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## **Study on the quasi-static and dynamic response of dielectric elastomer actuator under electrical loads**

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### **ABSTRACT**

The characteristics of five strain energy functions for dielectric elastomer (DE) are compared. A hyperelastic constitutive model based on second order Ogden strain energy function is achieved, which can accurately simulate the nonlinear large deformation of VHB4910 DE membrane under different tensile rates. According to the established model of DE, the quasi-static and dynamic response equations have been developed with regard to the dielectric constant changes of DE. The relation between the element's pre-stretch and DC voltage under equal bi-axial loading condition is analysed. Its dynamic response is predicted under constant and harmonic electrical loads. The results show that DE has an optimal pre-stretch rate corresponding to a minimum DC voltage inducing its extreme electro-deformation. The electrical strength has a significant influence on the dynamic response of DE. The vibration frequency of DE actuator diminishes as the electrical strength increases. The variation behaves nonlinear.

**Keywords:** Actuator, Dielectric elastomer, Dynamic response

**I-INCE Classification of Subject Number:** 38

### **1. INTRODUCTION**

As a typical kind of electroactive polymers (EAPs) with distinctive characteristics, dielectric elastomer (DE) has been considered as a smart soft material which can perform large size and shape deformation under electric loads. As a result, DE has been regarded as potentially useful material for actuator application due to its large actuation strain together with a fast response and a high energy density, which opens up a broad prospect in the fields of bionic mechanical design, aeronautical and space technology as

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well as vibration and noise control. The dielectric elastomer actuator (DEA), also called DE actuating element, consists of a DE membrane sandwiched between two compliant electrodes. When an electric field applied between the electrodes, compression in the thickness and stretching in the area of DE membrane will be achieved. Compared with conventional actuator based on electromagnetic principle such as shape memory alloy and electroactive ceramic, DEA has the advantages of low elastic stiffness and high dielectric constant which can produce high strain level with large deformation, considerable generated force, high energy conversion efficiency, lightweight and low-noise.[1, 2]

In recent years, a great amount of research has been carried out focusing on the electromechanical characteristics of DE, and the main theoretical methods are as follow[3]: mechanical property study based on hyperelastic and viscoelastic theory with material experiment, electrical deformation response study based on continuum mechanics and electrodynamic theory, failure mode and electromechanical instability study based on thermodynamic theory and electromechanical behavior based on finite element theory. In this paper, a comprehensive theoretical study is conducted in respects of DE hyperelastic constitutive relation, quasi-static and dynamic response considering DE dielectric constant changes under different electric loads which will provide theoretical guidance for the future DEA design.

## 2. HYPERELASTIC CONSTITUTIVE RELATION

According to the hyperelastic theory [4], DE can be regarded as a Green elastomer which has distinctive characteristics of geometric and material nonlinearity. Therefore, in order to obtain its nonlinear constitutive relation, we assume DE as a continuous, uniform and incompressible ideal material without the consideration of viscoelasticity and temperature factors. Based on continuum mechanics [5], the strain energy function of DE can be expressed with invariant  $I$  in the form of  $W(I_1, I_2, I_3)$  or stretch rate  $\lambda$  of  $W(\lambda_1, \lambda_2, \lambda_3)$ .

When in the state of homogeneous strain as shown in Figure 1, DE's principal stresses are [6]:

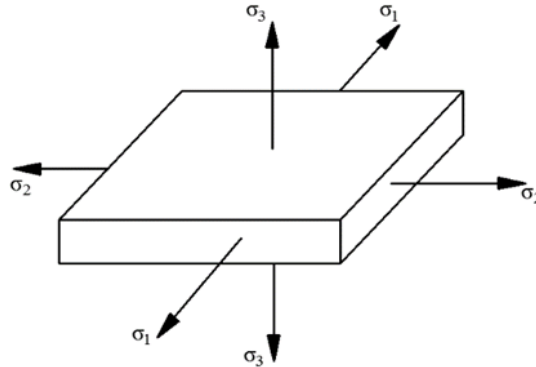


Figure 1 Axes of DE principal stress

$$\sigma_1 - \sigma_2 = 2(\lambda_1^2 - \lambda_2^2) \left( \frac{\partial W}{\partial I_1} + \lambda_3^2 \frac{\partial W}{\partial I_2} \right) \quad (1-a)$$

$$\sigma_2 - \sigma_3 = 2(\lambda_2^2 - \lambda_3^2) \left( \frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2} \right) \quad (1-b)$$

$$\sigma_3 - \sigma_1 = 2(\lambda_3^2 - \lambda_1^2) \left( \frac{\partial W}{\partial I_1} + \lambda_2^2 \frac{\partial W}{\partial I_2} \right) \quad (1-c)$$

$$\sigma_i = s_i \lambda_i \quad (i=1,2,3) \quad (2)$$

where  $\sigma_i$  is the real stress or cauchy stress in the three main direction,  $s_i$  is the nominal stress or engineering stress,  $\lambda_i$  is the stretch rate in the three main direction,  $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ , and  $I_2 = (\lambda_1 \lambda_2)^2 + (\lambda_2 \lambda_3)^2 + (\lambda_3 \lambda_1)^2$ . Besides, if the strain energy function is expressed with stretch rate  $\lambda$ , the cauchy stress  $\sigma_i$  of DE can also be expressed as [6]:

$$\sigma_i = \lambda_i \frac{\partial W}{\partial \lambda_i} - p \quad (3)$$

where  $p$  is known as hydrostatic pressure which can be derived from dynamics boundary conditions.

Suppose DE membrane is under the condition of uniaxial tension in the 1 direction illustrated in Figure 1, its strain  $\varepsilon_i$  and stretch rate  $\lambda_i$  meet the equation of  $\lambda_i = 1 + \varepsilon_i$ . Assume the stretch rate in the 1 direction is  $\lambda$  with cauchy stress  $\sigma$ , then:

$$\lambda_1 = \lambda, \quad \lambda_2 = \lambda_3 = \lambda^{-1/2}; \quad \sigma_1 = \sigma, \quad \sigma_2 = \sigma_3 = 0 \quad (4)$$

Considering Equation 1, 2 and 4, DE's stress can be derived as:

$$\sigma = 2 \left[ (1 + \varepsilon)^2 - (1 + \varepsilon)^{-1} \right] \left[ \frac{\partial W}{\partial I_1} + (1 + \varepsilon)^{-1} \frac{\partial W}{\partial I_2} \right] \quad (5)$$

$$s = 2 \left[ (1 + \varepsilon) - (1 + \varepsilon)^{-2} \right] \left[ \frac{\partial W}{\partial I_1} + (1 + \varepsilon)^{-1} \frac{\partial W}{\partial I_2} \right] \quad (6)$$

From Equation 5 and 6, once employing different models of strain energy function together with fitting the stress-strain data of uniaxial tension experiment, the material parameters of DE constitutive model can be obtained.

A uniaxial tension experiment of VHB4910 DE membrane was conducted under different tensile rates [6]. The calculation results and the experimental data are shown in Figure 2 and 3.

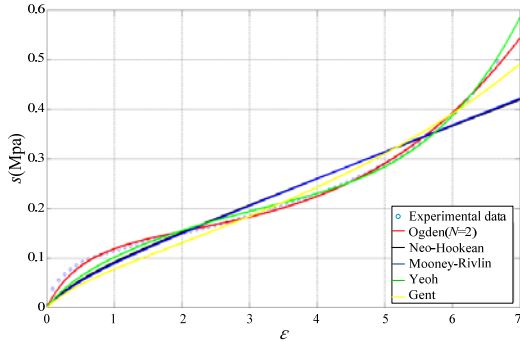


Figure 2 Comparison of uniaxial tension stress-strain curves using different constitutive models under tensile rate 400mm/min

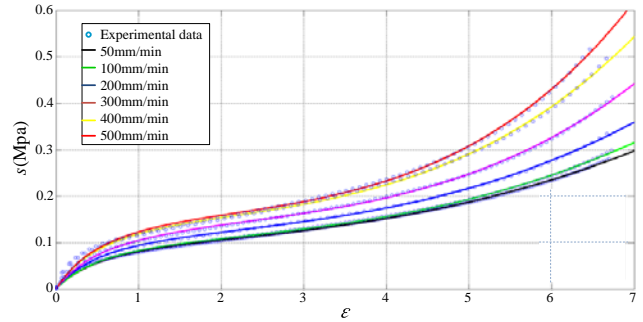


Figure 3 Comparison of uniaxial tension stress-strain curves using Ogden(N=2) constitutive model under different tensile rates

From Figure 2, the fitting curves of Neo-Hookean and Mooney-Rivlin model are similar straight lines far away from the experimental data which cannot describe DE's nonlinear large deformation. The fitting result of Gent model becomes better but still cannot predict the deformation precisely. The Yeoh model consisting of three material parameters can illustrate the S-shape characteristic of DE membrane as well as the constitutive relation when  $\lambda > 2$ , but deviates from the experimental data when  $\lambda < 2$ . The second order Ogden model fits the whole experimental data which can accurately predict the nonlinear stress-strain relation of DE. The constitutive equations and material

parameters of the five strain energy model under tensile rate 400mm/min are given in Table 1.

Table 1 Material parameters of different constitutive models under tensile rate 400mm/min

Strain Energy Model	Constitutive equation	Material parameters
Neo-Hookean	$W = \frac{\mu}{2}(I_1 - 3)$ $\sigma = \mu \left[ (1 + \varepsilon)^2 - (1 + \varepsilon)^{-1} \right]$ $s = \mu \left[ (1 + \varepsilon) - (1 + \varepsilon)^{-2} \right]$	$\mu=0.053\text{Mpa.}$
Mooney-Rivlin	$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3)$ $\sigma = 2 \left[ (1 + \varepsilon)^2 - (1 + \varepsilon)^{-1} \right] \left[ C_{10} + (1 + \varepsilon)^{-1} C_{01} \right]$ $s = 2 \left[ (1 + \varepsilon) - (1 + \varepsilon)^{-2} \right] \left[ C_{10} + (1 + \varepsilon)^{-1} C_{01} \right]$	$C_{10}=0.027\text{Mpa,}$ $C_{01}=-0.267 \times 10^{-2}\text{Mpa.}$
Yeoh	$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$ $\sigma = 2 \left[ (1 + \varepsilon)^2 - (1 + \varepsilon)^{-1} \right] \cdot \left\{ C_{10} + 2C_{20} \left[ (1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3 \right] + 3C_{30} \left[ (1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3 \right]^2 \right\}$ $s = 2 \left[ (1 + \varepsilon) - (1 + \varepsilon)^{-2} \right] \cdot \left\{ C_{10} + 2C_{20} \left[ (1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3 \right] + 3C_{30} \left[ (1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3 \right]^2 \right\}$	$C_{10}=0.030\text{Mpa,}$ $C_{20}=-0.269 \times 10^{-3}\text{Mpa,}$ $C_{30}=3.523 \times 10^{-6}\text{Mpa.}$
Gent	$W = -\frac{\mu}{2} J_m \ln \left( 1 - \frac{\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2} - 3}{J_m} \right)$ $\sigma = \mu \frac{(1 + \varepsilon)^2 - (1 + \varepsilon)^{-1}}{1 - \frac{(1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3}{J_m}}$ $s = \mu \frac{(1 + \varepsilon) - (1 + \varepsilon)^{-2}}{1 - \frac{(1 + \varepsilon)^2 + 2(1 + \varepsilon)^{-1} - 3}{J_m}}$	$\mu=0.044\text{Mpa,}$ $J_m=212.$
Ogden	$W = \sum_{i=1}^N \frac{\mu_i}{\alpha_i} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3)$ $\sigma = \sum_{i=1}^N \mu_i \left[ (1 + \varepsilon)^{\alpha_i} - (1 + \varepsilon)^{-\alpha_i/2} \right]$ $s = \frac{1}{1 + \varepsilon} \sum_{i=1}^N \mu_i \left[ (1 + \varepsilon)^{\alpha_i} - (1 + \varepsilon)^{-\alpha_i/2} \right]$	when $N=1$ , $\mu_1=0.039\text{Mpa,}$ $\alpha_1=2.168;$ when $N=2$ , $\mu_1=0.190\text{Mpa,}$ $\alpha_1=0.959, \mu_2=0.147 \times 10^{-3}$ $\text{Mpa, } \alpha_2=4.773;$ when $N=3$ , $\mu_1=-0.859 \times$ $10^{-2} \text{Mpa, } \alpha_1=4.773,$ $\mu_2=0.874 \times 10^{-2} \text{Mpa,}$ $\alpha_2=4.773, \mu_3=0.190\text{Mpa,}$ $\alpha_3=0.959.$

From the constitutive equation in Table 1, the Ogden model can be simplified as Neo-Hookean model when  $N=1$ ,  $\alpha_1=2$ , and as Mooney-Rivlin model when  $N=2$ ,  $\alpha_1=2$ ,  $\alpha_2=-2$ . Therefore, the Ogden strain energy function can delineate large deformation of DE with favorable universality and accuracy. From Figure 3, the hyperelastic constitutive model based on second order Ogden strain energy function can accurately simulate the nonlinear large deformation of VHB4910 DE membrane under different tensile rates.

### 3. QUASI-STATIC RESPONSE

According to reference [7], when put between two parallel compliant electrodes, DE will be subject to a vertical equivalent Maxwell stress  $p_E$ :

$$p_E = -\varepsilon_0 \varepsilon_r \left( \frac{U}{\lambda_3 z_0} \right)^2 \quad (7)$$

where the negative sign means compression in the vertical direction,  $\varepsilon_0$  is the vacuum dielectric constant usually equals to  $8.85 \times 10^{-12}$  F/m,  $U$  is the voltage exerted between the two electrodes,  $z_0$  is the initial thickness of DE membrane and  $\varepsilon_r$  is the relative dielectric constant.

It is worth noting that the dielectric constant is generally considered unchanged for ideal DE in most theoretical analysis. However, recent studies show that the dielectric constant of DE changes with stretch rate, excitation frequency, electrode material and temperature [8]. A dielectric experiment of VHB4910 DE membrane was conducted to consider the abovementioned dielectric constant change [9], where 100Hz of excitation frequency and silver grease electrode employed under biaxial equal tension condition. Thus the dielectric constant of DE on account of stretch rate is derived:

$$\varepsilon_r = 4.76 [1 - 0.051(\lambda_1 + \lambda_2 - 2)] \quad (8)$$

In order to analyze the voltage-induced quasi-static response of DEA, the states of initial, pre-stretched and electrical deformed are proposed as shown in Figure 4.

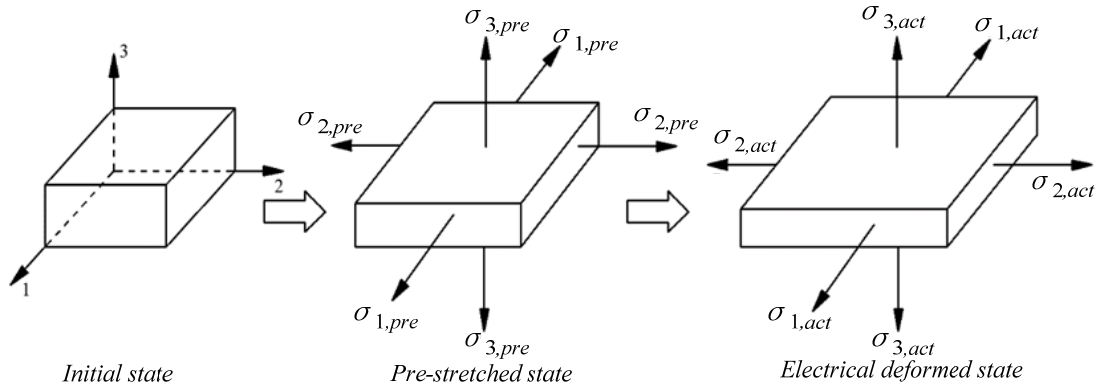


Figure 4 The process of voltage-induced deformation of DE actuating element

For the pre-stretched state of biaxial equal tension in the 1 and 2 directions, the pre-stretched stress  $\sigma_{pre}$  of DE is:

$$\sigma_{pre} = \sigma_{1,pre} = \sigma_{2,pre} = \lambda_{pre} \frac{\partial W}{\partial \lambda_{pre}} - \lambda_3 \frac{\partial W}{\partial \lambda_{3,pre}} \quad (9)$$

For the electrical deformed state, suppose the planar voltage-induced deformation is  $\varepsilon$  and the pre-stretched stress remains constant, thus:

$$\lambda_{1,act} = \lambda_{2,act} = \lambda_{pre} \cdot (1 + \varepsilon), \lambda_{3,act} = \frac{1}{\lambda_{pre}^2 (1 + \varepsilon)^2} \quad (10)$$

$$\sigma_{3,act} = \lambda_{3,act} \frac{\partial W}{\partial \lambda_{3,act}} + \lambda_{pre} \frac{\partial W}{\partial \lambda_{pre}} - \lambda_{3,pre} \frac{\partial W}{\partial \lambda_{3,pre}} - \lambda_{1,act} \frac{\partial W}{\partial \lambda_{1,act}} \quad (11)$$

Using the achieved constitutive model based on second order Ogden strain energy function, the equilibrium equation of DE actuating element is:

$$\sum_{i=1}^2 \mu_i \cdot \left[ \lambda_{pre}^{-2\alpha_i} \cdot (1 + \varepsilon)^{-2\alpha_i} + \lambda_{pre}^{\alpha_i} - \lambda_{pre}^{-2\alpha_i} - \lambda_{pre}^{\alpha_i} \cdot (1 + \varepsilon)^{\alpha_i} \right] = -\varepsilon_0 \varepsilon_r \left( \frac{U}{\lambda_{3,act} z_0} \right)^2 \quad (12)$$

Therefore, the quasi-static response of DE actuating element can be obtained solving Equation 12 under the excitation of DC voltage considering the change of dielectric constant. Based on Equation 8 and 12, the quasi-static response of planar DEA is calculated as shown in Figure 5 and 6.

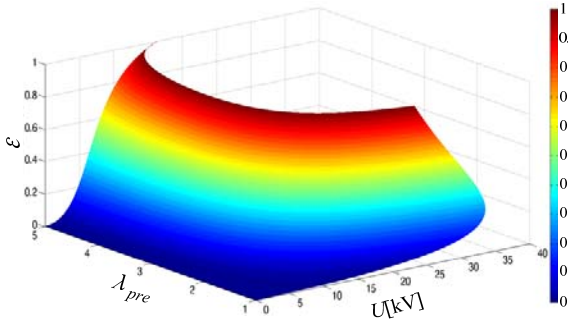


Figure 5 3D graph of planar strain, pre-stretch and DC voltage

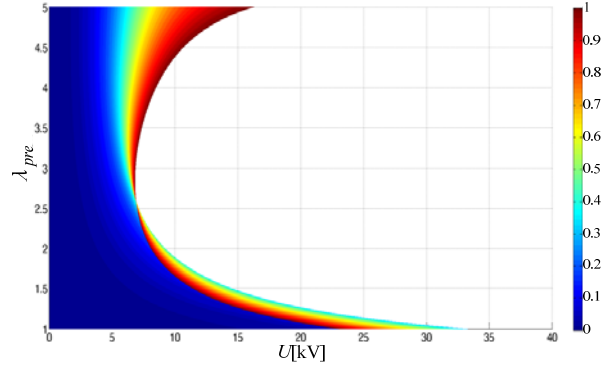


Figure 6 Contour lines of planar strain

From Figure 5 and 6, the pre-stretch rate and DC voltage are crucial to determine the electrical deformation characteristics of DE actuating element, and a maximum planar strain of 100% can be obtained under biaxial equal tension condition. Convenient for observation, here are the curves of planar strain and voltage under different pre-stretches, shown in Figure 7 and 8.

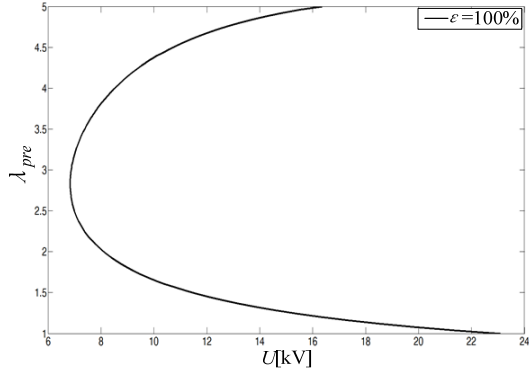


Figure 7 The curve of pre-stretch and voltage of 100% planar strain

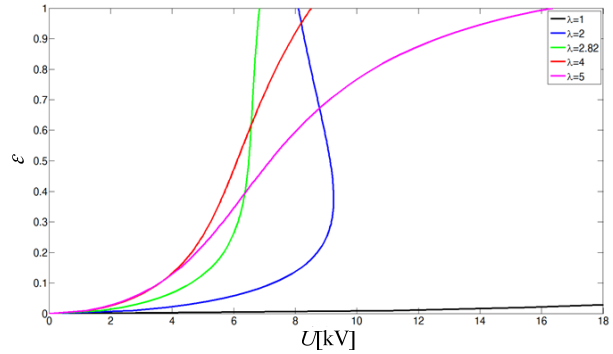


Figure 8 The curves of planar strain and voltage under different pre-stretches

From Figure 7, the curve of pre-stretch and voltage has an inflection point, i.e. there is a pre-stretch rate  $\lambda_{pre}^{opt}$  of 2.82 corresponding to a minimum voltage of 6.84kV which leads to 100% planar strain. Here we call  $\lambda_{pre}^{opt}$  the optimal pre-stretch rate. From Figure 8, distinct deformation can be observed with the increase of voltage under different pre-stretches. The electrical deformation increases with the pre-stretch rate increasing when

the voltage is less than 7kV, and increases at first and then decreases with the pre-stretch rate increasing when the voltage is larger than 7kV.

A comparison is made using the above quasi-static model with the dielectric constant remains 4.7, shown in Figure 9 and 10. It can be concluded that the change of dielectric constant is a key factor to influence the voltage-induced deformation of DEA. For instance, the optimal pre-stretch rate of DE actuating element rises to 3.95 while the minimum voltage of 100% electrical strain decreases to 4.58kV without the consideration of dielectric constant change. When exerting the same DC voltage, the electrical deformation of DE with regard to the change of dielectric constant is less than the deformation without the regard to the change of dielectric constant. Moreover, the abovementioned deformation difference becomes bigger when the pre-stretch rate increases.

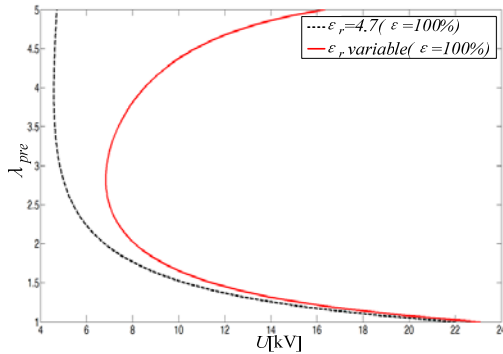


Figure 9 The curve of pre-stretch and DC voltage of 100% planar strain

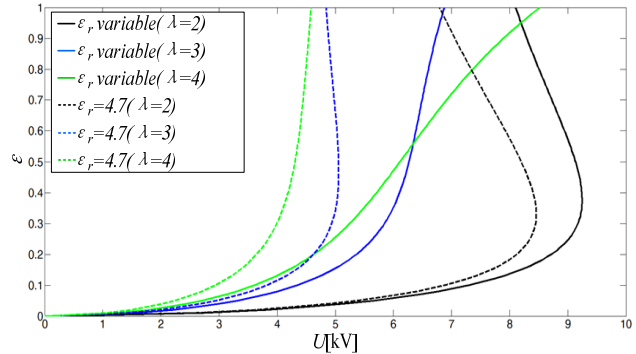


Figure 10 Comparison of the curves of planar strain and DC voltage under different pre-stretches

#### 4. DYNAMIC RESPONSE

To predict the voltage-induced dynamic response of DE, an dynamic analytical model is established as shown in Figure 11 with the size of  $2L \times 2L \times 2H$ . The material coordinate  $(X, Y, Z)$  represents the position of arbitrary point in the DE actuating element, while spatial coordinate  $(x, y, z)$  indicates the position of any point in the space, and the two coordinate system coincide at the center point. Considering geometric symmetry, a simplified 1/8 model with the size of  $L \times L \times H$  can be used for calculation. Assume the stretch rate in the thickness direction is  $\lambda$ , the dynamic response of DE actuating element can be derived [10]:

$$x(X, t) = \frac{L}{\sqrt{\lambda(t)}} X, \quad y(Y, t) = \frac{L}{\sqrt{\lambda(t)}} Y, \quad z(Z, t) = \lambda(t) Z \quad (13)$$

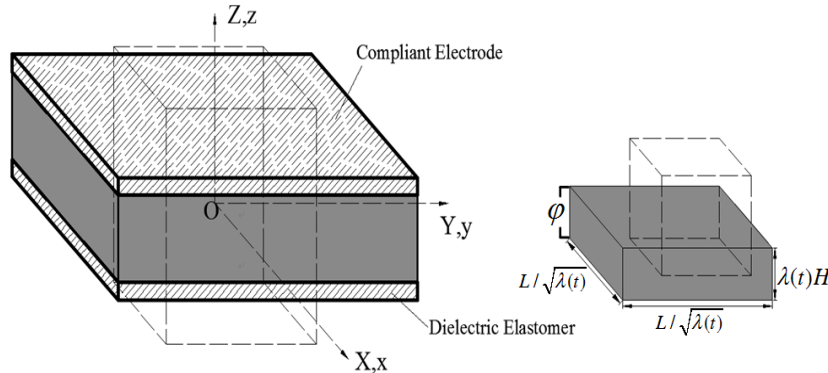


Figure 11 Dynamic analytical model of DE actuating element

According to thermodynamic energy theory, the change of Helmholtz free energy of DE actuating element equals to the sum of the change of electric field energy and the work of inertial force, i.e.:

$$V\delta F = \varphi\delta Q + \delta K \quad (14)$$

where  $F$  is the Helmholtz free energy per volume,  $V$  is the material volume,  $\varphi$  and  $Q$  is the electric potential and electric charge quantity respectively, and the work of inertial force is  $\delta K$ . Owing to electromechanical coupling, the Helmholtz free energy of DE actuating element can be obtained using the Ogden model:

$$F = \sum_{i=1}^N \frac{\mu_i}{\alpha_i} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) + \frac{1}{2} \varepsilon \frac{E_0^2}{\lambda^2} \quad (15)$$

where  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  represent the three stretch rates with  $\lambda_1 = \lambda_2 = 1/\sqrt{\lambda}$ ,  $\lambda_3 = \lambda$ ,  $\mu_i$  and  $\alpha_i$  are the corresponding material parameters,  $E_0$  is the nominal electric-field strength with  $E_0 = \varphi/H$  and the dielectric constant  $\varepsilon = \varepsilon_0 \varepsilon_r = 8.85 \times 10^{-12} \times 4.76[1 - 0.051(\lambda_1 + \lambda_2 - 2)]$ . Besides, the change of electric field energy and the work of inertial force are:

$$\varphi\delta Q = -2\varepsilon\varphi E_0 L^2 \frac{1}{\lambda^3} \delta\lambda = -2\varepsilon E_0^2 H L^2 \frac{1}{\lambda^3} \delta\lambda \quad (16)$$

$$\delta K = -\int_{\Omega} \rho \ddot{\mathbf{u}} \cdot \delta \mathbf{u} d\Omega = -\int_{\Omega} \rho \ddot{x} \delta x d\Omega - \int_{\Omega} \rho \ddot{y} \delta y d\Omega - \int_{\Omega} \rho \ddot{z} \delta z d\Omega \quad (17)$$

Where  $\mathbf{u} = \mathbf{x} - \mathbf{X}$  is the displacement vector with acceleration  $\ddot{\mathbf{u}}$ , material density  $\rho$  and system volume  $\Omega$ . Recall Equation 13 on Equation 17:

$$\delta K = -\rho H L^2 \delta\lambda \left( \frac{1}{6} L^2 \frac{1}{\lambda^3} \ddot{\lambda} + \frac{1}{3} H^2 \ddot{\lambda} - \frac{1}{4} L^2 \frac{1}{\lambda^4} \dot{\lambda}^2 \right) \quad (18)$$

Based on the above analysis, a second order differential motion equation of  $\lambda(t)$  i.e. the dynamic response model can be derived:

$$\ddot{\lambda} - \frac{3}{2} \frac{1}{(L^2 + 2H^2 \lambda^3)} \dot{\lambda}^2 + \frac{6 \left[ \sum_{i=1}^N \mu_i \left( \lambda^{\alpha_i+2} - \lambda^{\frac{\alpha_i}{2}+2} \right) + 8.4252 \times 10^{11} \left( \lambda^{\frac{1}{2}} - 1 \right) E_0^2 \right]}{\rho (L^2 + 2H^2 \lambda^3)} = 0 \quad (19)$$

Once given the boundary condition under different loads, the dynamic response of DE actuating element can be achieved by solving Equation 19. Based on the above dynamic response equation, a numerical analysis is conducted with the help of ode45 function in Matlab. Here we take VHB4910 DE membrane for example. Assume the initial condition is  $\lambda(0)=1.0$  and  $\dot{\lambda}(0)=0$ , namely exerting voltage from the original size. The constitutive model used in the numerical analysis is the abovementioned second order Ogden model with material density  $\rho=960\text{kg/m}^3$ .

When under the nominal electric field strength of 10kV/mm, 25kV/mm and 31.3kV/mm with  $H=0.5\text{mm}$  and  $L=5\text{mm}$ , the dynamic response of DE actuating element can be achieved, shown in Figure 12 and 13.

From Figure 12, electric-field strength is the key factor to determine the frequency and amplitude of DE actuator. The vibration frequency of DE actuator diminishes as the electrical strength increases. The amplitude enlarges and frequency decreases with the increase of electric-field strength, and maximum amplitude of 49% can be calculated with the critical electric-field strength of 31.3kV/mm. In addition, because of the material viscoelasticity, electromechanical interaction and other factors, the relation between vibration amplitude and electric field strength presents nonlinear as shown in Figure 13. When the electric field strength increases, the gradient of the curve of the vibration amplitude and electric field strength increases, i.e. the speed of the increasing amplitude is faster. When the electric field strength comes to the limited value, 51%



electrical deformation will be obtained, and electromechanical instability will occur if the electric field strength keeps increasing.

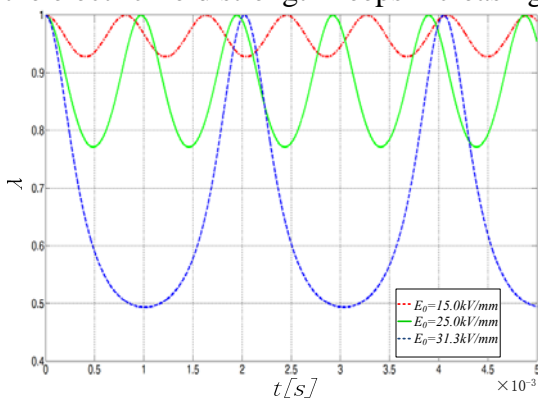


Figure 12 Dynamic response under different electric loadings

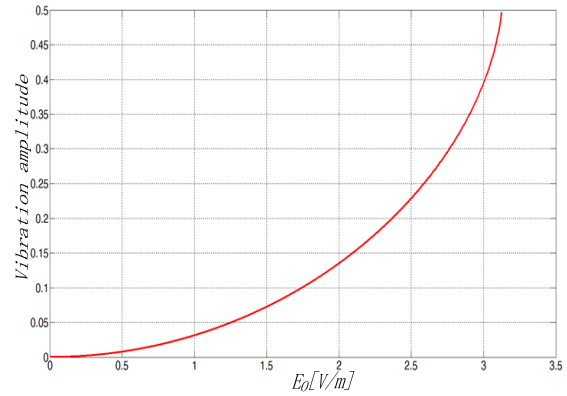


Figure 13 Curve of the vibration amplitude and electric field strength of DE actuating element

When the electric-field strength varies with the time, the dynamic response of DE actuating system is more complicated. Assume harmonic nominal electric-field strength:

$$E_0 = E_A \sin 2\pi f t \quad (20)$$

where  $E_A$  is the amplitude of electric-field strength and  $f$  is the frequency. With  $H=0.5\text{mm}$  and  $L=5\text{mm}$ , the frequency-amplitude response in the frequency range of (1Hz, 1kHz) can be achieved when  $E_A=20\text{kV/mm}$  and  $25\text{kV/mm}$ , shown in Figure 14 and 15.

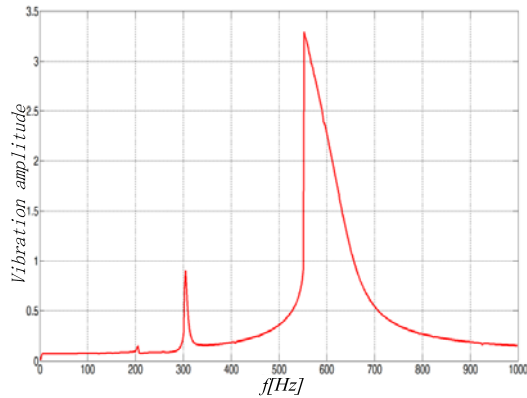


Figure 14 Frequency-amplitude response of DE actuating element under  $E_A=20\text{kV/mm}$

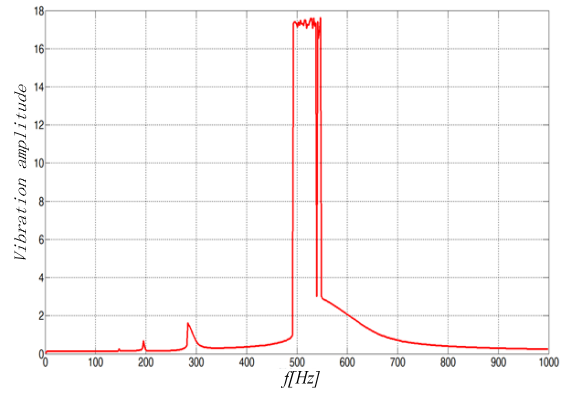


Figure 15 Frequency-amplitude response of DE actuating element under  $E_A=25\text{kV/mm}$

The Figure 14 and 15 show that peak values appear in the curve of frequency-amplitude response with  $E_A$  increasing. For instance, the peak values of 205Hz, 303Hz and 553Hz are obtained when  $E_A=20\text{kV/mm}$ . In addition, the frequency-amplitude response is out of convergence in the range of (490Hz, 550Hz) when  $E_A=25\text{kV/mm}$ , which indicates that the excitation frequency is close to the inherent frequency of DE actuating element and resonance is likely to happen.

For the frequency range near the resonance, the vibration of the system is dramatic but stable as you can see in Figure 16 and 17. Furthermore, compared with constant electrical loads, the amplitude of DE actuating system grows significantly under harmonic electrical loads with periodic characteristics.

It is known that reference [11] proposed a dynamic model based on Neo-Hookean model ( $\mu=0.053\text{Mpa}$ ) without the consideration of the change of DE dielectric constant ( $\epsilon_r=4.7$ ). The following comparison is made between the model in this paper and [11] as shown in Figure 18 and 19. The difference is obvious to find in Figure 18 and 19 with

the amplitude-nominal electric field strength and frequency-amplitude response of DE actuating element. Therefore, the above dynamic analytical model based on second order Ogden constitutive model considering the change of dielectric constant can predict the dynamic response of DE actuator more precisely.

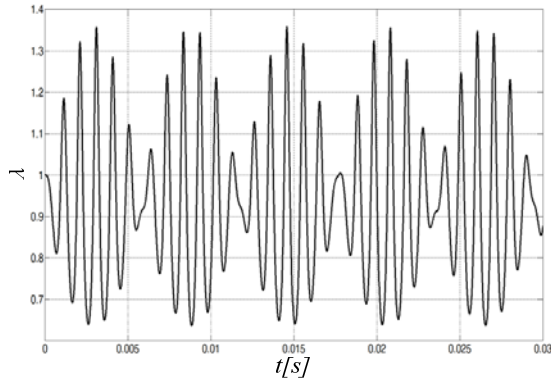


Figure 16 Dynamic response under  $E_A=25kV/mm$ ,  $f=480Hz$

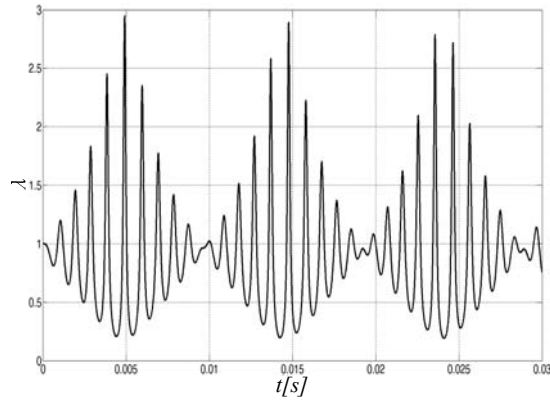


Figure 17 Dynamic response under  $E_A=25kV/mm$ ,  $f=560Hz$

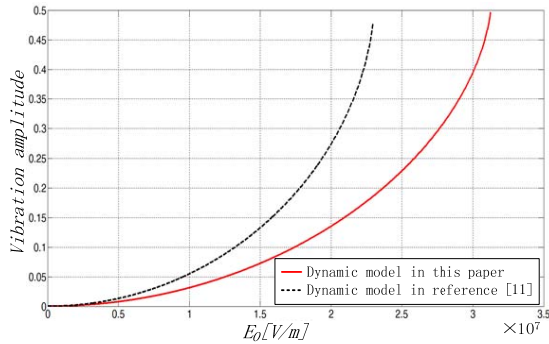


Figure 18 Comparison of dynamic response under different electric loadings

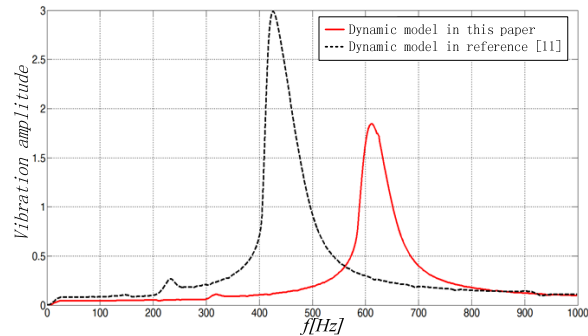


Figure 19 Frequency-amplitude response of DE actuating element under  $E_A=16kV/mm$

## 5. CONCLUSIONS

The quasi-static and dynamic responses of dielectric elastomer actuator under different electrical loads are achieved in this paper. First, the characteristics of five strain energy functions for VHB4910 DE membrane are compared. A hyperelastic constitutive model based on second order Ogden strain energy function is achieved, which can accurately simulate the nonlinear large deformation of VHB4910 DE membrane under different tensile rates. Second, according to the established model of DE membrane, the quasi-static response equation is developed considering the changes of DE dielectric constant. The results show that under the condition of equal bi-axial tension and DC voltage, DE has an optimal pre-stretch rate corresponding to a minimum DC voltage inducing its extreme electrical deformation. Besides, the electrical deformation of DE can be improved as pre-stretch rate increases in the low voltage range but things are different in the high voltage range. Third, the dynamic response equation is obtained with regard to the dielectric constant changes under constant and harmonic electrical loads. It is worth noting that the vibration frequency of DE actuator diminishes as the electrical strength increases and the variation behaves nonlinear.

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