Decomposition of cylindrical wave modes of fine-scale turbulence noise

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ABSTRACT

Decomposition of the sound field into cylindrical waves is one of the effective methods to understand the noise-generation process. In the present paper, a new mathematic function describing the dependence of cylindrical-wave modes of fine-scale turbulence noise on difference frequencies with Tam and Auriault's theory model is developed. Both axisymmetric and non-axisymmetric jets are analysed by the method of cylindrical wave decomposition. It turns out that the number of the dominant cylindrical wave modes is linearly related to the frequencies, which allows cutting down the computational time apparently. Comparisons between the calculated results with fewer cylindrical waves and experimental measurements operating subsonically through supersonically are carried out. Good agreements are observed over a wide range of frequencies and observation positions. Results of these comparisons indicate that the method of cylindrical wave decomposition is capable of predicting the far-field radiated noise accurately and efficiently.

Keywords: Turbulence noise, Cylindrical waves, Numerical method
I-INCE Classification of Subject Number: 23,76

1. INTRODUCTION

Cylindrical wave decomposition is intensively used for studying different kinds of acoustic noise, including noise of the aerodynamic origin[1, 2]. The spatial distribution of cylindrical waves was obtained by measuring the turbulence noise at finite azimuthal angles by Kopiev[3]. In order to obtain the contribution of the dominant cylindrical waves efficiently, the number of microphones in azimuthal direction was reduced by Georgy[4]. Cylindrical wave decomposition of turbulent noise has been studied extensively and deeply in experiments, but there is little research work in theory.

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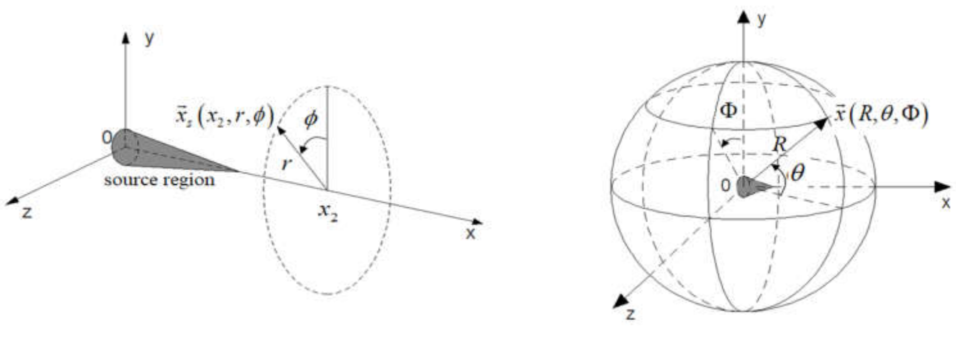
A theoretical model applicable to a variety of operating conditions was developed by Tam[5, 6], which could be applied to predict fine-scale turbulence noise in far field. In this theoretical method, the power spectrum was obtained by superimposing infinitely modes of cylindrical waves. The modes of the required cylindrical waves are supposed to be depended on the frequencies by Nishant[7]. The number of cylindrical-wave modes at finite frequencies was given empirically. For example, the number of cylindrical wave modes is 100 at f=12500Hz. However, the specific contribution of each mode of cylindrical waves with frequency is still unknown, which is not conducive to the calculation of jet noise in different jet conditions.

In this study, the cylindrical wave decomposition method is applied to analyse the contribution of various modes of cylindrical waves to the far-field radiation noise, in order to know more about the radiation characteristics of fine-scale turbulence noise. Based on the theoretical model, the numbers of the cylindrical-wave modes in both axisymmetric and non-axisymmetric jets are found to be proportional to frequencies. Finally, the anechoic chamber experiment is carried out subsonically and supersonically. It turns out that the calculated sound pressure levels with the fewer cylindrical waves are in good agreement with the measured data. At the same time, it is believed that this method has obvious advantages of computational efficiency to predict the far-field radiated noise of fine-scale turbulence.

2. CYLINDRICAL WAVE DECOMPOSITION

Cylindrical wave decomposition is a common method to analyse the directivity of noise sources and propagation. In this study, this method is used to analyse the radiation characteristics of fine-scale turbulent noise in the far field.



(a) Cylindrical coordinates in source region (b) Polar coordinates in the observation position

Figure 1. The schematic of two sets of coordinates.

The prediction of fine-scale turbulence noise is usually divided into two parts[5, 7]: the modelling of noise source and the solution of sound propagation. Analogized by the molecular thermal motion theory, a new sound source model was developed by Tam[5], which was accurate and not limited by axial symmetry[8, 9]. However, they did not take into account the influence of frequency on the characteristic scale of turbulence[10]. Therefore, a revised sound source model in this work is used as: $\left(\frac{\pi}{\ln 2}\right)^{\frac{3}{2}} \times \frac{\left(l_s(\omega)\right)^3 \exp\left[\left(l_s(\omega)\right)^2 \omega^2 / \left(4\ln 2\bar{u}^2\right)\right]}{\tau_s\left[1+\omega^2\tau_s^2\left(\bar{u}\cos\theta / a_\infty - 1\right)^2\right]}$, where ω denotes angular frequency; τ_s denotes turbulence time scale[5]; l_s denotes turbulence length scale[10]; θ denotes observation polar angle as shown in Figure 1(b); a_∞ denotes ambient sound velocity; u is the velocity components in the x-direction. And all of the mean flow variables are denoted by an overbar. After modelling the sound source, the Adjoint Green Function[6] is used to describe the propagation of the far-field radiated noise, and the power spectral density can be expressed as:

$$S(\vec{x},\omega)=4\pi\left(\frac{\pi}{\ln 2}\right)^{\frac{3}{2}}\iiint_{\infty}\left|p_a\left(x_2,r,\phi,\vec{x},\omega\right)\right|^2 d\phi\frac{A^2 l_s^3}{\tau_s}\left\{\frac{\exp\left[-\frac{\omega^2 l_s^2}{\bar{u}^2(4\ln 2)}\right]}{1+\omega^2\tau_s^2\left(1-\frac{\bar{u}}{a_\infty}\cos\theta\right)^2}\right\}dx_2 dr \qquad (1)$$

where the coordinate of sound source $\vec{x}_s\left(x_2,r,\phi\right)$ is shown as Figure 1(a). The Adjoint Green Function $p_a\left(x_2,r,\phi,\vec{x},\omega\right)$ is expressed as:

$$p_a\left(x_2,r,\phi,\vec{x},\omega\right)=\frac{-i\omega}{8\pi^2 a_\infty^2 R}\exp\left(i\frac{\omega}{a_\infty}\left(R-x_2\cos\theta\right)\right)\sum_{m=0}^{\infty}p_m(r)\cos m\phi \qquad (2)$$

$p_m(r)$ is expected to satisfy a second order homogeneous ordinary differential equation[6].

Equation 2 shows that Adjoint Green Function could be expressed as the superposition of infinitely many cylindrical waves: $\{1,\cos(\phi),\cos(2\phi),\ldots\ldots,\cos(m\phi),\ldots\ldots\}$, where the mode of the cylindrical wave m is a nonnegative integer. The image function of power spectral density can be derived as $S_m(\omega)$ in the domain of cylindrical wave modes, because $\{1,\cos(\phi),\cos(2\phi),\ldots\ldots,\cos(m\phi),\ldots\ldots\}$ is completely orthogonal on the interval of $[0,2\pi)$. $S_m(\omega)$ is deduced as:

$$S_m(\omega)=\frac{\displaystyle\iint_{Area_{x-r}}\left\{\frac{\exp\left[\frac{-\omega^2 l_s^3}{\bar{u}(4\ln 2)}\right]}{1+\omega^2\tau_s^2\left(1-\frac{\bar{u}}{a_\infty\cos\theta}\right)^2}\right\}^{\frac{1}{2}}\exp\left(i\frac{\omega}{a_\infty}\left(R-x_2\cos\theta\right)\right)p_m(r)dx_2 dr}{S_{\mathrm{ref}}(\omega_0)} \qquad (3)$$

where,

$$S_{\mathrm{ref}}(\omega_0)=\iint_{Area_{x-r}}\left\{\frac{\exp\left[\frac{-\omega_0^2 l_s^3}{\bar{u}(4\ln 2)}\right]}{1+\omega_0^2\tau_s^2\left(1-\frac{\bar{u}}{a_\infty\cos\theta}\right)^2}\right\}^{\frac{1}{2}}\exp\left(i\frac{\omega_0}{a_\infty}\left(R-x_2\cos\theta\right)\right)p_0(r)dx_2 dr \qquad (4)$$

$S_{\mathrm{ref}}(\omega_0)$ in the denominator for Equation 3 is used for normalization, which could be any value. Here we have $\omega_0=2\pi\frac{St_0\times U_j}{D_j}$ ($St_0=0.256$ is usually the peak Strouhal number; U_j denotes fully expanded jet velocity; and D_j denotes the diameter of the nozzle). As explained above, this new function could be applied to describe the contribution of different modes of cylindrical waves on the power spectrum.

By calculation, it is found that the value of $S_m(\omega)$ is approximately equal to 0 as $m>25$, which means, the contribution of cylindrical waves greater than 25th-mode to fine-scale turbulence noise could be ignored. In fact, the distribution of the first few modes of cylindrical wave in the total energy at different frequencies is even more important, which is conducive to reducing unnecessary calculation and predicting fine-scale turbulence noise accurately. For this reason, Equation 5 is developed:

$$\mathrm{F}(m,\omega)=\frac{\sum_{n=0}^{m}S_n(\omega)}{S_0(\omega)+S_1(\omega)+\ldots+S_{25}(\omega)}(m=0,\cdots,25) \tag{5}$$

$\mathrm{F}(m,\omega)$ describes the proportion of the first modes of the cylindrical waves in the total energy, after the cylindrical wave decomposition of the fine-scale turbulence noise as Equation 4. According to the result of $\mathrm{F}(m,\omega)$, the number of summation series in Equation 2 can be determined specifically, so as to achieve the purpose of rapid calculation.

3. AXISYMMETRIC JET

Taking the typical circular axisymmetric jet flow as an example, the fine-scale turbulent noise is decomposed into cylindrical waves according to the method in Section 2, and the relationship between frequencies and the number of the dominant modes of cylindrical waves is expected to be obtained.

The diameter of the circular axisymmetric nozzle is 25mm; the jet velocity is 0.7Ma; the temperature is 300K, and the ambient pressure is 102400Pa. The geometry of the nozzle is shown as follows:

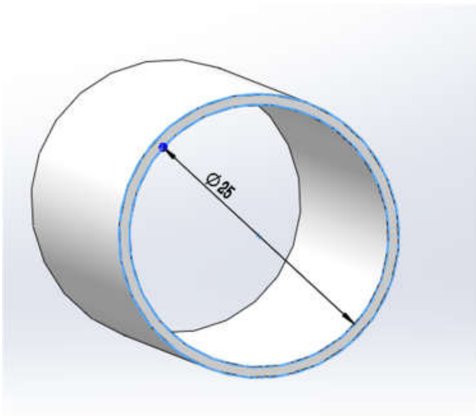


Figure 2. The geometry of the axisymmetric nozzle.

The flow field is simulated by commercial software, ANSYS FLUENT; and the sound computation is compiled by FORTRAN (more details could be found in Reference[11]). The results of function $\mathrm{F}(m,\omega)$ of different modes and frequencies are obtained as Figure 3.

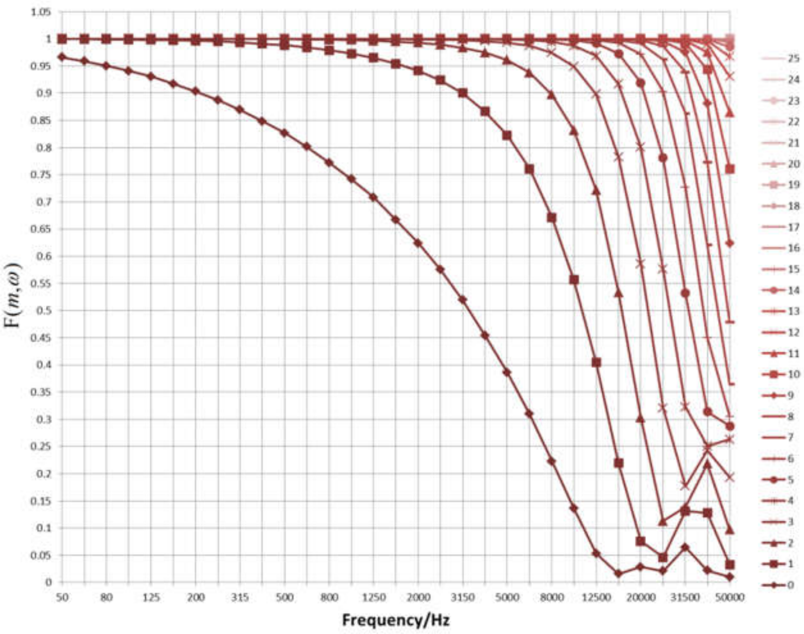


Figure 3. $\mathrm{F}(m,\omega)$ *of different modes and frequencies in axisymmetric jet.*

Assuming that $\mathrm{F}(m,\omega) \geq 0.95$ can represent all the energy at this frequency, the corresponding number of the cylindrical-wave modes at different frequencies is the value of N_m as $\mathrm{F}(m,\omega) = 0.95$. In this way, N_m is shown as the red line in Figure 4.

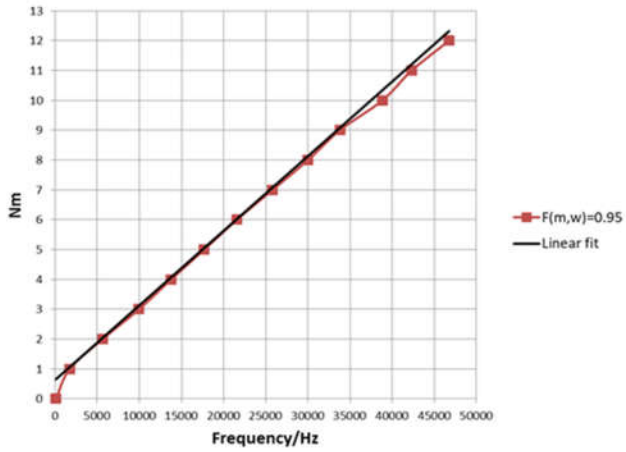


Figure 4. N_m *from* $\mathrm{F}(m,\omega) = 0.95$ *and linear fitting function.*

It shows that N_m from $\mathrm{F}(m,\omega) = 0.95$ has a linear proportional relationship with frequency *f* in Figure 4. A linear function is proposed as Equation 6 and shown by the black line in Figure 4.

$$N_m(f) = 0.00025f + 0.6337 \tag{6}$$

By comparing the power spectral density with $N_m(f) = 25$ and $N_m(f) = 0.00025f + 0.6337$ respectively, it is found that the latter only takes 20 minutes to calculate the far field sound field of each observation point, which is 10 times less than the former. Comparisons of sound field prediction are shown in Figure 5.

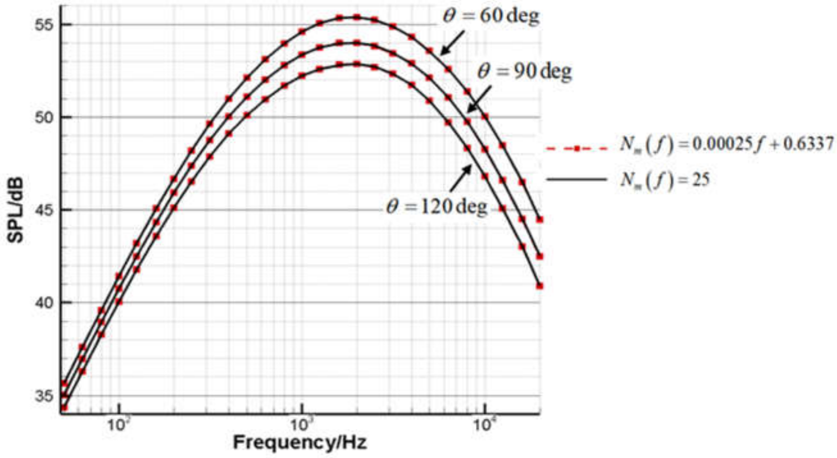


Figure 5 Sound pressure level calculated by the two methods.

It shows that the calculated results with the two methods are in good agreement. Obviously, the accuracy of the predicted results could be obtained by N_m in Equation 6. Besides that, N_m in Equation 6 has much more obvious computing advantages than the other method with the increase of observation points.

4. NON-AXISYMMETRIC JET

A large number of studies have shown that non-axisymmetric nozzles can enhance the mixing effect of turbulent flow, thus reducing the turbulence noise. Therefore, it is important to analyse the characteristics of the noise from non-axisymmetric jets deeply, and predict the sound pressure level accurately as well.

The jet generated by a nozzle with six rectangular lobes is taken as an example of non-axisymmetric jet. The geometry of the nozzle is shown as Figure 6.

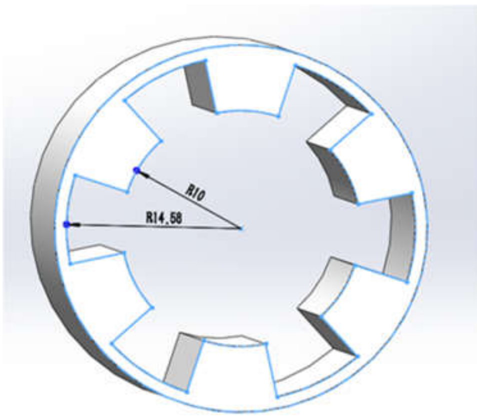


Figure 6. The geometry of the non-axisymmetric nozzle.

The inner ring radius of the nozzle is 10mm, and the outer ring radius is 14.58mm. The environmental parameters and the simulation methods for the flow field are the same as those in Section 3. It is noted that the function $\mathrm{F}(m,\omega)$ of the non-axisymmetric jet depends on the azimuthal angle ϕ of sound source, as shown in Figure 1(a). Without the loss of generality, $\phi = 0$ is taken as an example here. $\mathrm{F}(m,\omega)$ of different modes and frequencies is shown in Figure 7.

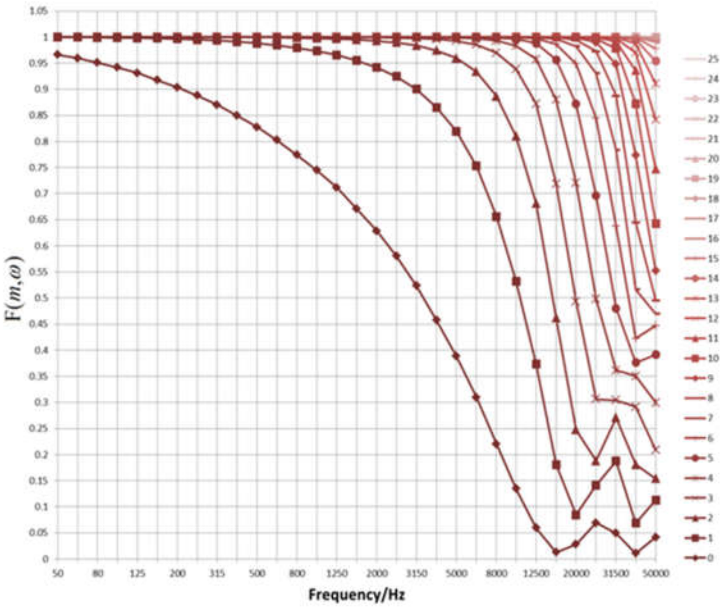


Figure 7. $\mathrm{F}(m,\omega)$ *of different modes and frequencies in non-axisymmetric jet.*

The number of the corresponding cylindrical-wave modes, namely N_m, could be obtained as $\mathrm{F}(m,\omega)=0.95$ at different frequencies. The relationship between N_m from $\mathrm{F}(m,\omega)=0.95$ and frequency is shown as the red line in Figure 8.

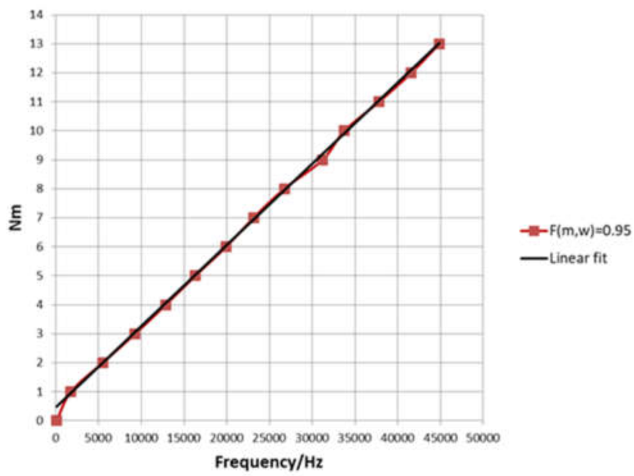


Figure 8. N_m *from* $\mathrm{F}(m,\omega)=0.95$ *and linear fitting function.*

It is also observed that N_m is linearly related to frequency *f* approximately, as the black line in Figure 8. A linear mathematic equation is used to fit N_m and *f*, which is expressed as:

$$N_m(f)=0.00028f+0.4682 \tag{7}$$

After a lot of calculation, the linear fitting functions obtained by other azimuthal angles are not significantly different from that obtained from $\phi=0$, so it can be considered that Equation 7 is applicable in all directions of azimuth. By comparing the power spectral density with $N_m(f)=25$ and $N_m(f)=0.00028f+0.4682$ respectively, it is also found that the computational time of the latter is 10 times less than that of the former. Comparisons of sound field prediction are shown in Figure 9.

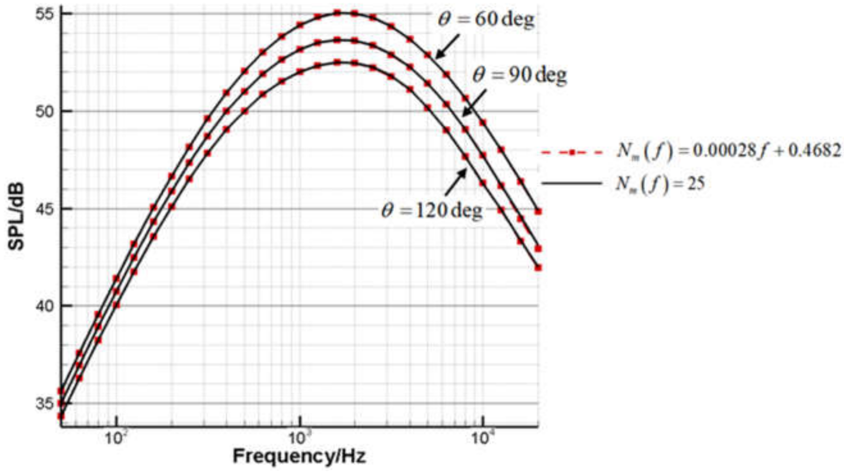


Figure 9. Sound pressure level calculated by the two methods.

Once again, as can be seen from the figure above, the two kinds of calculation results are in good agreement. Therefore, the accurate prediction of fine-scale turbulence noise can be obtained by Equation 7 for non-axisymmetric jets.

5. EXPERIMENTAL VERIFICATION

In order to verify the calculation method based on the distribution of cylindrical waves, a number of the jet noise experiments of circular nozzles operating subsonically through supersonically are carried out in the anechoic chamber. Descriptions of the anechoic facility, the jet simulator, and other details are provided in Reference[12]. The experimental environment and microphone layout are shown as:

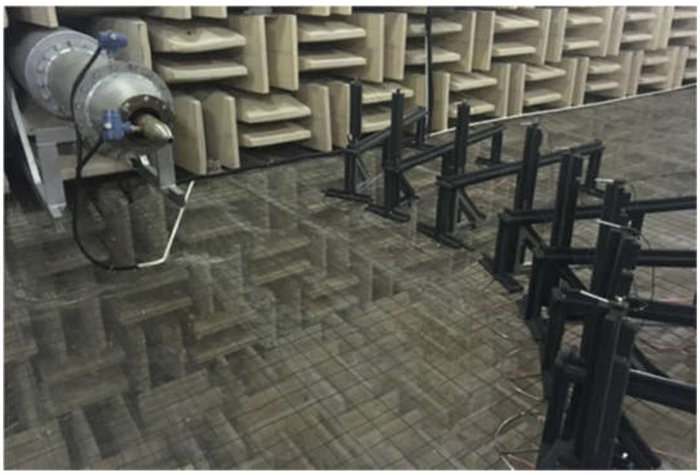
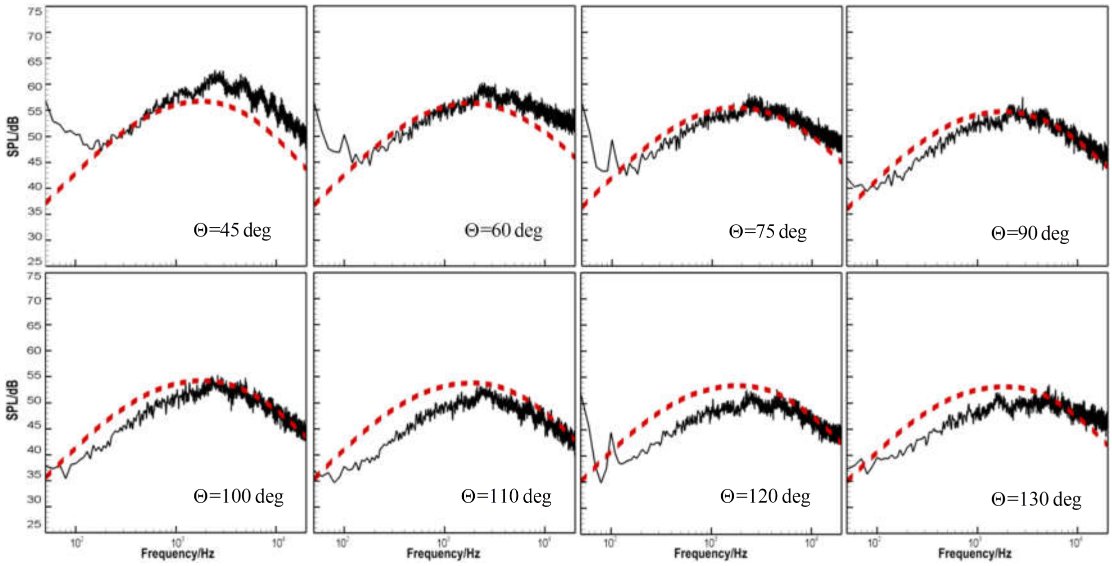
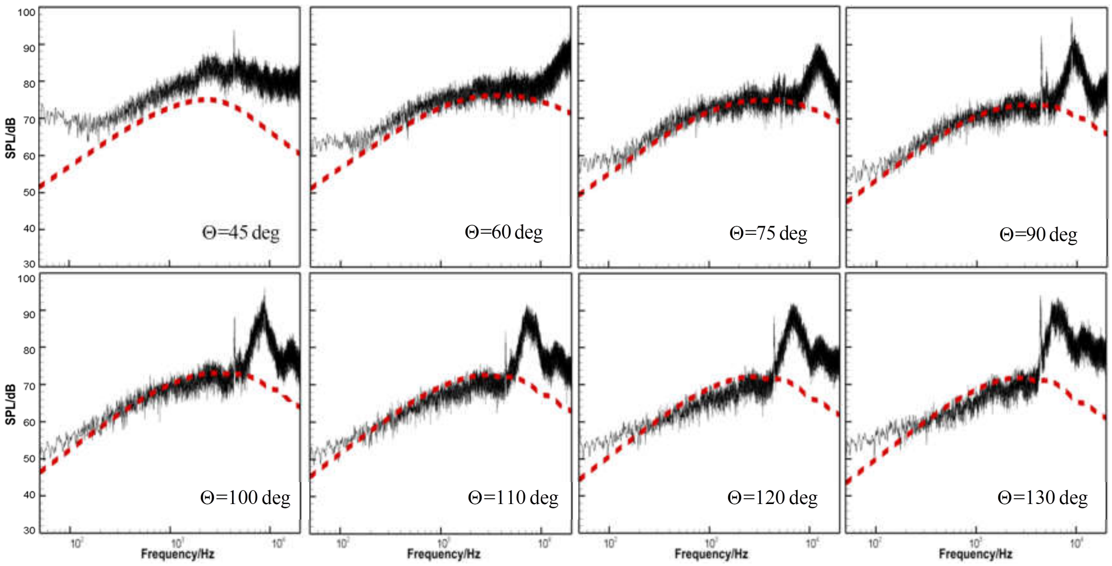


Figure 10. The photograph of the microphone array.

The diameter of the nozzle *Dj* is designed as 25mm. There are 8 microphones with a polar angle θ range from 45 to 130deg. The array is at a constant polar radius of *R*=50*Dj* from the origin of the coordinate system located on the jet centreline at the nozzle exit plane. The comparisons between the sound spectra calculated by N_m from Equation 6 and those measured by the experiments of 0.7Ma and 1.5Ma are shown as Figure 11, where $\Delta f = 1Hz$.



(a) 0.7Ma



(b) 1.5Ma

Figure 11. Jet noise spectra at R=50Dj.

(— experiment; ---- results from Equation 6)

Figure 11(a) indicates that good agreements between the results calculated by Equation 6 and experiment are found at the polar angles from 60deg to 130deg for subsonic noise. One possible reason for the loss of accuracy at 45deg is that this angle might be too close to the cone of silence[13], where the theory method is limited.

In Figure 11(b), the same good agreements are observed from 60deg to 130deg at the frequencies no more than 8 kHz for supersonic noise. At very high frequency, it is seen that there are noticeable differences between them. This is due to the broadband shock associated noise and the screech[14], which should be ignored in the comparison.

On considering that there are large variations in sound pressure level and spectral shape with jet-exit velocity over the test range, it is fair to conclude, based on a detailed examination of all of the comparisons, that there is good overall agreement between the results with the method of the cylindrical-wave decomposition and experiment data. It could also provide the solid evidence that the Equation 6 is capable of predicting fine-scale turbulence noise efficiently and accurately.

6. CONCLUSIONS

Cylindrical wave decomposition is a common method to analyse the radiation characteristics of noise sources. In this study, the method of cylindrical wave decomposition is applied to the analysis of the far-field acoustic radiation characteristics of fine-scale turbulent noise.

The far-field fine-scale turbulence noise generated by the axisymmetric nozzle and non-axisymmetric nozzles with 6 rectangular lobes are analysed in this method. It turns out that the dominant modes of the cylindrical waves are linearly related to the frequencies, which could be fitted by a linear function respectively.

Anechoic chamber experiments are carried out to verify the experimental data and prediction results calculated by this method. Due to the good agreement, it is believed that the method can be used to predict the far field noise of fine-scale turbulence efficiently and accurately. At last, it must be said that, although the initial results are promising, this method still needs to be tested over a wider range of jet working conditions.

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