

Indirect measurement of liner impedance under grazing flow using the two-port eduction method

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ABSTRACT

Acoustic wall treatments, such as liners, are commonly used to attenuate the noise propagating through duct systems. Such materials are typically characterized by their wall impedance, which depends on the sound pressure level and the flow conditions. Indirect measurement methods, relying on non-intrusive measurements and a model, are generally preferred to characterize this material parameter under representative conditions. The two-port impedance eduction method is one of these measurement techniques. It relies on acoustic pressure measurements to obtain a two-port model of a lined duct segment, which is then compared to a semi-analytical model for the acoustic transfer matrix to compute the liner impedance. Recently, an extension of this method has been proposed, which uses an iterative optimization method to reduce parameter uncertainties during both stages of the procedure. In this paper, this improved formulation of the two-port impedance eduction method is applied to a set of honeycomb liners under different grazing flow conditions to assess the accuracy, reliability and repeatability of the educed impedance.

Keywords: acoustic liners, impedance eduction

I-INCE Classification of Subject Number: 37

1. INTRODUCTION

As flow ducts are often a major transmission path for noise, their walls are often covered with acoustic wall treatments, such as honeycomb liners. The acoustic behavior of these wall treatments is typically characterized by their acoustic impedance Z . This is a frequency domain property defined as the ratio of the acoustic pressure to the normal velocity at the lined wall, which also depends on the mean flow velocity grazing over the liner facing sheet. Unfortunately, standardized measurement techniques [1] for the liner impedance rely on a simplified configuration with normal acoustic incidence and a quiescent medium, which is not representative for the real-life operating conditions. The

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in-situ method proposed by Dean [2] can be applied in realistic configurations, but it requires the cumbersome installation of microphones inside the liner material and it can only provide the local impedance in a point on the liner surface.

To overcome these drawbacks, a wide variety of indirect measurement methods have been developed. These impedance eduction techniques determine the liner impedance by comparing non-intrusive measurements on a realistic configuration and under representative conditions, to a model. One of these techniques is the two-port impedance eduction method [3-5], which is a robust method to obtain a first estimation of the impedance of a liner under grazing flow. It is based on the comparison between the measured two-port characteristics of a lined duct segment and a semi-analytical model, and relies on an iterative plane wave decomposition method to suppress the influence of measurement errors and uncertainties on the relevant physical parameters. This paper presents the results of the two-port impedance eduction method for a set of honeycomb liners with different perforated facing sheets.

This paper is structured as follows. Section 2 first briefly introduces the two-port impedance eduction method, followed by a description of the liner samples and the measurement approach in section 3. Thereafter, section 4 presents and discusses the results. Finally, section 5 concludes this paper with a summary of the main results and conclusions.

2. TWO-PORT IMPEDANCE EDUCTION METHOD

2.1 Two-port characterization of a duct component

A two-port model considers a duct component, mounted between an inlet duct and an outlet duct with rigid walls. For angular frequencies ω below the cut-off frequency of the first transversal mode of the duct, all high-order modes are evanescent and plane wave propagation can be assumed in these ducts. The two-port model then describes the flow-acoustic behavior of the component using a matrix relation between two sets of state variables, which characterize the acoustic field in these ducts.

Assuming plane wave propagation, the acoustic pressure and axial velocity at an axial position z in a duct can be expressed in frequency domain as:

$$p(\omega, z) = p^+(\omega) \exp(-jk^+z) + p^-(\omega) \exp(jk^-z) \quad (1)$$

$$u(\omega, z) = \frac{1}{\rho_0 c_0} (p^+(\omega) \exp(-jk^+z) - p^-(\omega) \exp(jk^-z)) \quad (2)$$

In these equations, ρ_0 is the fluid density, c_0 is the speed of sound, p^+ and p^- are the complex amplitudes of, respectively, the downstream and upstream propagating pressure wave and k^+ and k^- are the corresponding axial wavenumbers. In this paper, the wavenumbers in the ducts are modelled following reference [6], which accounts for the convective effects and the visco-thermal losses in the acoustic boundary layer:

$$k^\pm = k_0 \left(\frac{1}{1 \pm M_0} + (1 - j) \sqrt{\frac{\nu}{2\omega R^2}} \frac{1 + (\gamma - 1)/\text{Pr}}{(1 \pm M_0)^{3/2}} \right) \quad (3)$$

In this expression, $k_0 = \omega/c_0$ is the acoustic wavenumber, M_0 is the mean flow Mach number, $R = 2WH/(W + H)$ is the hydraulic radius of the rectangular duct with width W and height H , ν is the kinematic viscosity, γ is the ratio of specific heat constants and Pr is the Prandtl number.

The complex wave amplitudes p^+ and p^- , which determine the acoustic pressure and velocity field in a duct, can be computed from measured pressure spectra using the multiple microphone method [7]. This method requires microphone measurements at two or more axial positions z_i in the duct and solves the system of equations resulting from Equation (1):

$$\begin{bmatrix} p^+(\omega) \\ p^-(\omega) \end{bmatrix} = \begin{bmatrix} \exp(-jk^+z_1) & \exp(jk^-z_1) \\ \exp(-jk^+z_2) & \exp(jk^-z_2) \\ \vdots & \vdots \end{bmatrix}^{-1} \begin{bmatrix} p(\omega, z_1) \\ p(\omega, z_2) \\ \vdots \end{bmatrix} \quad (4)$$

If more than two microphone positions are used, the matrix inversion should be interpreted as a Moore-Penrose pseudo-inverse. Equation (4) then computes a least-squares solution of the overdetermined system of equations, which reduces the influence of noise on the measured pressure spectra.

The impedance eduction method relies on the transfer matrix formulation of the two-port model, which describes a linear matrix relation between the acoustic pressure p and axial velocity u at the inlet (\blacksquare_i) and outlet (\blacksquare_o) of the duct component:

$$\begin{bmatrix} p_o(\omega) \\ u_o(\omega) \end{bmatrix} = \begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} \begin{bmatrix} p_i(\omega) \\ u_i(\omega) \end{bmatrix} \quad (5)$$

For any measurement, the acoustic pressure and axial velocity at the inlet and outlet of the duct component can be computed using the expressions for the acoustic field (Equations (1) and (2)) and the result of the multiple microphone method (Equation (4)). The transfer matrix of a duct component can be determined by inverting Equation (5) using the acoustic pressure and axial velocity for two or more measurements, denoted by superscript $\blacksquare^{(1)}$, $\blacksquare^{(2)}$, etc.:

$$\begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} = \begin{bmatrix} p_o^{(1)}(\omega) & p_o^{(2)}(\omega) & \cdots \\ u_o^{(1)}(\omega) & u_o^{(2)}(\omega) & \cdots \end{bmatrix} \begin{bmatrix} p_i^{(1)}(\omega) & p_i^{(2)}(\omega) & \cdots \\ u_i^{(1)}(\omega) & u_i^{(2)}(\omega) & \cdots \end{bmatrix}^+ \quad (6)$$

These independent measurements can be carried out by varying the location of the excitation (multiple source method), the impedance of the outlet (multiple load method) or a combination of both [8].

In an experimental setting, Equations (4) and (6) are reformulated in terms of transfer functions between the microphone signals and a clean reference signal, such as the excitation signal sent to a loudspeaker [9]. These transfer functions are obtained from crosspower and autopower spectra, which allows using spectral averaging techniques. This suppresses the influence of uncorrelated noise on the microphone signals, including the contribution of aerodynamic pressure fluctuations

2.2 Two-port model of the lined duct segment

The two-port impedance eduction method considers a rectangular duct of width W and height H , where a liner of length L covers one wall of the duct over the full width of the cross-section. The impedance eduction method compares the measured acoustic transfer matrix of this lined duct segment to a semi-analytical model. As shown schematically in Figure 1, this model comprises a transfer matrix $[T_L]$, representing the lined segment, and two transition matrices $[T_{tr}]$, accounting for the hard wall – soft wall transition at the liner leading and trailing edge:

$$[T] = [T_{tr}]^{-1}[T_L][T_{tr}] \quad (7)$$

The matrix $[T_L]$ is the analytical two-port model for a finite segment of an infinite lined duct. The coefficients of this transfer matrix were derived in reference [3], assuming that the acoustic field is dominated by the least attenuated mode:

$$\begin{aligned} T_{L,11} &= \frac{Z^+ \exp(-jk_z^+ L) + Z^- \exp(jk_z^- L)}{Z^+ + Z^-} \\ T_{L,12} &= \frac{Z^+ Z^- (\exp(-jk_z^+ L) - \exp(jk_z^- L))}{Z^+ + Z^-} \\ T_{L,21} &= \frac{\exp(-jk_z^+ L) - \exp(jk_z^- L)}{Z^+ + Z^-} \\ T_{L,22} &= \frac{Z^- \exp(-jk_z^+ L) + Z^+ \exp(jk_z^- L)}{Z^+ + Z^-} \end{aligned} \quad (8)$$

Where Z^\pm is introduced as shorthand notation for:

$$Z^\pm = \frac{k_0}{\rho_0 c_0 (k_0 \mp M k_z^\pm)} \quad (9)$$

The transfer matrix coefficients depend only on k_z^+ and k_z^- , the axial wavenumbers of the least attenuated mode in the lined duct. These wavenumbers are determined by the liner impedance Z through a dispersion equation, obtained from the Ingard-Myers boundary condition [10, 11]:

$$Z = -j \frac{k_0}{k_x^\pm} (k_0 - M k_z^\pm)^2 \cot(k_x^\pm H) \quad (10)$$

This equation also involves the transversal wavenumbers k_x^\pm in the direction perpendicular to the liner, which can be derived from the corresponding axial wavenumbers using the compatibility relations:

$$k_x^\pm = \sqrt{(k_0 \mp M k_z^\pm)^2 - (k_z^\pm)^2} \quad (11)$$

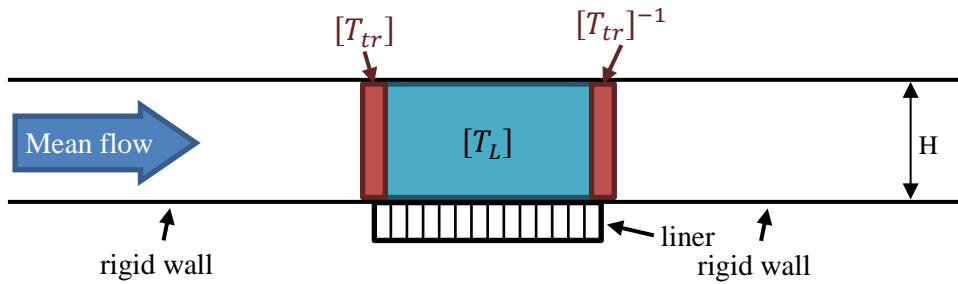


Figure 1 – Schematic overview of the semi-analytical model

The transition matrices $[T_{tr}]$ in equation (7) represent the hard wall – soft wall transition between the liner and the rigid inlet or outlet duct. The two-port impedance reduction method assumes that this transition is symmetric. Although this is questionable in the presence of a mean flow through the duct, it is a reasonable assumption for the low Mach numbers considered in this paper.

2.3 Iterative plane wave decomposition

As illustrated by Equations (3) – (4) and Equations (8) – (10), both the semi-analytical model and the multiple microphone method depend on the speed of sound and the mean

flow Mach number. Any errors or uncertainties on these parameters will therefore have a direct impact on the educed impedance. To suppress this influence, reference [5] proposed to replace the multiple microphone method by an iterative plane wave decomposition technique [12, 13], which fully exploits the available measurement data by optimizing the parameters during the computing the complex wave amplitudes.

The iterative plane wave decomposition method is based on Equation (1). For a measurement with N_m microphones and N_ω frequency lines, this expression yields a system of $N_m \times N_\omega$ equations. This overdetermined system of equations needs to be solved for the $2N_\omega$ complex wave amplitudes, the speed of sound and the mean flow Mach number. This can be done efficiently using an iterative approach. First, an estimate of the complex wave amplitudes is computed for each frequency using the conventional multiple microphone (Equation (4)). The best available estimate for the speed of sound and the mean flow Mach number, typically based on the measured temperature and flow rate, is used in this computation. A second step computes a correction for these parameters by treating the wave amplitude estimates as known constants and by solving one Gauss-Newton iteration for the large overdetermined system of equations, obtained from Equation (1) for all microphones and frequency lines. This correction is then applied to the speed of sound and the mean flow Mach number, where relaxation factors can be used to speed up convergence. The updated parameters are then used to compute a new estimate of the complex wave amplitudes with the multiple microphone method, and this result is used to obtain a new correction for the parameters. These steps are repeated alternately until a convergence criterion is met. For the measurements discussed in this paper, convergence is reached in less than 2000 iterations, which requires no more than a minute of computational time on a standard laptop.

2.4 Impedance eduction method

The two-port impedance eduction method computes the liner impedance by minimizing the difference between the measured transfer matrix and the semi-analytical model, summarized in section 2.2. A robust cost function for this optimization was proposed in reference [5]. Using some algebraic manipulations, it can be shown the trace of the semi-analytical model for the acoustic transfer matrix, given by Equation (7), doesn't depend on any of the transition matrix coefficients:

$$\text{Trace}([T]) = \text{Trace}([T_{tr}]^{-1}[T_L][T_{tr}]) = \exp(-jk_z^+L) + \exp(jk_z^-L) \quad (12)$$

Together with the dispersion equation (Equation (10)) and the compatibility relations (Equation (11)), this forms a system of five equations with five unknowns: the axial wavenumbers k_z^+ and k_z^- , the corresponding transversal wavenumbers k_x^+ and k_x^- and the sought liner impedance. The final step of the two-port impedance eduction method therefore consists in solving this determined system of equations.

3. LINER SAMPLES AND EXPERIMENTAL CONFIGURATION

This paper reports the results of the two-port impedance eduction method for liner samples with a length of $L = 200$ mm. All liners have the same honeycomb backing with a depth of $h = 17.5$ mm, and a facing sheet with circular perforations in a staggered pattern. The perforation diameter and distance between consecutive rows of perforations is different for all facing sheets, as detailed in Table 1. To assess the repeatability of the measurement results, two liner samples are manufactured using facing sheet A.

Table 1 – Liner sample parameters

	A	B	C	D
Plate thickness t [mm]	1.0	1.0	1.0	1.0
Perforation diameter d [mm]	1.9	1.7	2.0	1.9
Distance between rows [mm]	5.0	4.0	4.0	3.0
Open area σ [%]	5.67	7.09	9.82	15.75

The impedance of these liners is measured in a medium at rest and under a grazing flow with a velocity of 25 m/s ($M=0.07$) and 50 m/s ($M=0.014$). Only the linear regime of the liner is considered in this study. It is therefore verified for all measurements that the amplitude of the acoustic excitation is kept sufficiently low to ensure a linear behavior of the liner.

The measurements are carried out using the flow-acoustic test facilities of KU Leuven [12]. The liner sample is mounted between ducts with a rectangular cross section of 40 mm by 90 mm. The validity of the two-port model is limited to the cut-off frequency of the first transversal mode in these ducts. In a medium at rest, this implies that the frequency $f < c_0/(2W) \approx 1900$ Hz. The measurement ducts are equipped with four flush mounted microphones (PCB 378C10) and a loudspeaker is mounted at the extremity of each duct. A constant flow through the measurement ducts is generated by a rootsblower and led through a heat exchanger to ensure a constant ambient temperature at the inlet of the measurement section. To achieve higher flow rates, a second rootsblower is added in parallel.

The two-port characterization is based on a combined two source and two load method, using two loudspeaker locations and two termination impedances. This results in four measurements for each liner and each flow condition. All measurements are carried out using a stepped sine excitation from 200 Hz to 1800 Hz with a step of 50 Hz. A Scadas III system and Test.Lab Rev.15 software are used for the data acquisition and the first signal processing. For each frequency, the microphone signals and the excitation signal are recorded for 60 s at a sampling rate of 8192 Hz. The required autopower and crosspower spectra are computed using Welch's method with 60 averages and the transfer function at the excitation frequency is stored for further processing, as discussed in section 2.

4. RESULTS AND DISCUSSION

4.1 Comparison between impedance eduction results and semi-empirical models

The acoustic impedance of the liners can be predicted using semi-empirical models, such as the models of Guess [14] or Elnady [15]. These models express the impedance of a liner as a function of the measurement conditions and the geometrical parameters listed in Table 1. As all measurements have been carried out in the linear regime of the liners, the non-linear terms accounting for the effect of high sound pressure levels are not considered in this paper.

Figure 2 compares the educed impedance of the liners to these semi-empirical models. In a medium at rest, the difference between the models of Guess and Elnady is barely noticeable and only the liner impedance according to Elnady is therefore shown in the graphs. Although some discrepancies can be observed at lower frequencies, a good overall agreement between model and measurements is obtained for all liner samples.

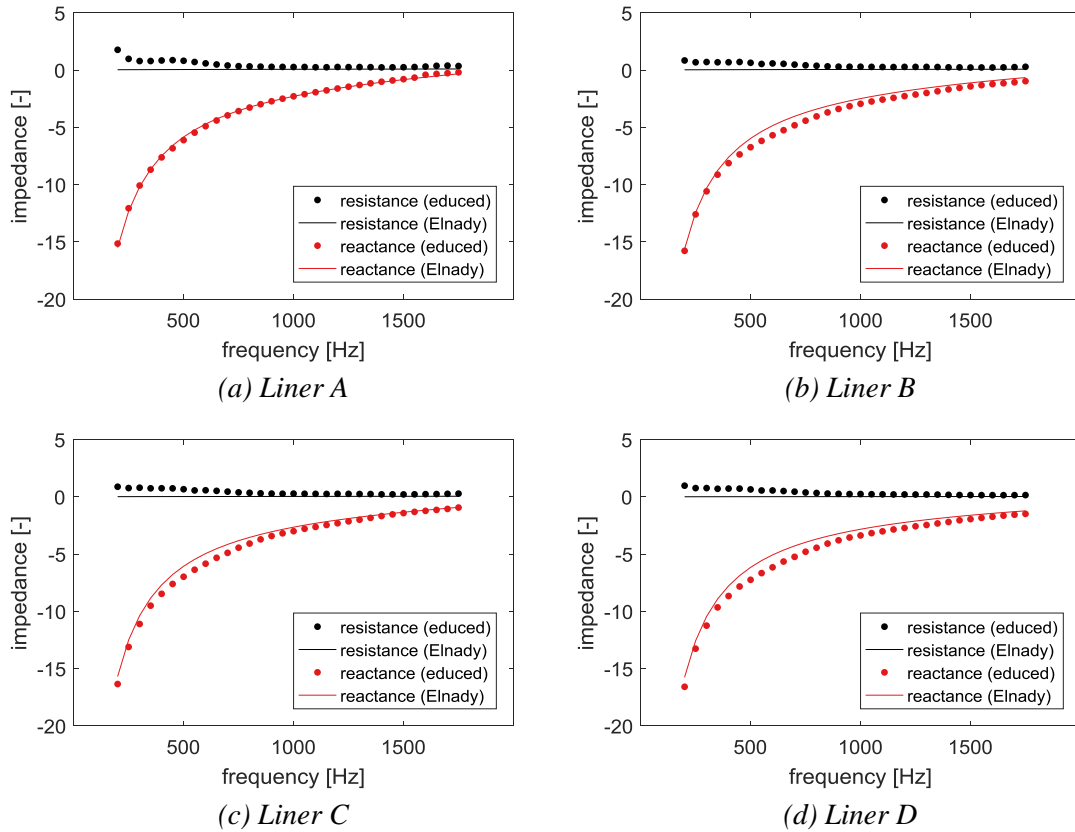


Figure 2: Educated impedance of liners A, B, C and D in a medium at rest (symbols) and liner impedance according to the semi-empirical model of Elnady [15] (lines).

It is well-known that the presence of a grazing flow over the facing sheet influences the impedance of the liner. A higher grazing flow velocity typically results in an increase of the resistance and a lower reactance. This behavior is clearly observed for liner A in Figure 3, which compares the educed impedance in a medium at rest to the results under a grazing flow with a Mach number of $M = 0.07$ and $M = 0.14$. Similar results are obtained for the other liners.

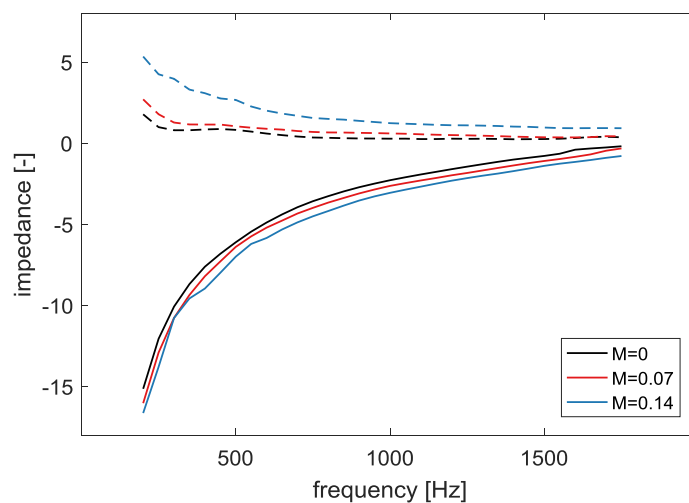


Figure 3: Educated resistance (dashed lines) and reactance (full lines) of liner A in a medium at rest and under a grazing flow with a Mach number of $M=0.07$ and $M=0.14$.

The models of Guess [14] and Elnady [15] account for the effects of the grazing flow using empirical corrections. For the resistance θ , both models use a correction term, which is constant over the entire frequency range and proportional to the mean flow Mach number. However, both models use a different proportionality constant:

$$\theta_{\text{Guess}} = \theta_{\text{Guess}, M=0} + 0.3M (1 - \sigma^2)/\sigma \quad (13)$$

$$\theta_{\text{Elnady}} = \theta_{\text{Elnady}, M=0} + 0.5M/\sigma \quad (14)$$

Figure 4 shows the measured and predicted change of the resistance of the liners due to the presence of a grazing flow. At higher frequencies, the model of Elnady overestimates the increase of the resistance, while the mean flow correction of Guess provides a reasonable estimate of the increase of the resistance. This suggests that the value of 0.5 for the proportionality constant is too high for the liners studied in this paper. This observation agrees with many empirical impedance models available in literature, such as the model of Bauer [16] and a more recent publication using Elnady's model [17], which set the value of the proportionality constant to 0.3.

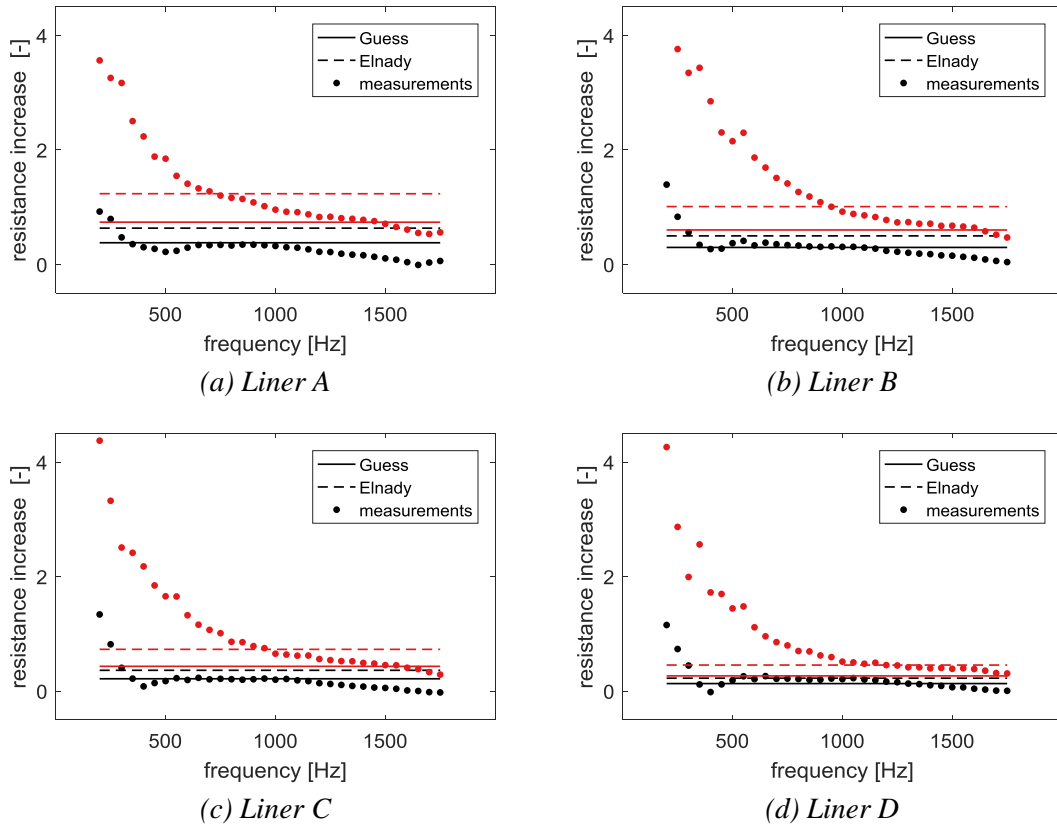


Figure 4: Effect of a grazing flow on the resistance of liners A, B, C and D: difference between the resistance under a grazing flow with a Mach number of $M=0.07$ (black) or $M=0.14$ (red) and the resistance in a medium at rest, obtained from the measurements (symbols) and the models of Guess [14] (full lines) and Elnady [15] (dashed lines).

At lower frequencies, both Guess' and Elnady's models severely underestimate the increase of the resistance. This observation has been reported on several occasions and more complex impedance models, such as reference [18], therefore use a frequency dependent mean flow correction for the liner resistance.

As for the resistance, the model of Elnady uses a frequency independent correction factor, proportional to the grazing flow Mach number, to account for the effect of the grazing flow on the reactance:

$$\chi_{Elnady} = \chi_{M=0} - 0.3M/\sigma \quad (15)$$

The Guess model follows a different approach and models the liner reactance as the sum of a viscous contribution, the radiation reactance and the cavity reactance, where the effect of grazing flow is comprised in the expression for the orifice end correction δ :

$$\chi_{Guess} = \frac{\sqrt{8\nu\omega}}{\sigma c_0} \left(1 + \frac{t}{d}\right) + \frac{k_0}{\sigma} (t + \delta) - \cot(k_0 h) \quad (16)$$

$$\delta = \frac{0.85d(1 - 0.7\sqrt{\sigma})}{1 + 305M^3} \quad (17)$$

Figure 5 shows the measured change of the reactance due to the grazing flow and the empirical corrections of the models of Guess and Elnady. The effect of the differences in the speed of sound, caused by temperature variations between the measurements with and without grazing flow, has been accounted for in all curves. This explains the apparent frequency dependency of constant correction terms in the models. The results clearly show that the model of Guess underestimates the decrease of the reactance over the entire frequency range. The correction term of Elnady on the other hand provides a surprisingly good prediction of the grazing flow effect.

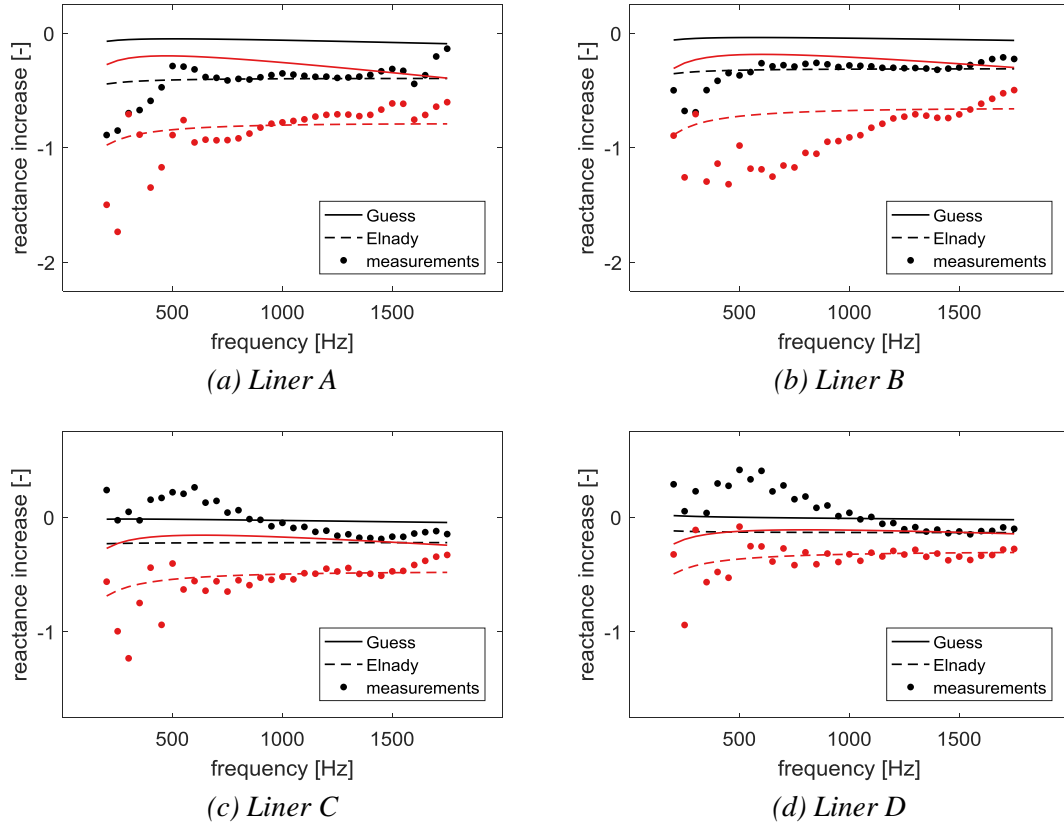


Figure 5: Effect of a grazing flow on the reactance of liners A, B, C and D: difference between the reactance measured under a grazing flow with a Mach number of $M=0.07$ (black) or $M=0.14$ (red) and the reactance measured in a medium at rest (symbols), compared to the models of Guess [14] (full lines) and Elnady [15] (dashed lines).

Considering the reasonable agreement between the educed impedance and the semi-empirical models shown in Figures 3, 4 and 5, it can be concluded that the educed impedance concurs with the literature, providing confidence in the reliability of the results of the two-port impedance eduction method.

4.2 Repeatability of the impedance eduction measurements

To assess the repeatability of the impedance eduction results, two liners with facing sheet A have been manufactured and measured. Additionally, the measurements for one of these liners have been repeated after completely dismounting and remounting the test rig. As expected, the educed impedance for the two identical liners are in excellent agreement and the results of the two independent measurements for the same liner can barely be distinguished in Figure 6. This illustrates that the two-port method is as a robust and reliable technique to measure the acoustic impedance of a locally-reacting liner in the presence of a grazing flow.

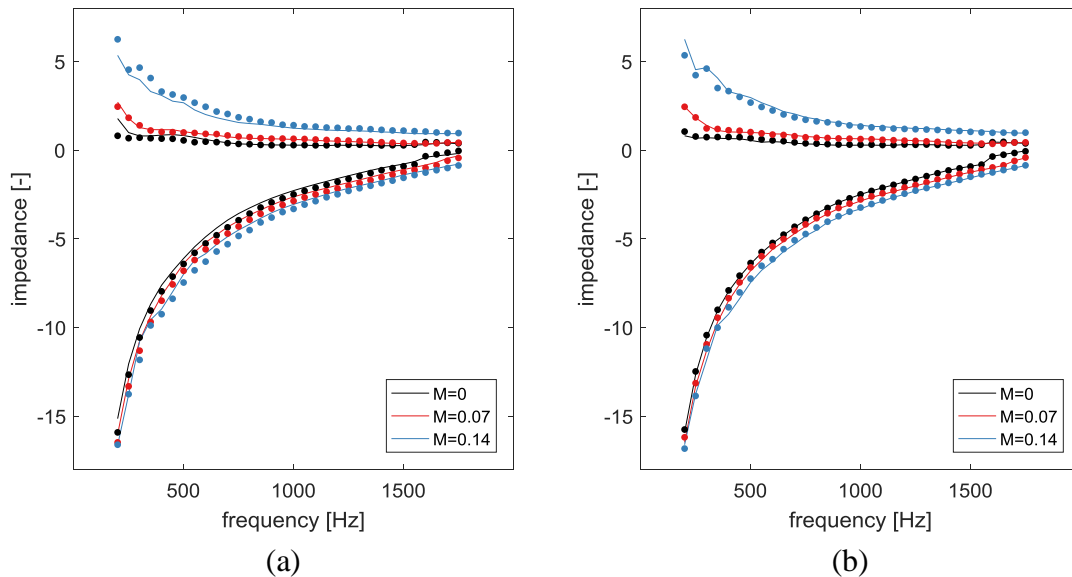


Figure 6: Impedance eduction results at different grazing flow velocities: (a) comparison between two liners with facing sheet A; (b) comparison between two independent measurements for the same liner with facing sheet A.

5. CONCLUSIONS

This paper characterizes the impedance of honeycomb liners with different perforated facing sheets under grazing flow using the two-port impedance eduction method. For all liners, the measured impedance in a medium at rest agrees well with semi-empirical models available in literature. Also the effect of grazing flow, which increases the liner resistance and decreases the reactance, is well captured by the impedance eduction method and corresponds to the empirical correction terms used in the models. Additionally, the excellent agreement between the educed impedance for two identical liner samples and between repeated measurements on a unique sample illustrates the repeatability of the results. Based on these observations, it can be concluded that the two-port method is a robust and reliable method to educe the impedance of a locally reacting liner under a low Mach number grazing flow.

6. ACKNOWLEDGEMENTS

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