

## **On the modelling of reactive silencers with narrow side-branch tubes**

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### **ABSTRACT**

**This work represents a theoretical study of the sound propagation in a waveguide loaded by an array of flush-mounted narrow side-branch tubes, forming a simple low-frequency reactive silencer. The individual tube-lengths and the distances between the adjacent tubes may vary in order to optimize the transmission loss in a given frequency range. The transmission properties of the silencer are calculated using the transfer matrix method, and the finite element method. A simple heuristic evolutionary algorithm, together with an analytical mathematical model (the transfer matrix method) is employed for the determination of the optimal silencer parameters. The numerical results are validated against the finite element simulation.**

**Keywords:** Reactive silencer, Heuristic optimization

**I-INCE Classification of Subject Number:** 34

### **1. INTRODUCTION**

Reduction of low-frequency noise propagating in ducts is an important issue, especially in modern buildings. This problem is traditionally treated by means of silencers. Dissipative ones employ porous absorbing materials to convert acoustic energy into heat; in reactive silencers, the basic principle is the reflection of acoustic energy on the impedance

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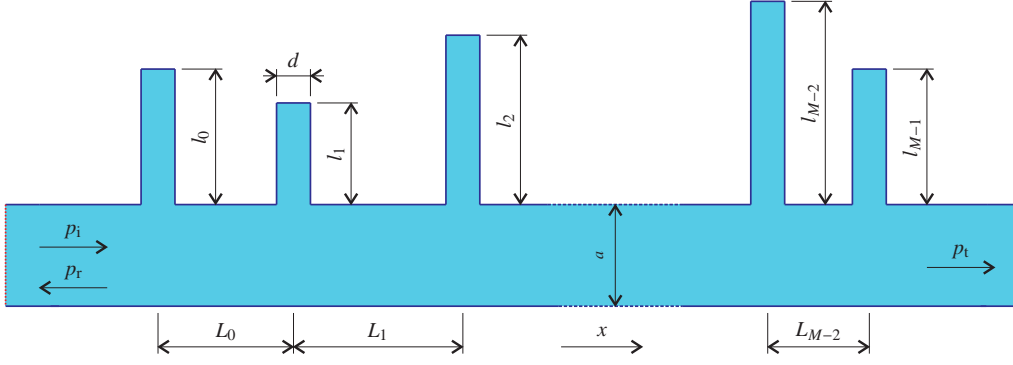


Figure 1: Geometry of a waveguide loaded with flush-mounted narrow side-branch tubes.

mismatch. Typical examples of the reactive silencers are the expansion chamber [1], or side-branch resonators. For example, these resonators can be represented by Helmholtz resonators (HRs), or side-branch tubes (SBTs). As it follows from the core principle, the resonators work efficiently only in a relatively narrow frequency range. However, several attempts have recently been made to extend the performance of side-branch resonators into a wider frequency range.

For example, detuned HRs were arranged in series, parallel, or both [2–6] to broaden the silencer efficiency frequency range. These arrays were also optimized [2, 7]. Arrays of side-branch tubes of varying lengths according to a simple relations were examined in works [8, 9]. The opening of a broad stop-band resulting from the hybridization of tubes' resonances with Bragg interference was further studied in [10]. The lengths and separating distances of side-branch tubes flush-mounted to a waveguide were optimized to maximize the minimum value of the transmission loss ( $TL$ ) over a given frequency range in a previous work [11]. This work deals with the transmission properties of a fluid-filled waveguide (duct) loaded by an optimized array of flush-mounted narrow side-branch tubes.

## 2. MATHEMATICAL MODEL

The studied geometry is shown in Fig. 1. It consist of an infinite rectangular duct with height  $a$ , loaded with flush-mounted  $M$  side-branch resonators (tubes) with the same widths  $d < a$ . Due to the symmetry, the configuration can be considered a two-dimensional one. The lengths of the individual resonators are  $l_0, l_1, \dots, l_{M-1}$ , the distances between the adjacent resonators are  $L_0, L_1, \dots, L_{M-2}$ . The system is considered to be filled with quiescent air at normal conditions, all the thermoviscous losses are neglected.

It is assumed that the frequency-range of interest is below the cut-on frequency, so that plane-wave approximation can be made. Under this approximation, it is assumed that there is an incoming plane-wave impinging upon the silencer  $\tilde{p}_i = \tilde{p}_{i0} e^{i(\omega t - kx)}$ , where  $\tilde{p}_{i0}$  is its complex amplitude,  $\omega$  is the angular frequency and  $k$  is the wavenumber; the incoming wave partially reflects as a reflected wave  $\tilde{p}_r$  and partially transmits through the silencer as the transmitted wave  $\tilde{p}_t$ .

The transmission properties of the silencer are calculated using the transfer matrix method (TMM), see e.g. [11, 12]; it consists of two building blocks: pieces of an uniform

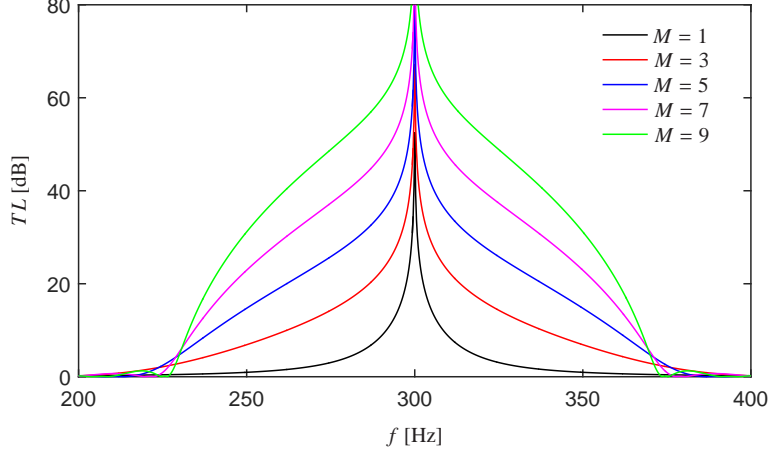


Figure 2: Transmission loss of a silencer employing Bragg scattering.

waveguide with lengths  $L_i$ , for which the transfer matrix reads

$$\begin{bmatrix} \tilde{p}(x) \\ \tilde{v}(x) \end{bmatrix} = [\mathbf{T}_{g,i}] \begin{bmatrix} \tilde{p}(x + L_i) \\ \tilde{v}(x + L_i) \end{bmatrix}, \quad [\mathbf{T}_{g,i}] = \begin{bmatrix} \cos kL_i & iZ_0 \sin kL_i \\ iZ_0^{-1} \sin kL_i & \cos kL_i \end{bmatrix}, \quad (1)$$

where  $\tilde{v}$  is the complex velocity in the main waveguide, and  $Z_0$  is the characteristic impedance, and the side-branch resonators, for which the transfer matrix reads

$$\begin{bmatrix} \tilde{p}(x-) \\ \tilde{v}(x-) \end{bmatrix} = [\mathbf{T}_{t,i}] \begin{bmatrix} \tilde{p}(x+) \\ \tilde{v}(x+) \end{bmatrix}, \quad [\mathbf{T}_{t,i}] = \begin{bmatrix} 1 & 0 \\ (d/a)Z_t^{-1} & 1 \end{bmatrix}, \quad (2)$$

where  $Z_t = -iZ_0 \cot kl_i$  is the input impedance of the  $i$ -th side-branch resonator. The total transfer matrix is calculated as a product of the individual transfer matrices

$$[\mathbf{T}] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = [\mathbf{T}_{t,0}] [\mathbf{T}_{g,0}] [\mathbf{T}_{t,1}] [\mathbf{T}_{g,1}] \dots [\mathbf{T}_{t,M-2}] [\mathbf{T}_{g,M-2}] [\mathbf{T}_{t,M-1}]. \quad (3)$$

Employing the coefficients of the total transfer matrix (3), the transmission coefficient is calculated as

$$\mathcal{T} = \frac{2}{T_{11} + T_{12}/Z_0 + T_{21}Z_0 + T_{22}}, \quad (4)$$

and the transmission loss as

$$TL = -20 \log |\mathcal{T}|. \quad (5)$$

### 3. NUMERICAL RESULTS

In all the numerical examples below, the main waveguide height  $a = 15$  cm, and side-branch resonators' width  $d = 4$  cm. The numerical results employing TMM have been verified by comparison with the finite element simulations, good agreement between the data has been found.

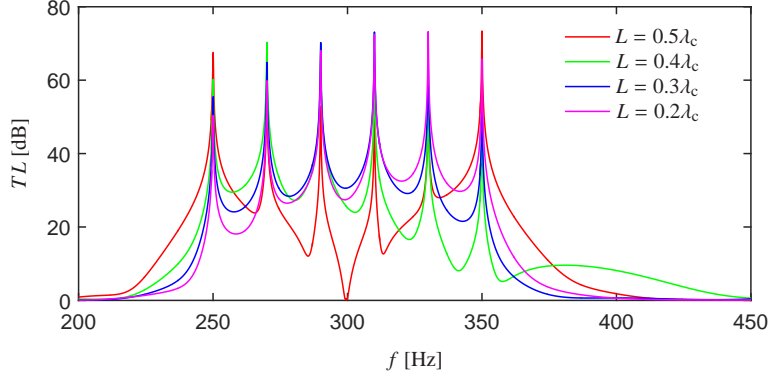


Figure 3: Transmission properties of a silencer with differently tuned SBTs and different distances between the adjacent SBTs.

### 3.3.1. Non-optimized silencers

Figure 2 shows the transmission properties of a silencer employing Bragg scattering, see e.g. [3, 4, 10]. In this case, the lengths of the individual resonators are the same  $l_i = l$ , and correspond to  $\lambda_c/4$  of the frequency  $f_c = 300\text{Hz}$ , the distances between the individual resonators are also the same  $L_i = L$  and correspond to  $\lambda_c/2$  of  $f_c$ . It can be seen in Fig. 2 that with the increasing number  $M$  of the resonators, a bell-shaped stop-band begins to form with the frequency-range  $\Delta f \approx 150\text{Hz}$ . It can be shown, that if  $L \neq 2l$ , the stop-band structure is deformed.

Another way of broadening the silencer frequency range employs the tubes of different lengths (resonant frequencies), see e.g. [8, 10]. Figure 3 shows the case of a silencer with  $M = 6$  SPTs tuned at resonance frequencies

$$f_i = (250, 270, 290, 310, 330, 350)\text{ Hz}.$$

The distances between the individual SBTs, are the same, and set to  $L = \alpha\lambda_c$ , where  $\lambda_c$  corresponds to  $f_c = 300\text{Hz}$ , and  $\alpha = 0.2, 0.3, 0.4, 0.5$ .

It can be seen in Fig. 3 that the transmission properties of the silencer strongly depend on the distance between the SBTs, and for  $L = \lambda_c/2$ , there is even null  $TL$  in the middle of the stop-band.

### 3.3.2. Optimized silencers

It has been shown in Figs. 2 and 3, that the transmission loss of a silencer with SBTs strongly depends on the lengths  $l_i$  of the individual SBTs and the distances  $L_i$  between them.

In order to find an optimum configuration, the following optimization procedure has been adopted, see [11]. The performance of the silencer has been appraised employing the minimum value of  $TL$  in a pre-defined frequency range, so the objective function for the maximization is

$$Q(l_0, l_1, \dots, l_{M-1}, L_0, L_1, \dots, L_{M-2}) = \min \{TL(f_i), i = 0, 1, 2, \dots, N - 1\}, \quad (6)$$

where  $f_i = f_c - \Delta f/2 + \Delta f \times i/(N - 1)$ ,  $f_c$  being the center frequency, and  $\Delta f$  the frequency range. For the maximization of the objective function (6), a variant of

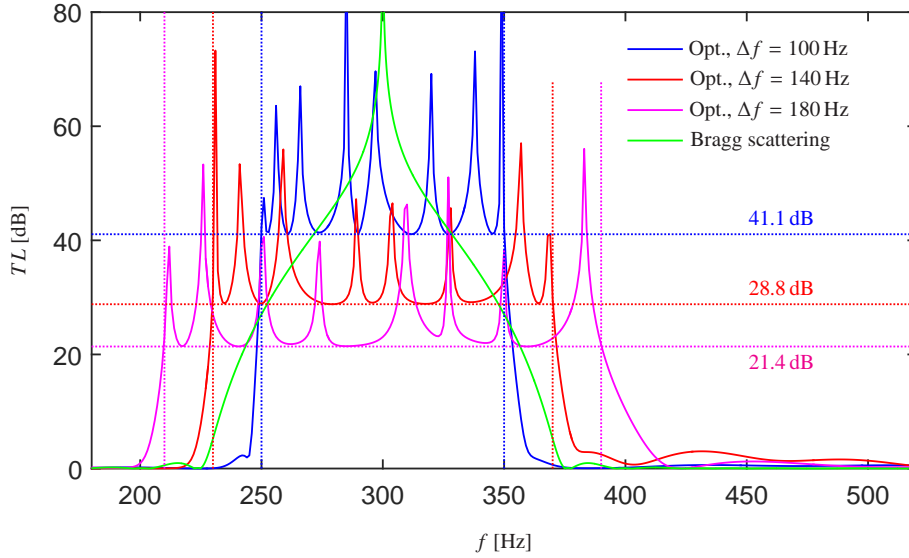


Figure 4: Transmission loss of optimized silencers with  $M = 8$  SBTs,  $f_c = 300$  Hz and different bandwidths  $\Delta f$ .

heuristic evolutionary algorithm  $(\lambda, \mu) - ES$  – Evolution strategies, see e.g. [13], has been employed.

Figure 4 shows the transmission loss of optimized silencers with  $M = 8$  SBTs,  $f_c = 300$  Hz and different bandwidths  $\Delta f$ , together with the silencer employing the Bragg scattering ( $M = 8$ ). It can be seen that the optimization process leads to the same values of  $TL_{\min}$  in the targeted frequency range for the given  $M$ , and that the  $TL_{\min}$  decreases with the increasing bandwidth  $\Delta f$ . Compared to the silencer employing Bragg scattering, the stop-band is more controlled and “flat”. The conducted parametric study has revealed, that for the given parameters (geometrical dimensions, frequency range), the minimum transmission loss is a linear function of the number of SBTs.

#### 4. CONCLUSIONS

Within this work, reactive silencers composed of an array of narrow side-branch tubes were studied. A one-dimensional mathematical model employing the transfer matrix method under the plane-wave approximation was used. It was demonstrated that the transmission properties of a silencer of this type strongly depend on the lengths of the individual resonators as well as on the distances between them. A simple optimization algorithm was implemented for the maximization of the minimum transmission loss in a pre-defined frequency range. Compared to non-optimized silencers, the optimized ones have relatively “flat” transmission loss within the targeted frequency range. For the given parameters, the minimum transmission loss is a linear function of the number of the resonators.

#### 5. ACKNOWLEDGEMENTS

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