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Closure Mechanism Investigation of Axisymmetric and Helical Screech Modes in Underexpanded Round Jets.

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ABSTRACT

This work provides an experimental contribution to the study of the Screech phenomenon. Two jets undergoing axisymmetric (*A2*) and helical (*C*) modes are considered, corresponding to $M_j=1.13$ and *1.5*, respectively. Acoustic measurements using a microphone and Particle Image Velocimetry (PIV) are employed in combination with Proper Orthogonal Decomposition (POD) in order to obtain information about the velocity fluctuations fields. We investigate the Screech closure mechanism by extracting the signature of upstream *travelling* instabilities using the spatial Fourier transform of the complex-valued leading POD eigenfunctions. The spatial support of these instabilities is compared to the respective theoretical upstream neutral waves modes of the jet, predicted by a Vortex-Sheet model. A close agreement between the experimental and the theoretical data is obtained for the axisymmetric Screech mode *A2*, confirming that these upstream waves may be involved in the closure mechanism of this one, as already mentioned in recent works. In addition, the signature of these kinds of instabilities for the helical mode *C* shows a reasonable concordance with theoretical data, suggesting that this Screech mode may also be closed by upstream neutral modes.

Keywords: Screech, Underexpanded Jets, Closure Mechanism
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1. INTRODUCTION

The expansion process of underexpanded supersonic jets in quiescent atmosphere leads to the formation of a train of shock-cell structures within the potential core that interacts with the jet turbulence inside of the mixing layer, producing "shock-associated noise". Under special self-resonance conditions these imperfectly expanded jets emit very intense pure tones known as Screech which was first pointed out by Powell[1,2]. Screech, as well as broadband shock-associated noise (BBSAN), is part of the shock noise and is a tonal component that is highlighted by one or several peaks in the noise spectrum. Moreover, it is known that due to primarily upstream directivity, these intense tones are able to cause structural damages and failure by fatigue in the airplane components[3,4].

Since the early works of Powell[1,2] screeching jets have been widely studied[5], however mostly from the acoustic point of view. The principle of its generation is cyclic. It consists of a looping behaviour with 4 phases: 1) turbulent perturbations (instabilities waves) increasing within the mixing layer and convecting downstream. They constitute the energy source of the screech feedback loop[6]; 2) interaction between shock and turbulence generating upstream pertubation; 3) backward propagation of these waves, and; 4) instabilities re-excitation close to jet exit, restarting the loop, where at this location the thin layer makes instabilities more receptive to excitation[7]. Therefore, Screech noise is the result of a resonance phenomenon between downstream waves, associated to Kelvin-Helmholtz instabilities, and upstream travelling waves propagating outside of the flow.

As the Mach number M_j increases the Screech phenomena presents a curious behaviour for round and elliptical jets: frequencies jumps and modal structures shifts. These "stages", visualized as "jumps" in the Screech frequency curve, are known as Screech modes and are classified according to the jet mode instability. Powell[1] identified, for round jets, 4 Screech modes named *A*, *B*, *C* and *D*. Powell *et al.*[8] employed a Schlieren system for visualisation and microphones to determine the mode organization: *A1* and *A2* are axisymmetric (or varicose mode), *B* is flapping (or sinuous), *C* is helical and *D* is flapping (or sinuous) too, which confirms previous Powell's observation. Similar results were also observed by Davies & Oldfield[9]. Nowadays, the mode *D* may be considered as a mode *B* extension[10]. It means that extrapolating mode *B* in the Strouhal x M_j curve, the frequencies for *B* and *D* modes match together.

Concerning the Screech closure mechanism, Shen & Tam[11] proposed that the feedback phenomenon may be closed by two kinds of disturbances propagating upstream, suggesting that 2 different closure mechanisms may exist. The first mechanism, proposed by Tam *et al.*[12], consists of interactions between the shock-cell and amplified hydrodynamic disturbances in the mixing layer that generate *acoustic* perturbations propagating upstream, outside of the jet shear layer, up to the nozzle lip and exciting new instabilities. From this model of feedback, Shen & Tam found a good agreement for the modes *A1* (axisymmetric) and *B* (flapping), meanwhile the results for the modes *A2* (axisymmetric) and *C* (helical) are poorly predicted. This is the *classical* or *standard* model of the Screech closure mechanism, also proposed by Powell[1].

The second mechanism is based on the presence of another kind of upstream instability waves, with a spatial support inside and outside of the flow, identified by Tam & Hu[13] and named here as *upstream jet neutral modes*. From this model, Shen & Tam could predict the frequency of modes *A2* (axisymmetric) and *C* (helical). Recently, Edgington-Mitchell *et al.*[14] showed the signature of this kind of instabilities for both axisymmetric Screech modes *A1* and *A2*, indicating that these upstream-travelling disturbances may close the Screech feedback mechanism of all axisymmetric Screech tones. Moreover, Gojon *et al.*[15], from numerical simulation, highlighted these kinds of waves for the jet undergoing helical dominant Screech mode *C*. However, no experimental observation is still available. Finally, according to Shen & Tam, the presence of these two feedback models could explain the coexistence of two Screech tones simultaneously.

The focus of the following work is then to investigate experimentally the signature of these upstream neutral instabilities in the jets undergoing two different dominant Screech modes (axisymmetric and helical), with the purpose of evaluate their possible involvement in the Screech closure mechanism. In order to reach this objective we employ acoustic measurements, PIV experiments and POD techniques. From the POD data we carried out a spatial Fourier decomposition of the velocity fluctuations fields, enabling to extract information about the signature of these upstream instabilities.

2. EXPERIMENTAL TECHNIQUES AND FACILITIES

The experiments were carried out at the P' Institute, a CNRS (*Centre National de la Recherche Scientifique)* laboratory. To investigate the supersonic flow, one circular convergent nozzle with *10 mm* outlet diameter and *1.0D* lip thickness was employed. The flow conditions considered are given in the table 1.

Table 1: analysed flow conditions. T_0=293K and $Re_j=\rho_j U_j D_j/\mu$.

NPR	M_j	*Mode*	*St*	U_j *(m/s)*	Re_j
2.23	*1.13*	*A2*	*0.65*	*347*	*2.9E+05*
3.67	*1.5*	*C*	*0.32*	*427*	*4.44E+05*

The acoustic data acquisition were performed using a ¼" GRAS microphone (46BP model) located at 46D from the nozzle exit and at 90° according to the jet axis. The signal acquisition is performed by a PXIe-1071 system of National Instruments with an acquisition frequency of *200 kHz.* The acoustics measurements had the purpose of monitor the Screech dominant modes during the experiments.

2.1 Particle Image Velocimetry

The PIV system consisted in a laser Quantel Twins Ultra *30mJ*, type *Nd-YAG*, with a *532 nm* wavelength and a pulse duration of *7 ns* yielding *1mm* light sheet thickness. The images were acquired by an Image Pro *4M* camera with *2048* x *2048 pixels* resolution employing a Tokina ATX Pro *100mm* lens and *f/4* aperture (f-stop). A set of 2000 snapshots was acquired for each jet condition. The interframe time was set to *1μs* and an acquisition frequency of *7Hz* was employed between images pairs that, due to characteristic frequencies of the flow, did not allow time resolved analysis. The cross-correlation was performed with initial interrogation window size of 32 pixels that was reduced to a final window of 16 pixels, employing 50% overlap.

2.2 Spatial Fourier Decomposition (κ-ω_s Decomposition)

To extract information about the upstream neutral waves in the jet flow we transform the POD spatial functions in the Fourier domain in order to separate the upstream and downstream propagating instabilities. This method was previously employed by Edgington-Mitchell *et al.*[14] for axisymmetrics *A1* and *A2* screeching jets.

The coherent fluctuations associated with the Screech tones in the PIV data were obtained by a snapshot Proper Orthogonal Decomposition (POD). Considering that in screeching jets the first two POD modes represent the coherent structures associated with the aeroacoustic feedback process[16] and that this modal pair represents a *periodic phenomenon* at the angular Screech frequency[17] ($\omega_s=2\pi f_s$), it is possible to obtain the velocity field ($U^c(x,y,t)$) associated with these structures by forming a low order reconstruction:

$$U^c(x,y,t) = \sum_{n=1}^{2} a_n(t)\Phi_n(x,y) \quad (1)$$

where $\Phi_n(x,y)$ is the n^{th} eigenfunction (spatial mode), $a_n(t)$ represents the n^{th} eigenvector (temporal coefficient). If both, a_1 and a_2, and, Φ_1 and Φ_2, are in phase quadrature, representing a convective motion, we can form the complex coefficients $\alpha = a_1 - ia_2 = \hat{a}e^{-i\omega_s t}$ and the eigenfunctions $\varphi = \Phi_1 + i\Phi_2$, allowing the previous equation to be rewriten as:

$$U^c(x,y,t) = \mathcal{R}e(\hat{a}e^{-i\omega_s t}\sum_{\kappa}\hat{q}_k^c(y)e^{i\kappa x}) \quad (2)$$

where $\mathcal{R}e()$ is the real part of the complex number and

$$\hat{q}_k^c(y) = \sum_{x}\varphi(x,y)e^{-ikx} \quad (3)$$

The Equation 3 above is the spatial Fourier transform of the complex POD function φ. Applying this transform, the wavenumbers spectrum associated with the streamwise velocity fluctuations $\hat{u}_k^c$ at the Screech frequency can be obtained. Since the wavenumbers spectrum is known, the Screech instabilities can be separated according to their propagation direction where negative phase speeds require $\kappa<0$, contrary to the positive case where $\kappa\geq0$. As such, the downstream and upstream propagative coherent velocity fluctuations, $u_d^c(x,y,t)$ and $u_u^c(x,y,t)$, respectively, are obtained using the following Equations:

$$U^c(x,y,t) = \mathcal{R}e(\hat{a}e^{-i\omega_s t}\left[\sum_{\kappa<0}\hat{q}_k^c(y)e^{i\kappa x} + \sum_{\kappa\geq0}\hat{q}_k^c(y)e^{i\kappa x}\right]) \quad (4)$$

and:

$$u_u^c = \mathcal{R}e\left(\hat{a}e^{-i\omega_s t}\hat{u}_u^c\right) = \mathcal{R}e\left(\hat{a}e^{-i\omega_s t}\sum_{\kappa<0}\hat{q}_k^c(y)e^{i\kappa x}\right) \quad (5)$$

$$u_d^c = \mathcal{R}e\left(\hat{a}e^{-i\omega_s t}\hat{u}_d^c\right) = \mathcal{R}e\left(\hat{a}e^{-i\omega_s t}\sum_{\kappa\geq0}\hat{q}_k^c(y)e^{i\kappa x}\right) \quad (6)$$

Therefore, this work focuses on the analysis of the upstream coherent propagative waves ($\hat{u}_u^c$) observed experimentally. Then, the signature of these waves are compared to the theoretical streamwise velocity eigenfunction associated to the upstream jet neutral modes which will be briefly presented in the next section.

3. UPSTREAM NEUTRAL JET MODES

The upstream-travelling neutral jet modes were pointed out in the work of Tam & Hu[13]. Considering a simple vortex-sheet model they shown the possible presence of these waves in supersonic flows and noticed that these instabilities have a signature *inside* and *outside* of the jet, enabling upstream information propagation even for supersonic jets. This enables a feedback closure mechanism. It is important to recall that these waves form the pair mode (*m,n*), corresponding to the azimuthal and the radial order of the instability mode, respectively. The radial order *n*, as observed by Towne *et al.*[18], plays a role determining the number of the anti-nodes (lobes) in the pressure distribution. In other words, for each azimuthal wavenumber *m*, there is a countably infinite set of solutions *n=1,2,3,…*. An important feature of these instabilities is that they only exist at narrow frequency ranges. This is their main difference to acoustic waves which exist for all frequencies.

As mentioned, the upstream-travelling jet modes may be evaluated employing a cylindrical vortex-sheet model which considers a supersonic flow expanding in a fluid at rest and these waves are part of the solution of the vortex-sheet dispersion relation first derived by Lessen *et al.*[19]:

$$\mathcal{D}_j(\kappa, \omega, M, T, m) = 0 \qquad (7)$$

with

$$\mathcal{D}_j = \frac{1}{(1-\frac{\kappa M}{\omega})^2} + \frac{1}{T}\frac{I_m\left(\frac{\gamma_i}{2}\right)[\frac{\gamma_0}{2}K_{m-1}\left(\frac{\gamma_0}{2}\right) + mK_m\left(\frac{\gamma_0}{2}\right)]}{K_m\left(\frac{\gamma_0}{2}\right)[\frac{\gamma_i}{2}I_{m-1}\left(\frac{\gamma_i}{2}\right) + mI_m\left(\frac{\gamma_i}{2}\right)]} \qquad (8)$$

where $\gamma_0 = \sqrt{\kappa^2 - \omega^2}$ and $\gamma_i = \sqrt{\kappa^2 - \frac{1}{T}(\omega - M\kappa)^2}$. In the above equations I_m and K_m are the m^{th} order modified Bessel functions of the first and second kind, respectively, and *m* is the azimuthal wavenumber. *M* is the acoustic Mach number (U_j/a_∞) and *T* is the jet temperature ratio (T_j/T_∞). The jet Mach number is a function of the acoustic Mach number and the temperature ratio: $M_j = U_j/a_j = M/\sqrt{T}$. It needs to be stressed that this model has been widely used for stability analysis of subsonic and supersonic jets among which one can cite Michalke[20], Towne *et al.*[18] and Jordan *et al.*[21]. It is important to notice that the dispersion relation depends on the jet Mach number. Subsequently the frequencies and wavenumber of the waves solutions of the Equation 8 are also dependent on *Mj*. In other words, the condition of existence of upstream neutral waves depends on the flow conditions considered and on the frequency. Solving the Equation 8, we obtain the dispersion curve as shown in fig. 1 for $M_j=1.5$, where we present the roots of the dispersion relation in the κ-ω plane. Moreover, for a given κ-ω pair, the corresponding eigenfunctions are obtained for the streamwise velocity component:

$$\hat{u}(r) = \begin{cases} \dfrac{-\kappa I_m(\gamma_i r)}{M\kappa - \omega} & for\ 0 \le \dfrac{r}{D} \le 0.5 \\ \dfrac{\kappa K_m(\gamma_o r) I_m(\frac{\gamma_i}{2})}{\omega K_m(\frac{\gamma_o}{2})} & for\ \dfrac{r}{D} \ge 0.5 \end{cases} \qquad (9)$$

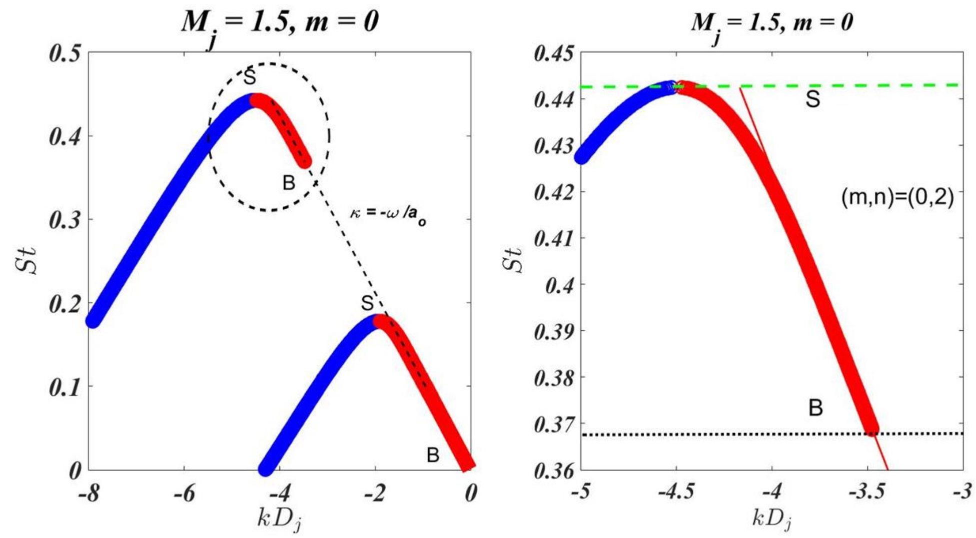

Figure 1: Left: dispersion relations of axisymmetric jet neutral modes for Mj =1.5 and Temperature ratio (T) equals to 1. Dashed line represents the sonic condition (κ=-ω/a_0), S and B represent the saddle and branch points, respectively. Right: allowable frequencies of upstream neutral waves (m,n=0,2). Green dashed line represents the upper frequency limit (saddle point) and black dashed line represents the bottom frequency limit (branch point). Taken from Mancinelli[22] with authorization and modified.

It is possible to notice in fig. 1 that, even for negative phase velocity ($\kappa<0$), the neutral modes are constitued by instabilities with diferent group velocity propagating downstream (κ+) and upstream (κ-). The direction of energy propagation is given by the sign of the slopes of the dispersion curves, i.e. the group velocity. For instance, waves with $\Delta St/\Delta\kappa D_j >0$ are able to propagate energy downstream and those with $\Delta St/\Delta\kappa D_j <0$ propagate energy upstream of the jet. The points of zero slope ($\Delta St/\Delta\kappa D_j=0$) represents saddle points (*S*) where the instabilities κ+ and κ- coalesce[18]. Saddle points are important as they represent cut-on frequencies region in the jet resonance study. In other words, they give the upper limit of the allowable frequencies for which the upstream neutral jet waves κ- can exist. On the other hand, the bottom limit of these waves existence is given by the branch points (*B*), representing the cut-off frequencies for the jet resonance. They give the lower limit of the allowable frequencies for which the upstream neutral jet waves can exist. We consider here the fully expanded jet diameter D_j to get the nondimensional wavenumber κD_j.

Thus, the focus of the following work is to investigate experimentally the presence of these upstream neutral instabilities in the jet for various *NPR* in order to evaluate their possible involvement in the Screech closure mechanism.

4. RESULTS

4.1 Axisymmetric Screech Mode (A2)

The spatial Fourier transform (Equation 3) is applied to the composed function $\varphi = \Phi_1 + i\Phi_2$. Then, the spatial wavenumbers associated to the streamwise component of velocity ($\hat{u}_k^c$) are obtained. In the fig. 2 we plot the amplitude of $\hat{u}_k^c$ as a function of the spatial wavenumber kD_j where the sign of kD_j determines the sign of the phase velocity, indicating the possible direction of the propagation. In other words, the sign of kD_j may indicate the waves travelling direction. From this figure it is possible to notice a remarkable energy for positive phase speed (κ>0), at a velocity lower than the speed of sound (κD_j *=4.1)* and associated to propagation of Kelvin-Helmholtz instabilities. There is also a noticeable energy in the negative *k* region, maybe associated to upstream propagation (dashed ellipse in figure). Note that this propagation occurs at a phase velocity near to the sound velocity (vertical dashed line, at κD_j *=-4.1*). Since the energy is located outside and inside of the jet, this suggests the presence of the upstream jet neutral instabilities (*k-)*.

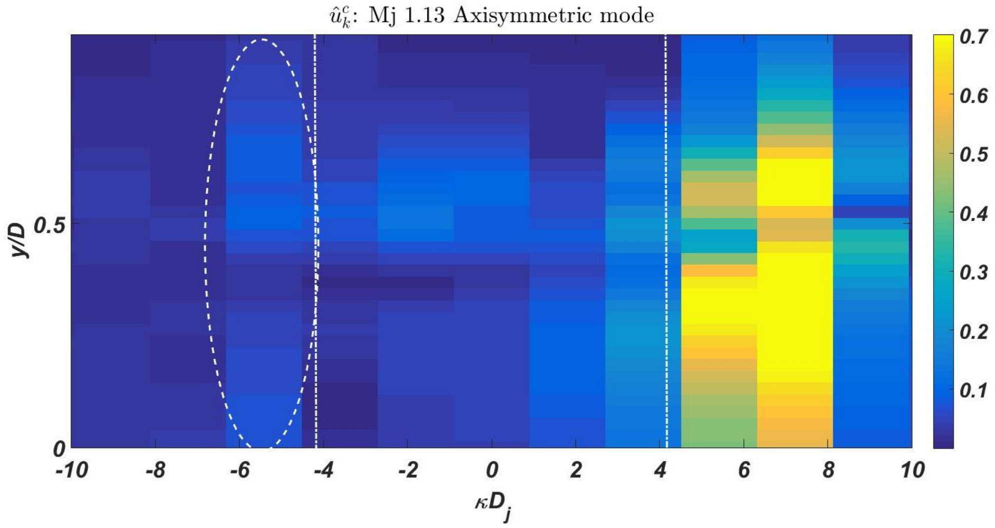

Figure 2: wavenumber spectrum of the coherent streamwise velocity ($\hat{u}_k^c$). Vertical dashed lines represent the wavenumbers associated to the speed of sound at the Screech frequency in the upstream (negative values) and downstream (positive values) directions.

As the wavenumber spectrum is known, the upstream coherent waves $\hat{u}_u^c$ can be obtained. In order to reconstruct these waves, one discrete value of kD_j=-5.45 is considered before Fourier inversing. This point was chosen because it is the closest availabe to the wavenumber of the sound waves kD_j*=-4.1,* where we expect, as showed in fig. 2, the jet modes to lie in. The upstream-travelling instability, maybe associated to the wavenumber experimentally evaluated, is depicted in fig. 3. The results show that the upstream instability waves present a strong energy level in the jet axis (*r/D=0)*, decreasing and increasing in the radial direction before reaching the mixing layer (*r/D=0.5).*

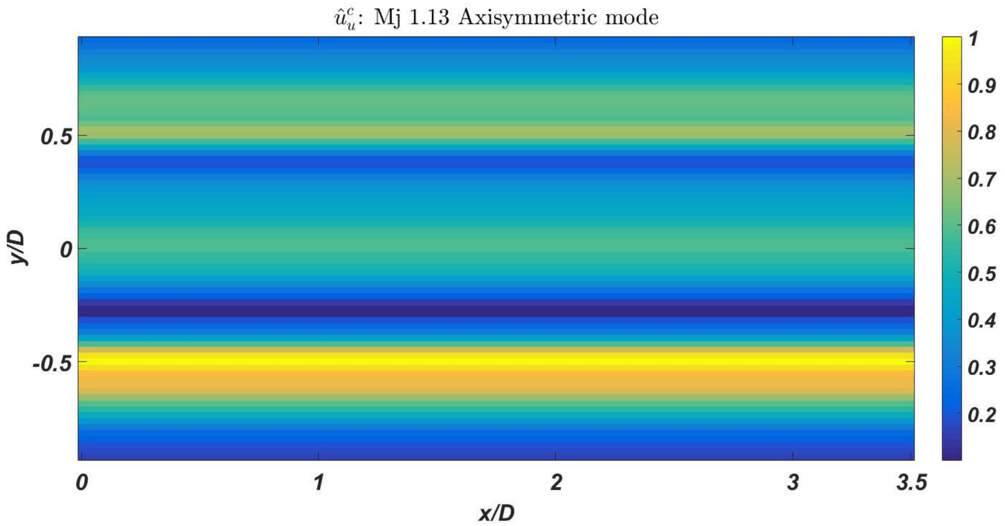

Figure 3: amplitude of the upstream-travelling waves component associated to the negative wavenumber kD_j=-5.45, normalized by the overall maximum value.

In order to validate the results obtained, a comparison between the experimental data and the model was carried out, where the upstream-travelling wave ($\hat{u}_u^c$) profile is compared to the vortex-sheet eigenfunction, provided from Equation 9. As such, we chose the eigenvalue κ at the Screech frequency *St=0.65* in the dispersion relation corresponding to an upstream-travelling neutral waves of the family (*m=0,n=2*), as can be seen in fig. 4. The experimental profile of the upstream instability at the *x/D=1.0* is compared to the theoretical one, in fig. 5.

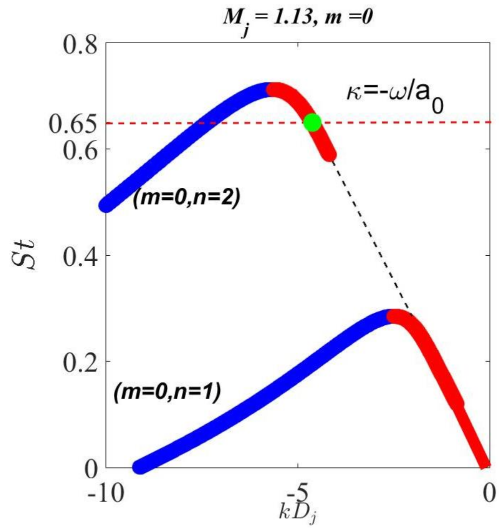

Figure 4: solutions of the cylindrical vortex-sheet dispersion relation. Chosen point (green) in the family of waves (m=0,n=2).

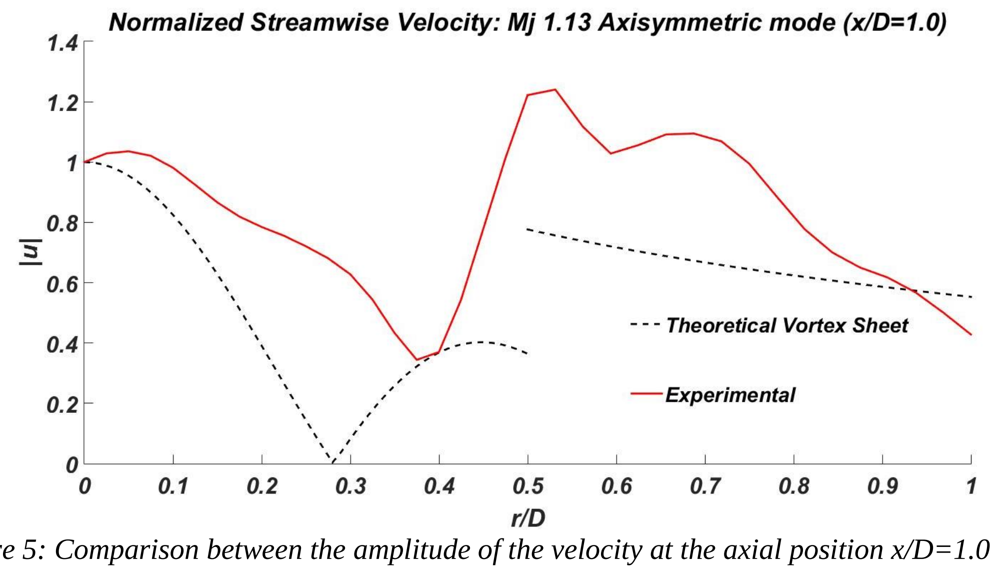

Figure 5: Comparison between the amplitude of the velocity at the axial position x/D=1.0 of the experimental upstream-travelling waves and the theoretical vortex-sheet eigenfunction for (m=0, n=2). Mj=1.13, the velocities are normalized by their value at the jet axis (r/D=0).

The experimental result presents a reasonable agreement with the theoretical model, suggesting the presence of upstream travelling neutral waves propagating inside and outside of the flow. Thus, this result corroborates with the previous work[14], indicating that the upstream-travelling jet neutral instabilities (*m=0,n=2*) may play an important role in the Screech closure mechanism for the axisymmetrics modes.

4.2 Helical Screech Mode (C)

Employing the spatial Fourier transform of the complex function φ formed with the two first POD eigenfunctions $\varphi = \Phi_1 + i\Phi_2$, we obtain the wavenumber spectra of the streamwise velocity components ($\hat{u}_k^c$). The amplitude of $\hat{u}_k^c$ is presented in fig. 6. From this figure it is possible to remark a strong instability, represented by high energy levels, propagating downstream (κ>0) with a phase velocity lower than the speed of sound ($\kappa D_j = 2.5$) and again associated to Kelvin-Helmholtz instabilities. Similarly to the axisymmetric case, one can observe a noticeable energy in the negative *k* region, as shown by the dashed ellipse in the plot. These waves are propagating with a negative phase velocity and they have a velocity almost equal to that of the speed of sound. Moreover, we can see that they have a support inside and outside of the jet core. This suggests the presence of the upstream neutral jet modes (*k-)* in the velocity fluctuations.

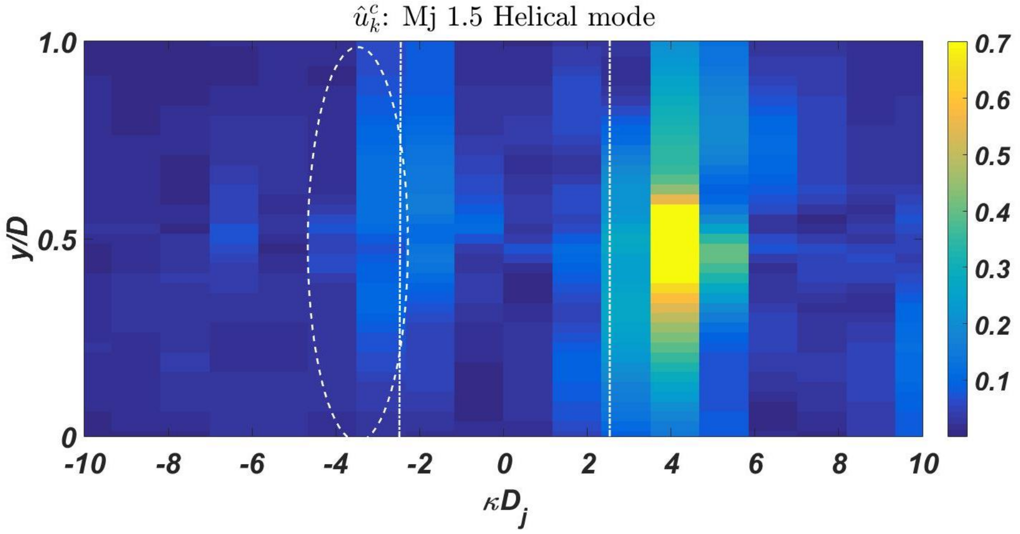

Figure 6: wavenumber spectrum of the coherent streamwise velocity ($\hat{u}_k^c$). Vertical dashed lines represent the wavenumbers associated to the speed of sound at the Screech frequency in the upstream (negative values) and downstream (positive values) directions.

Again, as the wavenumbers are known, the upstream coherent waves $\hat{u}_u^c$ can be obtained. For the upstream travelling waves reconstruction, we consider one discrete value of kD_j=-2.92 to inverse the Fourier transform of $\hat{u}_k^c$. As for the axisymmetric case, the points were chosen due to fact that they are the closest to the sound waves wavenumber $kD_j = -2.5$. The upstream-travelling waves are depicted in fig. 7. The topology of the image reveals meaningful information about these waves: 1) it is possible to remark a discontinuity in the mixing layer ($r/D = \pm 0.5$); 2) there is a high intensity lobe located between the jet axis and the mixing layer and 3) outside of the mixing layer ($r/D > \pm 0.5$) it is possible to notice that, likewise in the axisymmetric case, these instabilities present a remarkable energy that decay moving away from the jet boundary.

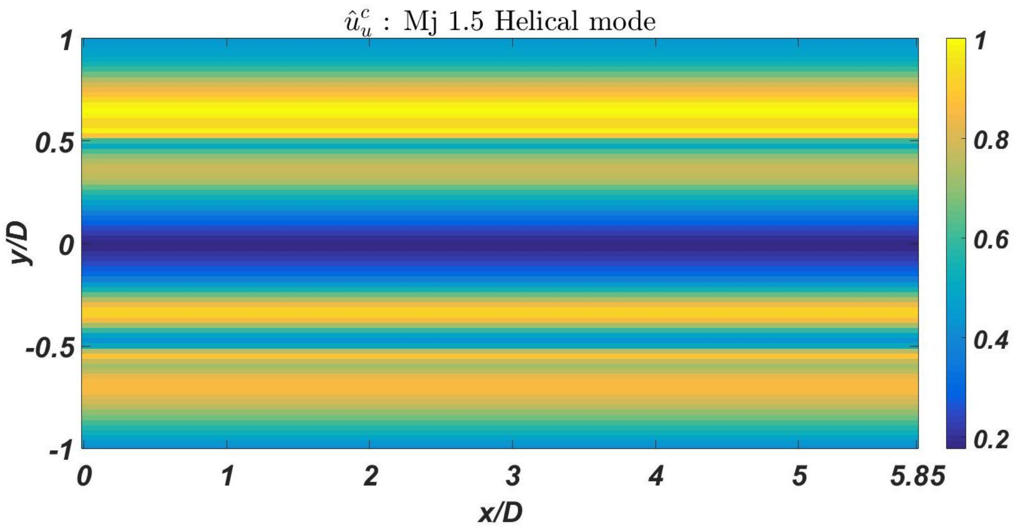

Figure 7: amplitude of the upstream-travelling waves component associated to the negative wavenumber kD_j=-2.92, normalized by the overall maximum value.

A comparison between the experimental data and the model is carried out. As such, an eigenvalue κ (ω) is chosen close to the saddle point, representing an upper limit of the allowable frequencies for the upstream travelling waves of family (*m=1,n=1*), as one can see in the fig. 8. Moreover, this point is very close of the Screech frequency (*St=0.32*). In this figure, the chosen point (κ (ω)) is represented by the green ellipse. Then, the experimental profile of the upstream instability ($\hat{u}_u^c$), at *x/D=5.0*, is compared to the theoretical one, as one can observe in fig. 9.

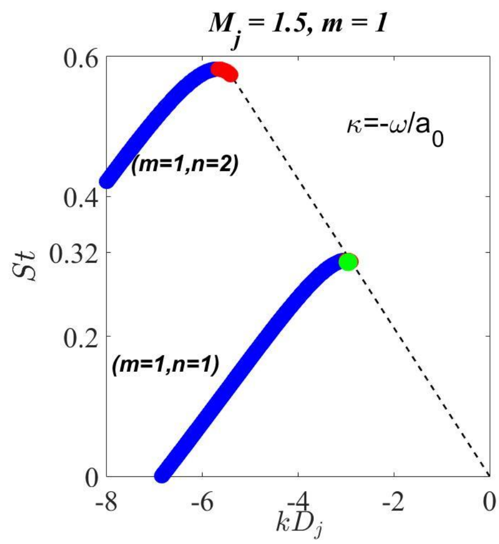

Figure 8: solutions of the cylindrical vortex-sheet dispersion relation. Chosen point (green) in the family of waves of κ (m=1,n=1).

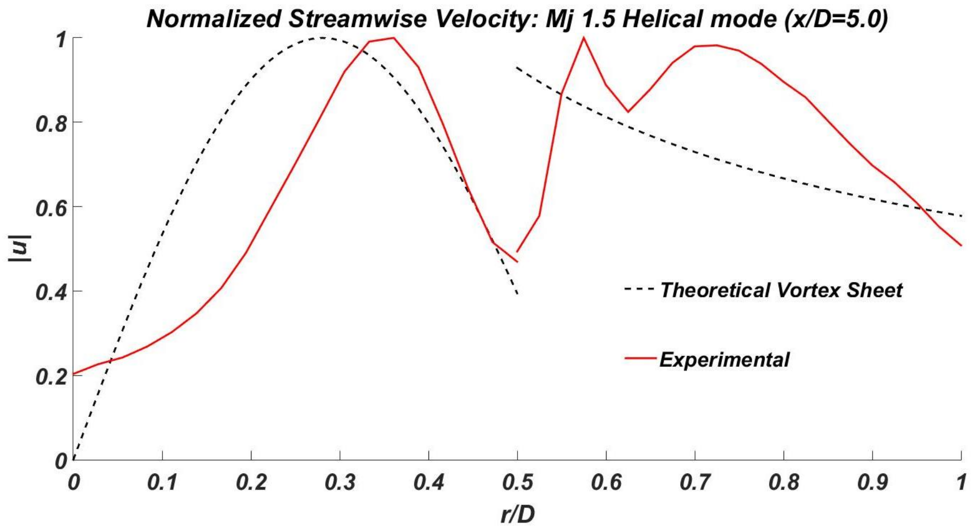

Figure 9: Comparison between the amplitude of the velocity at the axial position x/D=5.0 of the experimental upstream-travelling waves and the theoretical vortex-sheet eigenfunction for k(m=1,n=1), M_j=1.5. The velocities are normalized by the maximum value inside of the jet.

In this figure, one can notice a reasonable agreement between the experimental and theoretical velocities, suggesting the existence of helical neutral jet modes propagating upstream in the jet flow. This observation agrees with numerical results of Gojon et al.[15], indicating that upstream-travelling jet neutral instability may be involved in the closure of the Screech mechanism for the helical *C* mode.

4. CONCLUSIONS

In this work the signatures of the upstream-travelling waves were observed in the jets undergoing axisymmetric (*A2*) and the helical (*C*) Screech modes.The radial support of these waves were compared to theoretical eigenfunctions predicted by the vortex-sheet dispersion relation and a reasonable agreement was found for both cases, suggesting that maybe the upstream jet neutral instabilities may play a role as a Screech closure mechanism, agreeing with recent works of Edgington-Mitchell *et al*. and Gojon *et al.* Therefore, these results may provide a contribution to understand the staging phenomenon present in screeching jets which is still an open question, where the physical mechanisms that rule this behaviour need further study.

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6. REFERENCES

1. Powell, A., "*On the mechanism of choked jet noise*," Proceedings of the Physical Society, Section B 66, 12, 1953, pp 1039–1056.
2. Powell, A., "*On edge tones and associated phenomena*," Acustica, Vol. 3, 1953, pp. 233–243.

3. Hay, J. A., and Rose, E. G., “*In flight shock cell noise*,” Journal of Sound and Vibration, Vol 11, 1970, pp 411–420.
4. Seiner, J. M., Manning, J. C., and Ponton, M. K., “*Model and full scale study of twin supersonic plume resonance,*” 25th AIAA Aerospace Sciences Meeting, 1987.
5. Raman, G., “*Supersonic jet screech: Half-century from Powell to the present*,” Journal of Sound and Vibration 225(3), 1999, pp 543-571.
6. Tam, C.K.W., “*Supersonic Jet Noise*”, Annual Review of Fluid Mechanics, vol. 27, 1995, pp. 17-43.
7. Kandula, M., “*Shock-Refracted Acoustic Wave Model for Screech Amplitude in Supersonic Jets,*” AIAA Journal, Vol. 46, No. 3, 2008, pp. 682-689.
8. Powell, A., Umeda, Y., and Ishii, R., “*Observations of the oscillation modes of choked circular jets,*” The Journal of the Acoustical Society of America, vol 92, 5, 1992, pp 2823–2836.
9. Davies, M. G., and Oldfield, D. E. S., “*Tones from a choked axisymmetric jet. I. Cell structure, eddy velocity and source locations*,” Acustica 12 (4), 1962, pp 257–266.
10. Assunção, T. L., “*Experimental Study of Underexpanded Round Jets: Nozzle Lip Thickness Effects and Screech Closure Mechanisms Investigation*”. PhD Thesis, Université de Poiters, 2018.
11. Shen, H., and Tam, C., “*Three-dimensional numerical simulation of the jet screech phenomenon,*” AIAA Journal, Vol 40, 2002, pp 33–41.
12. Tam, C. K. W., Seiner, J. M., and Yu, J. C., “*Proposed relationship between broadband shock associated noise and screech tones,*” Journal of Sound and Vibration, Vol 110, issue 2, 1986, pp 309–321.
13. Tam, C. K. W., and Hu, F., “*On the three families of instability waves of high-speed jets,*” J. Fluid Mech, Vol 201, 1989, pp 447–483.
14. Edgington-Mitchell, D., Jaunet, V., Jordan, P., Towne, A., Soria, J., and Honnery, D. R., “*Upstream-travelling acoustic jet modes as a closure mechanism for screech*,” Journal of Fluid Mechanics, Vol. 855,R1, 2018.
15. Gojon, R., Bogey, C., and Mihaescu, M., “*Oscillation Modes in Screeching Jets*,” AIAA Journal, Vol. 56, No. 7, 2018, pp. 2918-2924.
16. Edgington-Mitchell, D., Honnery, D. R., and Soria, J., “*The underexpanded jet mach disk and its associated shear layer,*” Physics of Fluids, 26, 2014.
17. Jaunet, V., Collin, E., and Delville, J., “*POD-Galerkin advection model* for *convective flow*: *application* to a *flapping rectangular supersonic jet*,” Exp Fluids, vol. 57, no 5, 2016, p. 1-13.
18. Towne, A., Cavalieri, A. V. G., Jordan, P., Colonius, T., Schmidt, O., Jaunet, V., and Brès, G., “*Acoustic resonance in the potential core of subsonic jets*,” Journal of Fluid Mechanics, Vol 825, 2017, pp. 1113-1152.
19. Lessen, M., Fox, J., and Zien, H., “*The Instability of Inviscid Jets and Wakes in Compressible Fluid*,” Journal of Fluid Mechanics, Vol. 21, No. 1, 1965, pp. 129–143.
20. Michalke, A., “*A note on the spatial jet-instability of the compressible cylindrical vortex sheet*”. 1970.
21. Jordan, P., Jaunet, V., Towne, A., Cavalieri, A. V. G., Colonius, T., Schmidt, O., and Agarwal, A., “*Jet-flap interaction tones*”. Journal of Fluid Mechanics, Vol 853, pp 333-358, 2018.
22. Mancinelli, M., “*Etude de l’aérodynamique et de la stabilité de deux jets supersoniques*”, Post Doc project, CNES, internal report, 2018.