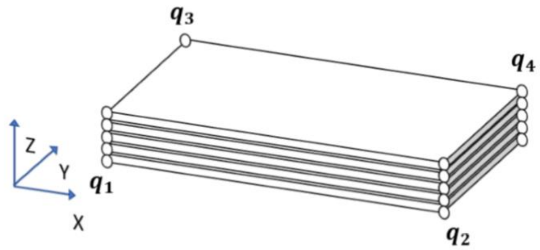

Figure 1 – An example of FE model of a periodic cell and the ordering of the hypernodes on the corners.

Assuming for simplicity a cell extracted from an homogeneous-in-plane structure, the subset of nodes at the corners (hypernodes) can be ordered as in Fig. 1. The periodicity conditions can be imposed by simply connecting the nodes each other using complex propagating constraints λ_x and λ_y [8-10]. In this way, the degrees of freedom belonging to all the hypernodes can be connected to the ones of a single one:

$$\boldsymbol{q} = [\lambda]\,\boldsymbol{q_1};\ [\lambda] = [\,I \quad \lambda_x I \quad \lambda_y I \ \ \lambda_x\lambda_y I\,] \tag{1}$$

where λ_x and λ_y, which are the complex propagation constrants are:

$$\lambda_x = e^{-\mathrm{i}\,k_x L_x};\ \lambda_y = e^{-\mathrm{i}\,k_y L_y} \tag{2}$$

where L_x and L_y are the sizes of the cell in the plane X-Y, while k_x and k_y are the wavenumbers (for each wave type) in the X and Y direction, respectively. Identical conditions are applicable also to the force vectors. Exploiting the periodic link and multiplying the dynamic stiffness equation by the Hermitian of the periodicity matrix, the reduced dynamic stiffness equation can be derived:

$$[\lambda]^H[\,\boldsymbol{K} - \omega^2\boldsymbol{M}\,][\lambda]\,\boldsymbol{q}_1 = [\lambda]^H[\lambda]\boldsymbol{f} + [\lambda]^H[\lambda]\boldsymbol{e} \tag{3}$$

Where $\boldsymbol{f}$ and $\boldsymbol{e}$ are the nodal vectors of internal and external forces respectively; ω is the circular frequency. Because of the equilibrium of internal forces between consecutive cells, the term $[\lambda]^H[\lambda]\boldsymbol{f}$ in Equation 3 is null.

At this stage, when no external forces are applied, the problem in Equation 3 is representative of a three-parametric eigenproblem in λ_x, λ_y and ω, that can be solved by imposing two variables at the time [10]. When the propagation constants are imposed and the frequency derived, the problem becomes a standard linear eigenvalue problem. If one wavenumber and the frequency are fixed, deriving the other one from Equation 3, the problem becomes quadratic. For example, expliciting any hypernode component in the dynamic stiffness equation, Equation 3 takes the following form:

$$[(\boldsymbol{D}_{11} + \boldsymbol{D}_{22} + \boldsymbol{D}_{33} + \boldsymbol{D}_{44})\lambda_x\lambda_y + (\boldsymbol{D}_{12} + \boldsymbol{D}_{34})\lambda_x{}^2\lambda_y + (\boldsymbol{D}_{13} + \boldsymbol{D}_{24})\lambda_x\lambda_y{}^2 + \boldsymbol{D}_{32}\lambda_x{}^2 + \boldsymbol{D}_{23}\lambda_y{}^2 + (\boldsymbol{D}_{21} + \boldsymbol{D}_{43})\lambda_x + (\boldsymbol{D}_{31} + \boldsymbol{D}_{42})\lambda_y + \boldsymbol{D}_{14}\lambda_x{}^2\lambda_y{}^2 + \boldsymbol{D}_{41}]\,\boldsymbol{q}_1 = \boldsymbol{0} \tag{4}$$

Solving the quadratic eigenvalue problem in λ_x or λ_y, the dispersion curves of the media can be derived.

2.1 The Aerodynamic Model: Piston Theory

To simulate a one-sided mean flow, a specific aerodynamic theory has to be used. Here, the simplest aerodynamic theory for supersonic incompressible flows, the *Piston Theory* is used to simulate the aerodynamic operator [1,13]. This theory, valid from Mach > 1.5, assumes that the pressure fluctuations in any point of the system are independent from the others [1,13].
Using the notation of Equation 3, the self-excited force terms can be written as a function of the convective and continuity derivative, [1,3,4,13]:

$$\boldsymbol{e} = -\rho_0 a_0 \boldsymbol{A}_n \left(\frac{d\boldsymbol{w}}{dt} + U\frac{d\boldsymbol{w}}{dx}\right) \tag{5}$$

where $\boldsymbol{w}$ represents the out-of-plane displacements vector (coordinate Z in Fig. 1), ρ_0 is the fluid density, a_0 the sound speed, $\boldsymbol{A}_n$the nodal area vector and U the flow-speed. The out-of-plane displacements can be expressed by multiplying **q** for a matrix (**Δ**) of zeros and unitary values in the positions corresponding to the target degrees of freedom (in this case, the translations in Z).
In a periodic framework, the spatial derivative in Equation 5, is a function of the structural propagation constant (λ_x, assuming X as the flow direction), and can be expressed, using a simple numerical scheme for the first derivative, as in Equation 6.

$$\frac{d\boldsymbol{w}}{dx} = \boldsymbol{\Delta}[\lambda]\left(\frac{\lambda_x - 1}{L_x}\right)\boldsymbol{q}_1 \tag{6}$$

Assuming harmonic motion and substituting Equation 5 and 6 in Equation 3, the final dynamic stiffness equation is obtained:

$$[\lambda]^H \left[\boldsymbol{K} - \omega^2\boldsymbol{M} - \rho_0 a_0 \boldsymbol{A}_n \boldsymbol{\Delta}\left[i\omega - U\left(\frac{\lambda_x - 1}{L_x}\right)\right]\right][\lambda]\,\boldsymbol{q}_1 = [\lambda]^H[\lambda]\boldsymbol{f} = 0 \tag{7}$$

An altered stiffness of the structure, depending on the wave's propagation constants in the stream-wise direction, is observed. At the same time, an added viscous damping is added by the aerodynamic operator. The quadratic eigenvalue problem described in Equation 4, for the structure alone, becomes the one in Equation 8, when a one-sided supersonic flow is described using the *Piston Theory*.

$$\begin{aligned}&[(\boldsymbol{D}_{11} + \boldsymbol{D}_{22} + \boldsymbol{D}_{33} + \boldsymbol{D}_{44})\lambda_x\lambda_y + (\boldsymbol{D}_{12} + \boldsymbol{D}_{34} + 4\rho_0 a_0 \boldsymbol{A}_n\boldsymbol{\Delta}[\lambda]\,U/L_x)\,{\lambda_x}^2\lambda_y + \\ &(\boldsymbol{D}_{13} + \boldsymbol{D}_{24})\lambda_x{\lambda_y}^2 + \boldsymbol{D}_{32}{\lambda_x}^2 + \boldsymbol{D}_{23}{\lambda_y}^2 + (\boldsymbol{D}_{21} + \boldsymbol{D}_{43})\lambda_x + (\boldsymbol{D}_{31} + \boldsymbol{D}_{42})\lambda_y + \\ &\boldsymbol{D}_{14}{\lambda_x}^2{\lambda_y}^2 + \boldsymbol{D}_{41}]\,\boldsymbol{q}_1 = \boldsymbol{0}\end{aligned} \tag{8}$$

Where the dynamic stiffness matrix **D** is given by:

$$\boldsymbol{D} = [\,\boldsymbol{K} - \omega^2 \boldsymbol{M} - \rho_0 a_0 \boldsymbol{A}_n \boldsymbol{\Delta}\,[i\omega + U/L_x]\,] \quad (9)$$

3. RESULTS

First a validation with the reference case proposed by J.W. Miles in [7] is performed. The travelling wave approach proposed by Miles is first described, being not coincident to a classic *Piston Theory*. The same concepts and mathematical derivations followed in the previous section are used here for a coherence with the reference result described in [7].

Then, the sound transmission for a specific turbulent boundary layer model, the Cockburn – Robertson [14], is compared for a purely structural case and when aeroelastic effects are taken into account using the *Piston Theory*.

3.1 Validation: Bending Waves with Flow

J.W. Miles, in [7], discussed the flutter of a plane isotropic infinite panel in a two-dimensional compressible flow [7]. As furtherly discussed by Dugundji, [6], he utilizes an axis system which is fixed to the air at rest, and considers the infinite panel to be moving with velocity U in the negative X direction. The Equation of motion is:

$$D\, {\partial^4 w}/{\partial x^4} + m\left({\partial w}/{\partial t} - U\,{\partial w}/{\partial x}\right)^2 = \Delta p \quad (10)$$

where *D* is the bending stiffness of the plate, *m* the mass density per unit area and Δp the self-excited pressure distribution. The solution of Equation 10 leads to an expression of the bending wave speed relative to the panel (*c*+*U*):

$$c + U = \frac{1}{1+\mu}\left[\mu U \pm \sqrt{(1+\mu))c_0^2 - \mu U^2}\,\right];$$

$$\mu = \frac{\rho_0 \lambda}{2\pi\, m};\; c_0 = \sqrt{\frac{D}{m}\frac{2\pi}{\lambda}} \quad (11)$$

where μ represents the mass density ratio (for a one-sided flow), λ the structural wavelength and c_0 the wave speed in absence of flow [6,7]. Dynamic instability (travelling flutter condition) occurs for a wave speed equal to:

$$c + U = \frac{\mu U}{1+\mu} \quad (12)$$

The variation of flutter speed with the wavelength can be thus investiated since both the mass density ratio and the wave speed are functions of the wavelength [6,7].

The same test-case is reproduced using the method described in Section 2. A 2mm-thick aluminium infinite plate is studied. Within the periodic cell framework, a 2mm cube, representing the homogenised cell of the aluminium plate, is modelled using three SOLID45 elements in ANSYS. The resulting wave speed (bending waves) of the panel, for a 300 m/s flow, is compared to the one calculated using the travelling wave approach of Miles in Fig. 2.

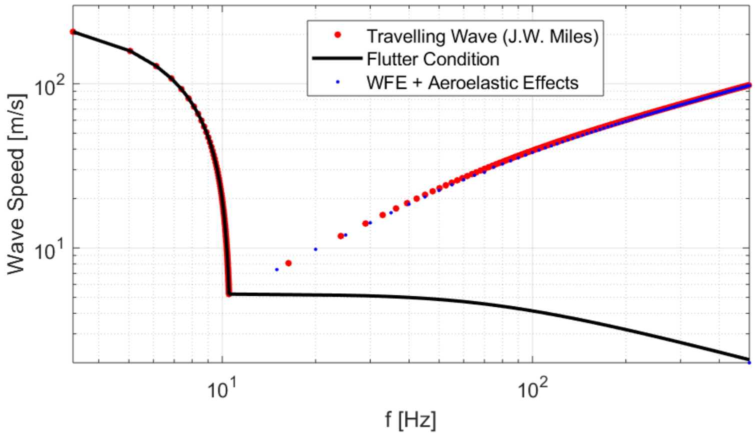

Figure 2 – The structural bending wave speed in presence of a 300 m/s flow on one side, modelled using the Piston Theory. The flutter condition is also reported in black.

The present approach (WFE + Aeroelastic effects) seems to closely follow the reference wave speed from [7], in the frequency regions characterised by dynamic stability. The modified quadratic eigenvalue problem in Equation 8 is here solved. The numerical solution, differently from the wave approach of Miles, includes different wave type (not solely bending). In Figure 3, the dispersion curves of the plate are compared for different flow regimes and, again, the present method is accurate in predicting the bending waves in the flow regimes of dynamic stability.

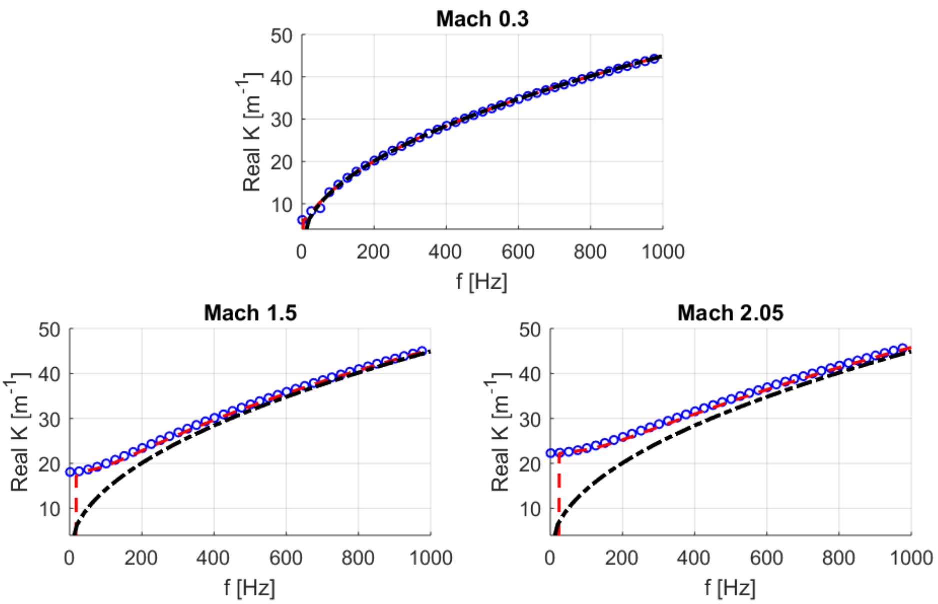

Figure 3 – The bending waves in a 2mm-thick aluminium panel for different flow regimes. O (blue): Present Approach; -- (red): Reference [7]; -.- (black): Analytical bending waves in absence of flow.

The more the flow speed increases, the more the effect of the flow on the structural waves is evident in a larger frequency range. The effects, as expected, seem to increase the wavenumber component of the bending waves, thus making the structure less stiff for fixed frequency. Moreover, coherently with the physics of the problem, the wavenumbers of the "affected" waves, return to the same values of the purely structural waves (no flow), at higher frequencies.

3.2 The Induced Noise by Supersonic TBL with Aeroelastic Effects

An immediate application of the presented approach is related to the calculation of the sound transmission. While the wavenumbers are to be derived from the eigenvalue problem, when the target is the dispersion curves, these can be imposed within a weighted integration in the wavenumber domain, to simulate a general load acting on the structure, by means of surface waves. This method is presented by the authors in [12].

The Cockburn-Robertson model, representing an external supersonic turbulent boundary layer (TBL), is considered and, as showed in Section 2, the *Piston Theory* is used for the self-induced component of the total load. This model is semi-empirical and obtained from measurements on spatial vehicles, thus is predictive only for high Mach numbers. The single point spectra of the TBL model investigated is:

$$\frac{\phi_p(f)}{q_\infty^2 \delta} = \frac{\frac{\langle p^2 \rangle}{q_\infty^2}}{\left(\frac{\delta f_0}{U}\right)\left[1+\left(\frac{f}{f_0}\right)^{0.9}\right]^2} \tag{13}$$

where

$$\langle p^2 \rangle \approx \left[\frac{0.006}{(1+0.14M^2)q_\infty}\right]^2 \tag{14}$$

f is the circular frequency, f_0 the characteristics frequency ($f_0 = 0.346\frac{U}{\delta}$), M is the local Mach number, δ the boundary layer thickness and q_∞ the dynamic pressure.

Figure 5 shows a comparison between two cases: absence of flow and with aeroelastic effects. The test structure is still a 2 mm-thick aluminium finite panel (0.7m x 0.5m) under a Mach 1.5 flow on one side. The effects at the lowest frequency bands seem the most evident.

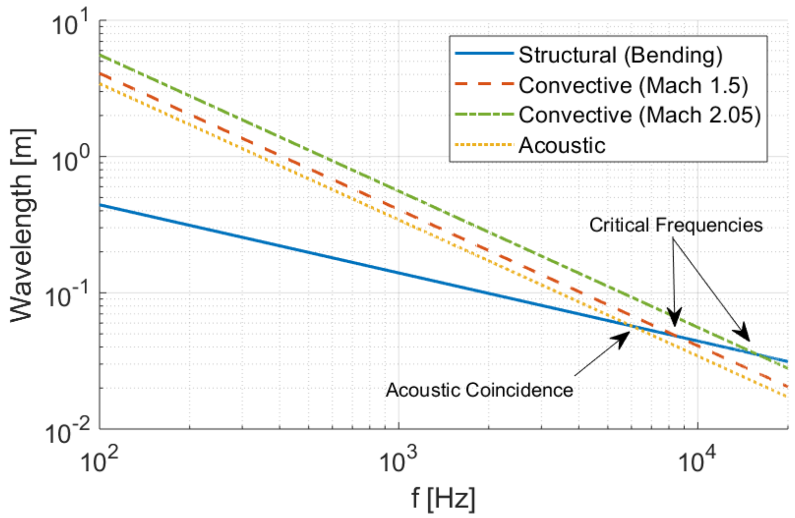

Figure 4 – Dispersion curves for the cases studied. Acoustic and aerodynamic coincidences are identified with respect to structural flexural waves.

An effect is also observed just before the acoustic coincidence (6 kHz; Fig. 4). Moving to higher frequencies, the aeroelastic effects seem to reduce; the passage around the aerodynamic coincidence (8 kHz; Fig. 4) is not influenced.

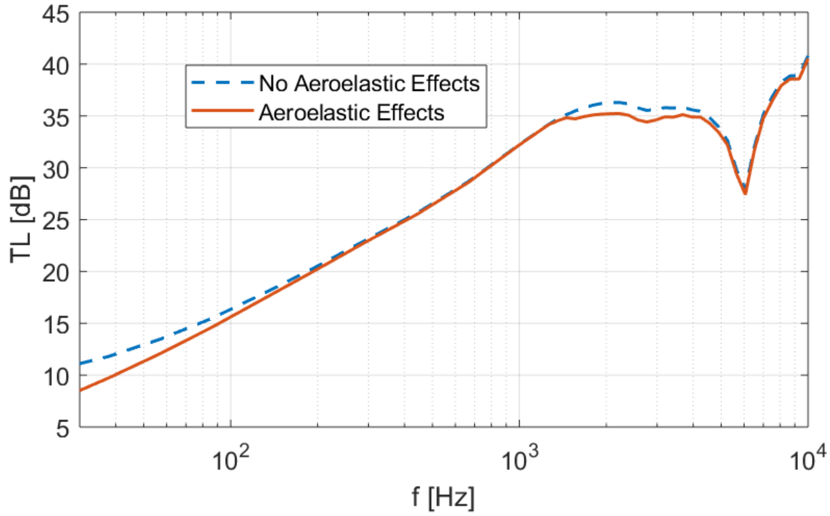

Figure 5 – The transmission loss for a 0.7x0.5 m^2 isotropic plate under supersonic TBL excitation (Mach 1.5) with and without aeroelastic effects.

A comparison for a higher Mach number (Mach 2.05) is shown in Figure 6. The classic aeroelastic effects at low frequency are still important, while the variations, close to the acoustic coincidence, are strongly reduced in this case and are almost negligible.

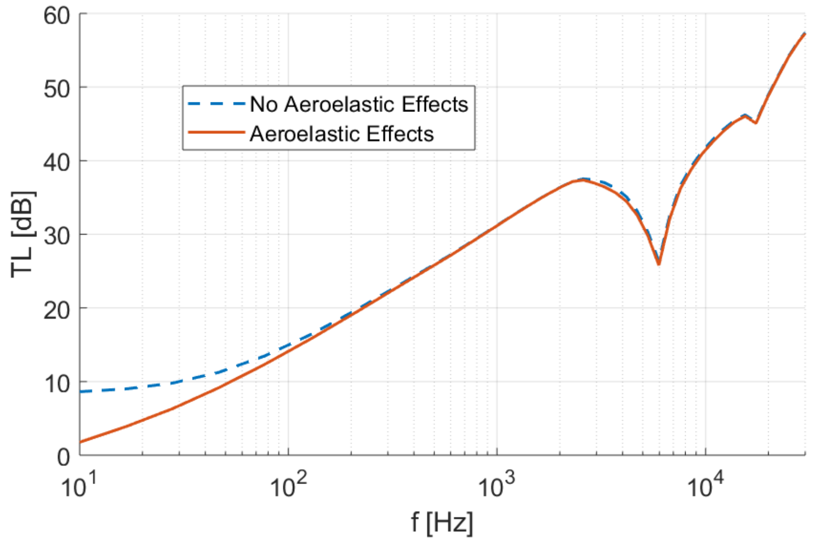

Figure 6 – The transmission loss for a 0.7x0.5 m^2 isotropic plate under supersonic TBL excitation (Mach 2.05) with and without aeroelastic effects.

Opposite considerations can be done for the averaged structural velocity of the plate. The stronger aeroelastic effects in the low frequency bandwidth are in accordance with the classic aeroelasticity of plates [1-8], and still prove that the phenomenon is somewhat a low-frequency one.

It is worth to underline how the results presented for the dispersion curves in Fig. 3 are not comparable with the ones for the transmission loss in Fig. 5 and 6. In fact, the aerodynamic model presented in [7] and used for the comparisons of Fig. 3 is not the same Piston Theory model used in Section 3.2.

4. CONCLUSIONS

The effects of one-sided mean flow on the structural elastic waves' propagation is investigated using a wave-based finite element method. While the structural operator is studied using a single periodic cell, the aerodynamic one is modelled using the simplest aerodynamic theory, valid for supersonic flows: the *Piston Theory*.

The spatial derivatives (convective terms) are expressed as a function of the elastic waves' propagation constants and the eigenvalue problem, arising from the imposition of the periodicity conditions on the structure, is modified with additional damping and stiffness terms connected to the flow-induced self-excited excitation components.

First, a validation using the travelling flutter approach, available in the literature, is performed. The sound transmission loss of the plate is then calculated and studied when aeroelastic effects are taken into account, for different flow regimes. Strong alterations are observed in the lowest frequency bands, due to corresponding variations in the bending waves' wavenumbers.

ACKNOWLEDGEMENTS

This project has received funding from the European Unions Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 675441. Prof. E. Dowell, from Duke University, is gratefully acknowledged for the fruitful discussions.

REFERENCES

1. E.H. Dowell, *Aeroelasticity of Plates and Shells*, Noordhoff International Publishing, Princeton, 1975.
2. R.L. Bisplinghoff, K. Ashley, R.L. Halfman, *Aeroelasticity*, Addison-Wesley Publishing Company, Inc., Cambridge (MA), 1955
3. E.H. Dowell, C.S. Ventres, *Flutter of Low Aspect Ratio Plates*, AIAA Journal 8 (6), 1162-1164, 1970.
4. J.W. Miles, *Supersonic Flutter of A Cylindrical Shell*, Journal of the Aeronautical Sciences 24, 107-118, 1957.
5. A. Iollo and M.D. Salas, *On The Propagation of Perturbations In Two Simple Aeroelastic Systems*, Journal of Sound and Vibration 222 (1), 152-162, 1999.

6. J. Dugundji, E.H. Dowell, B. Perkin, *Subsonic Flutter of Panels on Continuous Elastic Fundations – Experiments And Theory*, Massachusetts Institute of Technology, ASRL Technical Report No. 74-4, 1962.
7. J.W. Miles, *On the Aerodynamic Instability of Thin Panels*, Journal of the Aeronautical Sciences 23 (8), 771-791, 1956.
8. R.W. Leonard and J.M. Hedqkipeth, *On Panel Flutter And Divergence of Infinitely Long Unstiffened And Ring-Stiffened Thin-Walled Circular Cylinders*, NACA Technical Note 3638, 1956.
9. L. Brillouin, *Wave Propagation in Periodic Structures: Electric Filters And Crystal Lattices*, 2nd edition Dover Publications (Mineola, New York).
10. D. Mead, *Wave Propagation in Continuous Periodic Structures: Research Contributions from Southampton*, Journal of Sound and Vibration 190 (3), 495–524, 1996
11. E. Manconi, B. R. Mace, *Modelling Wave Propagation in Two Dimensional Structures Using Finite Element Analysis*, Journal of Sound and Vibration 318(45) (2008) 884–902.
12. F. Errico, M. Ichchou, S. De Rosa, O. Bareille, F. Franco, *A load approximation for the vibroacoustic analysis of periodic structures to fluid excitation*, ISMA 2018 Conference, Leuven, 2018
13. W.P. Rodden, E.F. Farkas, H. Malcom, A.M. Kliszewski, *Aerodynamic Influence Coefficients from Piston Theory: Analytical Development and Computational Procedure*, Aerospace Corp., 1962
14. W. Graham, *A Comparison of Models for the Wavenumber-Frequency Spectrum of Turbulent Boundary Layer Pressures*, Journal of Sound and Vibration 206(4), 541–565, 1997.